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ISLAMIC UNIVERSITY OF  
TECHNOLOGY  
(IUT)



Organisation of Islamic Cooperation

## **Effects on Natural frequency of Simply Supported Square Plate due to concentrated masses and Circular Cutout**

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November,2014

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Mechanical and Chemical Engineering

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An Undergraduate thesis submitted to the department of Mechanical &  
Chemical engineering of Islamic University of Technology, Board Bazar,  
Gazipur in partial fulfillment of the requirements for the degree

OF

**BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING**

## CANDIDATES DECLARATION

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma

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Dedicated  
To  
Our Beloved Parents



## ACKNOWLEDGEMENT

We are grateful to Almighty Allah (Subhanahu-Tala) who made it possible for us to finish the project successfully on time and without any trouble.

Firstly, we would like to express our sincerest appreciation and profound gratitude to our supervisor **DR. MD. ZAHID HOSSAIN**, Professor, Mechanical and Chemical Engineering Department, IUT, for his supervision, encouragement and guidance. It has been privilege for us, working with somebody with such ingenuity, integrity, experience and wittiness.

We would like to thank Md. Shahriar Islam, Lecturer, IUT for his help and insight. We would like to thank the following instructors who provided invaluable collaborative support and made our time at IUT Machine Workshop, exciting, fun and productive.

In particular Md. Matiar Rahman, senior operator, Md. Rajaul Karim, operator (Machine shop). Special thanks to Md Rakibul Hasan (mechanics lab) for his useful suggestions and availability whenever needed.

We would also like to convey gratitude to all other faculty members of the Department for their valuable advice in every stage for successful completion of this project. Their Teaching helped us a lot to start and complete this thesis work.

Of course, any errors are ours alone. We seek excuses if there is any mistake found in this report.

## ABSTRACT

In any operation performed by machine on any structure vibration may occur. This vibration can be devastating when the frequency matches with the natural frequency of that structure. Due to resonance the structure can be failed permanently. So it is always desired to reduce the vibration of the structure or maintain a low level. The natural frequency of a simply supported square plate can be modified by making cutout in the plate or by placing concentrated mass on the plate. In this project, our aim is to devise a way of predicting natural frequency for a simply supported square plate. To do that, we analyze vibration characteristics of a simply supported square plate by using ANSYS MECHANICAL APDL 14.0. Vibration characteristics is investigated by analyzing modal and harmonic analysis for the simply supported square plate. Then the analysis is done due to presence of cutout and concentrated mass. The position of cutout and concentrated masses are also an important factor for the vibration characteristics of the plate. By changing the position of cutout and concentrated masses our investigation is also performed.

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## **CHAPTER 1 : INTRODUCTION**

### **1.1 PLATE VIBRATION:**

Study of vibration of plates is an extremely important area owing to its wide variety of engineering applications such as in aeronautical, civil, and mechanical engineering. Since the members, viz., beams, plates, and shells, form integral parts of structures, it is essential for a design engineer to have a prior knowledge of the first few modes of vibration characteristics before finalizing the design of a given structure. In particular, plates with different shapes, boundary conditions at edges and various complicating effects have often found applications in different structures such as aerospace, machine design, telephone industries, nuclear reactor technology, naval structures, and earthquake-resistant structures. A plate may be defined as a solid body bounded by two parallel plates, flat surfaces having two dimensions far greater than the third. The vibration of plates is an old topic in which a lot of work has already been done in the past decades. In earlier periods, results were computed for simple cases only where the analytical solution could be found. The lack of good computational facilities made it almost impossible to get reasonable accurate results even in these simple cases. This may be the causes for why in spite of a lot of theoretical developments; numerical results were available only for a few cases. By the aid of fast and efficient algorithms complex plate vibration problems can be solved in a very short time and give comparatively accurate results. Methods like finite element methods, boundary integral equation methods, finite difference methods, and the methods of weighted residuals have made handling any shape and any type of boundary conditions possible. This analysis mainly focuses on numerical analysis. In numerical analysis ANSYS (Mechanical APDL 14.0) was used to find the natural frequencies and extract the mode shapes and to observe the response under external loading (Harmonic analysis) with and without spring-damper. By varying the spring and damper it has been tried to find a suitable means of reducing vibration.

### **1.2 IMPORTANCE OF PLATE VIBRATION**

Most human activities involve vibration in one form or other. Most prime have vibration problems due inherent unbalance in the engines. In turbines, vibration cause drastic mechanical failures. Plates are the most commonly used element in mechanical structures and machines such as aircrafts, ships and submarines. In designing a structure, plates are usually specified only to withstand applied static loads. However, this is inadequate for more accurate applications. Dynamic forces and random cyclic loads also threaten the stability of a system. There exist a large number of discrete frequencies at which a rectangular plate will undergo large amplitude vibration by sustained time varying forces of matching frequencies. Thus, the possibility of large displacement and stresses due to this recent type of excitation must be taken into account. By predicting the salient frequencies in which the vibration is maximum for specific plate structure active and appropriate measures, like passive and active methods can be installed adequately in response to the working range of the structure. When resonance occur the deflection of the structure is excessive and it can lead to devastating failures which can incur

excessive cost to repair or replace. So it's of great concern to study the structure beforehand and avoid such havoc. Hence vibration analysis of plate is imperative in today's structure designs.

### 1.3 LITERATURE REVIEW

In the recent past, it has been observed that the research in the field of vibration is unceasingly accruing immense importance in the modern science, due to the significant role in every field of applied sciences. As fundamental structural elements, plates of various geometries are widely used in various engineering fields such as, aerospace technology, missile technology, naval ship design and telephone industry etc. Due to the appropriate variation of plate thickness, these plates provide the advantage of reduction in weight and size, and also have significantly greater efficiency for vibrations as compared to the plate of uniform thickness. Thus the vibration characteristics of plates having variable thickness have attracted the interest of researchers.

Amba-Rao analyzed the vibration of a simply supported rectangular plate carrying a concentrated mass using a Fourier sine transform Amba-Rao [1]. Magrab investigated the vibration of a rectangular plate carrying a concentrated mass with two edges supported and the remaining edges either clamped or free. Several other authors discussed and investigated different methods to analyze the linear and non-linear vibration of plates with different geometry and boundary conditions [3-6]. Several work on the effect of spring mass system in the middle and other positions on a plate have been done by some author ,among them recently Ding Zhou [7] analyzed free vibration of rectangular plate with concentrating on the continuously distributed spring mass system. Some papers studied the free vibration of rectangular plates with elastic/rigid concentrated masses by using different methods such as the exact solution [9-10], the optimal Rayleigh-Ritz method [11] and the mode expansion method [12]. McMillan and Keane investigated the possibility of using attached masses to control the vibration of rectangular plates [13-14].

Incredible amount of research has been carried out on the free vibration of plates under various boundary conditions and various shapes of cutout by different analytical method. Moon Kyu applied the Independent Coordinate Coupling Method (ICCM) for a rectangular plate with a circular cutout and verified the results with finite element method [21]. Paramasivam used the finite difference plate with a rectangular hole by the classical Rayleigh-Ritz method [22]. Takahashi used the classical Rayleigh-Ritz method after deriving the total energy by subtracting the energy of the hole from the energy of the whole plate [8]. Lee and Kim carried out vibration experiments on the rectangular plates with a hole in air and water [18]. Pickett analyzed the vibration behavior of plates with holes [16]. G. Aksu and R. Ali determined the dynamic characteristic of plates with cutouts using Finite Difference Method [17]. Kwak, M. K. & Han, S. B. presented a new method of Independent Coordinate Coupling Method (ICCM) for the free vibration analysis of a rectangular plate with a rectangular or a circular hole [19]. This method utilizes independent coordinates for the global and local domains and the transformation matrix between the local and global coordinates .In the Rayleigh-Ritz method, the effect of the

hole can be considered by the subtraction of the energy for the hole domain in deriving the total energy.

## CHAPTER 2

### BASIC OF VIBRATION

#### 2.1 VIBRATION

Vibration is the mechanical oscillation of a particle, member, or a body from its position of equilibrium. It is the study that relates the motion of physical bodies to the forces acting on them. The basic concepts in the mechanics of vibration are space, time, and mass (or forces). When a body is disturbed from its position, then by the elastic property of the material of the body, it tries to come back to its initial position. In general, we may see and feel that nearly everything vibrates in Nature; vibrations may be sometimes. Very weak for identification. On the other hand, there may be large devastating vibrations that occur because of manmade disasters or natural disasters such as earthquakes, winds, and tsunamis. Natural and human activities always involve vibration in one form or the other. Recently, many investigations have been motivated by the engineering applications of vibration, such as the design of machines, foundations, structures, engines, turbines, and many control systems.

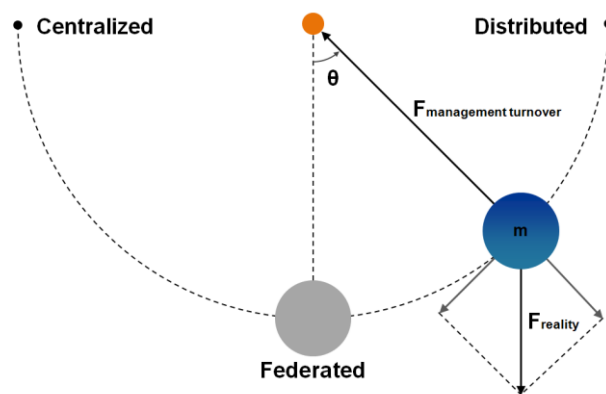


Figure 1:Vibration of a pendulum

#### 2.2 TYPES OF VIBRATION:

Vibration can be classified in several ways. Some of the important classifications are as follows.

##### 2.2.1 FREE VIBRATION AND FORCED VIBRATION

## **FREE VIBRATION:**

After an initial disturbance, if a system is left to vibrate on its own then it is called free vibration. In free vibration no external force is applied or acted on the system. Oscillation of a simple pendulum is an example of free vibration.

## **FORCED VIBRATION:**

If a system is subjected to an external force the resulting vibration is known as forced vibration. The oscillation that arises in machine such as diesel engine is an example.

## **2.2.2 LINEAR AND NON LINEAR VIBRATION**

### **LINEAR VIBRATION:**

If all basic components of a vibratory system the spring, the mass and the damper- behave linearly, the resulting vibration is known as the linear vibration. If the vibration is linear then the principle of superposition holds.

### **NON LINEAR VIBRATION:**

If any of the basic components of vibration behave nonlinearly then the vibration is called nonlinear vibration. For linear vibration the principle of superposition is not valid.

## **2.3 CAUSES OF VIBRATION**

The main causes of vibration are as follows:

- ❖ Unequal distribution of forces in a moving or rotating machinery
- ❖ External forces like wind, tides, blasts, or earthquakes
- ❖ Friction between two bodies
- ❖ Change of magnetic or electric fields
- ❖ Movement of vehicles, etc.

## **2.4 REQUIREMENTS FOR VIBRATION**

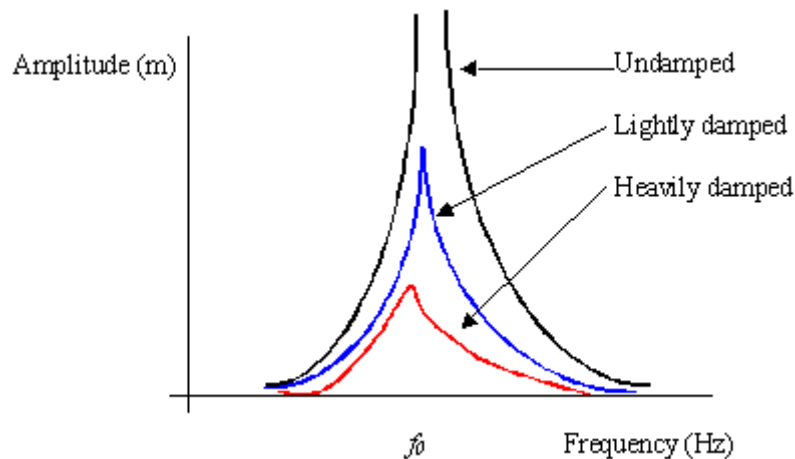
The main requirements for the vibration are as follows:

- ❖ There should be a restoring force.
- ❖ The mean position of the body should be in equilibrium.
- ❖ There must be inertia (i.e., we must have mass).



## 2.5 RESONANCE

A certain system has more than one natural frequency. If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs. When resonance happens, the amplitude of vibration will increase without bound and is governed only by the amount of damping present in the system and the system undergoes dangerously large oscillations. Therefore, in order to avoid disastrous effects resulting from very large amplitude of vibration at resonance the natural frequency of a system must be known and properly taken care of. Otherwise failures of such structures as buildings, bridges, turbines and airplane wings may be occurred.



## 2.6 MODAL ANALYSIS:

Modal analysis is a process of obtaining innate dynamic characteristics of a system in the forms of natural frequencies, damping factors and mode shapes and using them to formulate mathematical model for its dynamics behavior. The mathematical model is referred to as the modal mode of the system and the information for the characteristics is known as the modal data. Modal analysis is based upon the fact that the vibration response of the linear time-invariant dynamic system can be expressed as the linear combination of a set of simple harmonic motions called the natural modes of vibration. The natural modes of vibration are inherent to a dynamic system and are determined completely by its physical properties like mass, stiffness, damping and their spatial distribution. Each mode is described by the modal parameters: natural frequency, damping factor and characteristics displacement pattern, which is mode shape. The mode shape corresponds to a natural frequency.

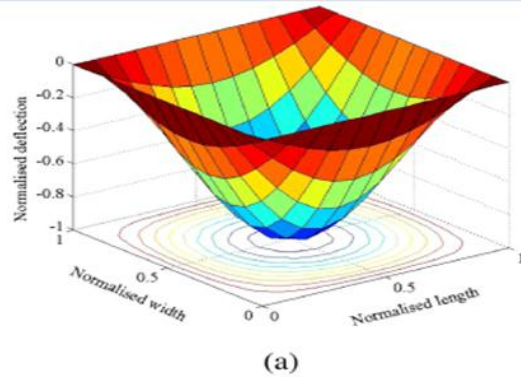
## 2.7 SHAPES OF DIFFERENT MODES:

A normal mode of an oscillating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. The motion described

by the normal modes is called resonance. The frequencies of the normal modes of a system are known as its natural frequencies or resonant frequencies. A physical object, such as a building, bridge or molecule, has a set of normal modes that depend on its structure, materials and boundary conditions.

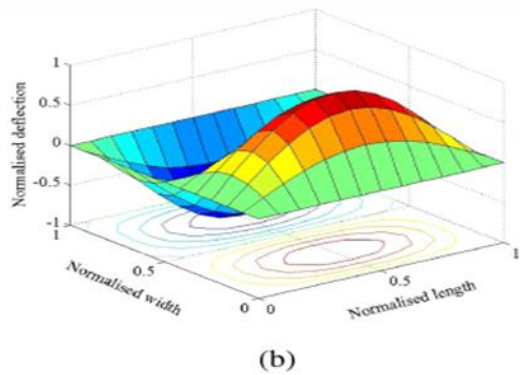
**First Mode:**

In the first mode all of the plates move upwards or in the downward simultaneously.



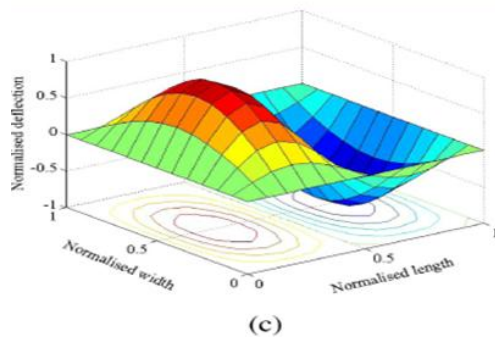
**Second Mode:**

In the second mode half of the plate moves upward and the rest of the plate move downward. This upward and downward alters simultaneously.



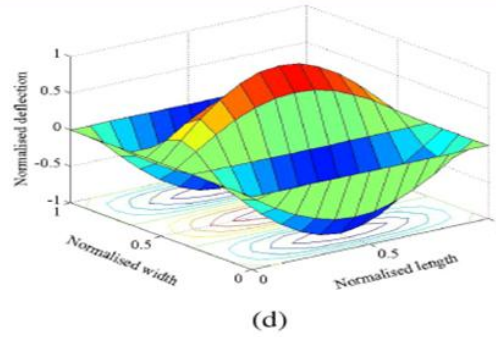
**Third Mode:**

In the third mode half of the plate moves upward and the rest of the plate move downward. This upward and downward alters simultaneously. The difference between 2<sup>nd</sup> mode and 3<sup>rd</sup> mode is: they are perpendicular to each other.



**Fourth Mode:**

In the fourth mode the middle of the plates move upward and the rest two parts move downwards.



**CHAPTER 3:**

**NUMERICAL ANALYSIS OF A RECTANGULAR PLATE WITHOUT SPRING-DAMPER USING ANSYS 14.0.**

**PROBLEM SPECIFICATION:**

- Plate = MILD STEEL
- Length = 60cm
- Width = 60cm
- Thickness = 2mm
- Density = 7850kg/m<sup>3</sup>

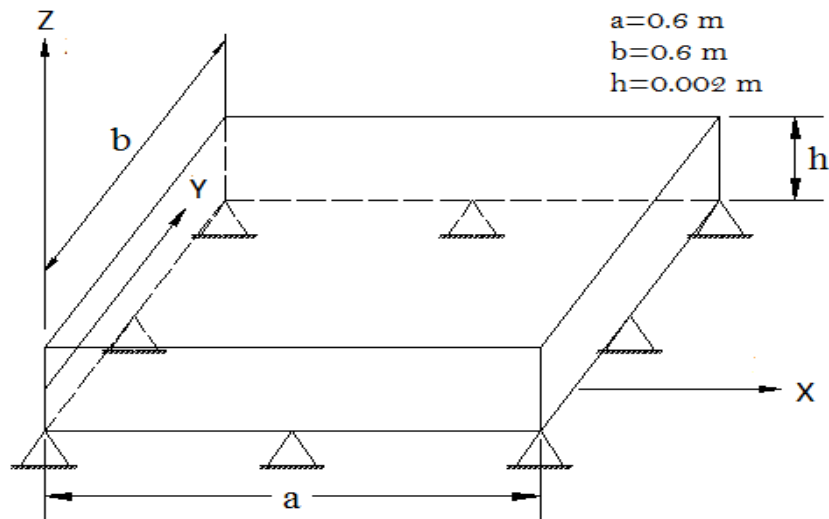


Figure 2: Model Of the Simply supported rectangular plate

We have divided our work on three cases. We have done the analysis based on this three cases.

- Case 1: Analysis of natural frequency and amplitude of the
- Case 2: Analysis of the behavior of vibration of the plate with different cutout and point load.
- Case 3: Behavior of vibration is analyzed in different positions of the plate.

## **ANSYS:**

ANSYS is engineering simulation software uses FEM to predict system response. Structural mechanics solution from ANSYS provide the ability to simulate every structural aspect of a product, including linear static analysis that simply provides stresses or deformations, modal analysis that determines vibration characteristics; through the advanced transient nonlinear phenomena involving dynamic effects and complex behavior. The ANSYS Mechanical (APDL) software suite is used for both modal and harmonic analysis.

## **ANSYS INPUT:**

### **Modal Analysis:**

No of modes to extract: 10

No of modes to expand:10

## **NUMERICAL WORK:**

The material of the plate was chosen to have the mechanical properties defined by

Youngs Modulus :  $210e^{09}Nm^{-2}$

Poissons Ratio : 0.3

Density :  $7850kg/m^3$

The element used for 3-D modeling of solid structures is SOLID 185. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, stress stiffening, creep, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials.

# BUILDING THE MODEL IN MECHANICAL APDL:

## Plate with concentrated mass:

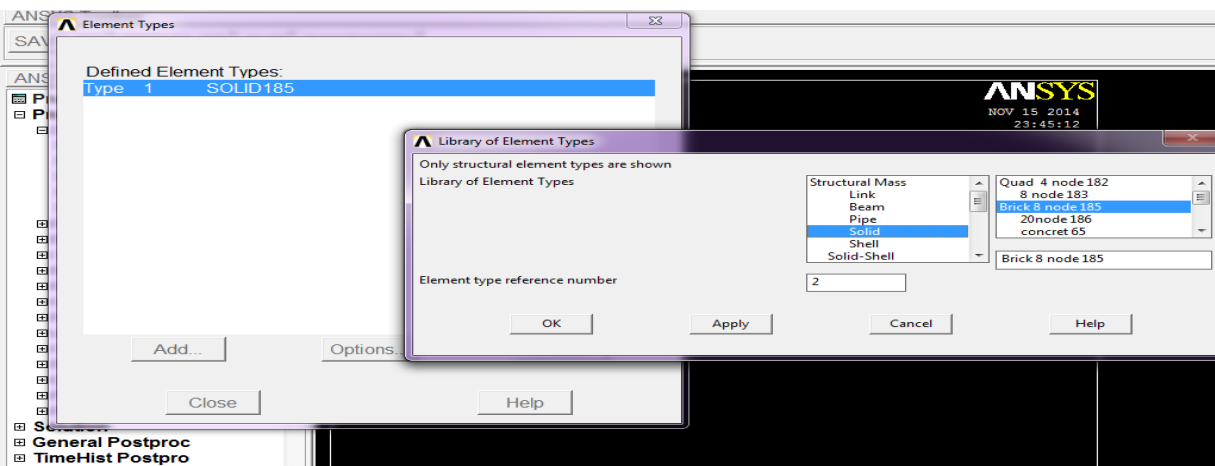
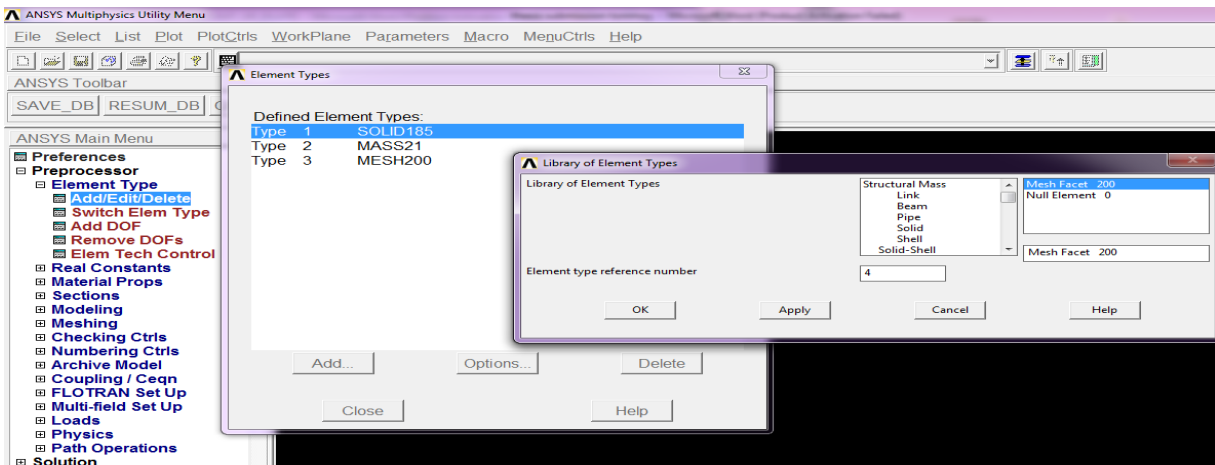
The steps followed in ANSYS APDL14, are given bellow:

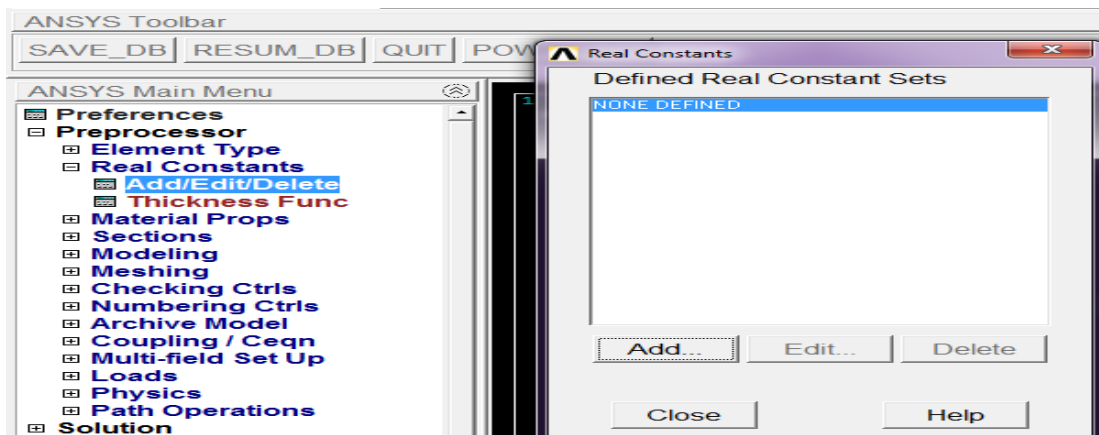
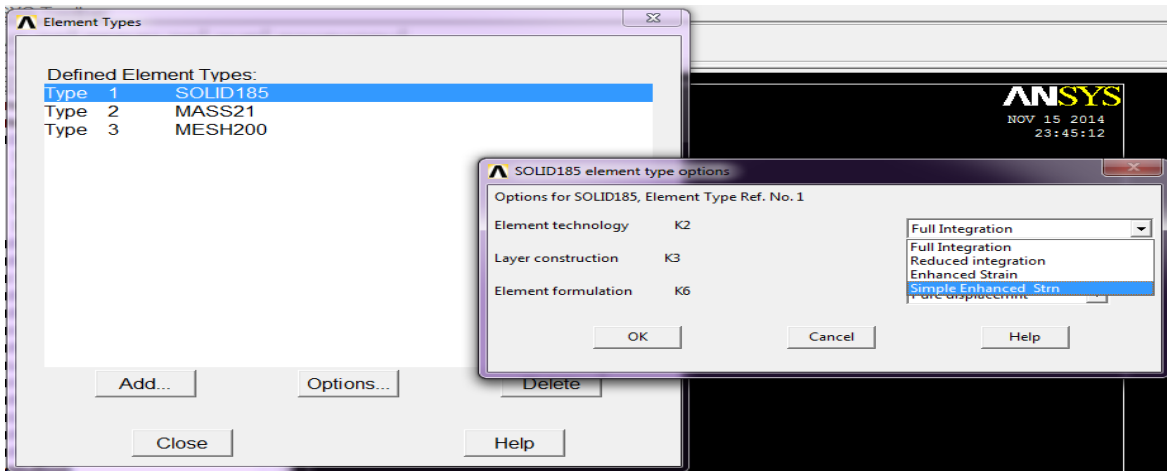
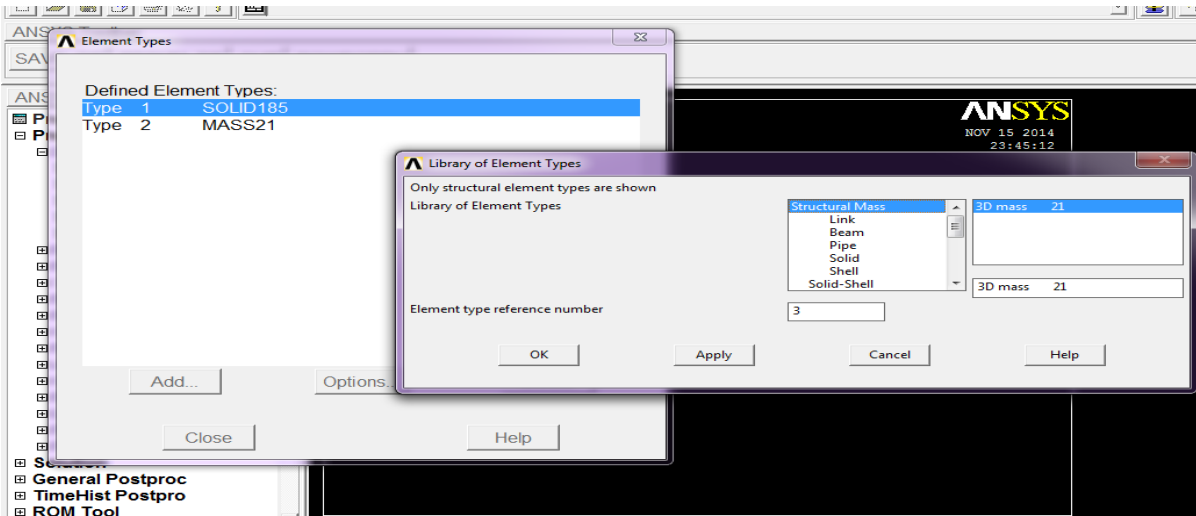
GUI>Preferences>Structural

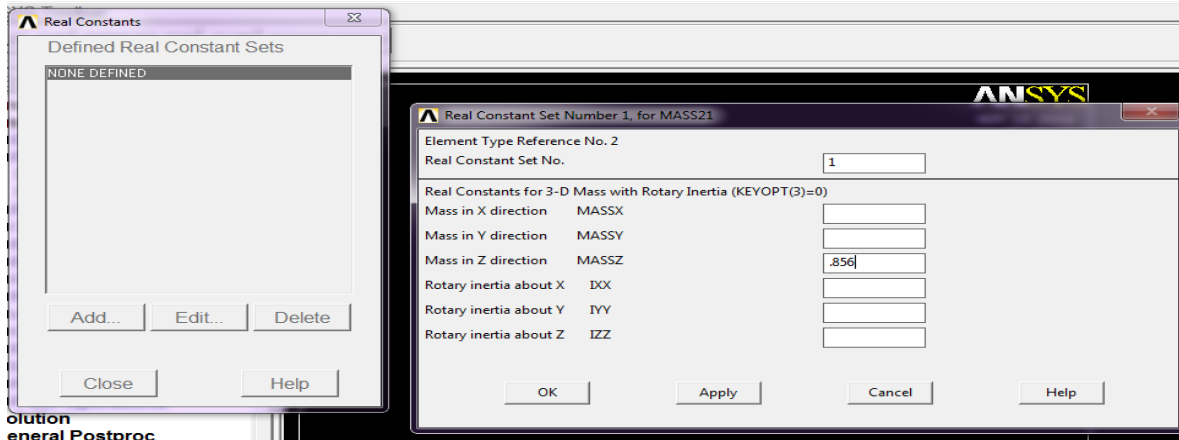
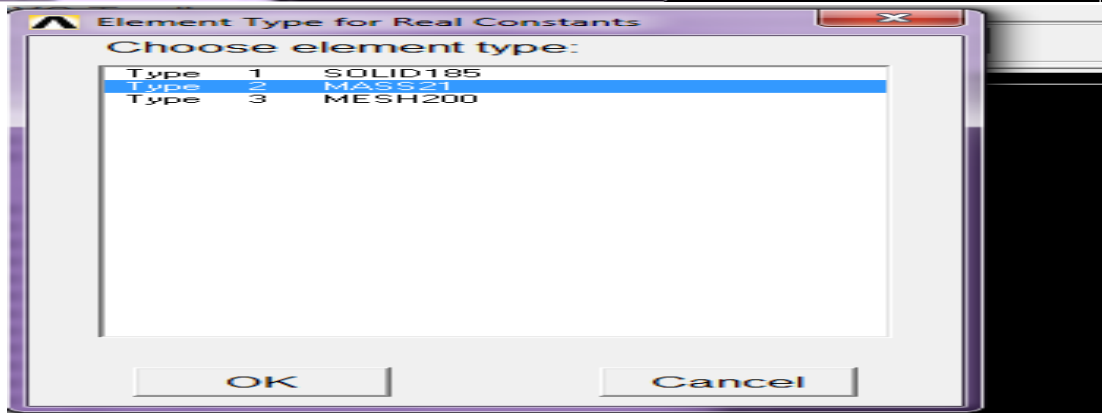
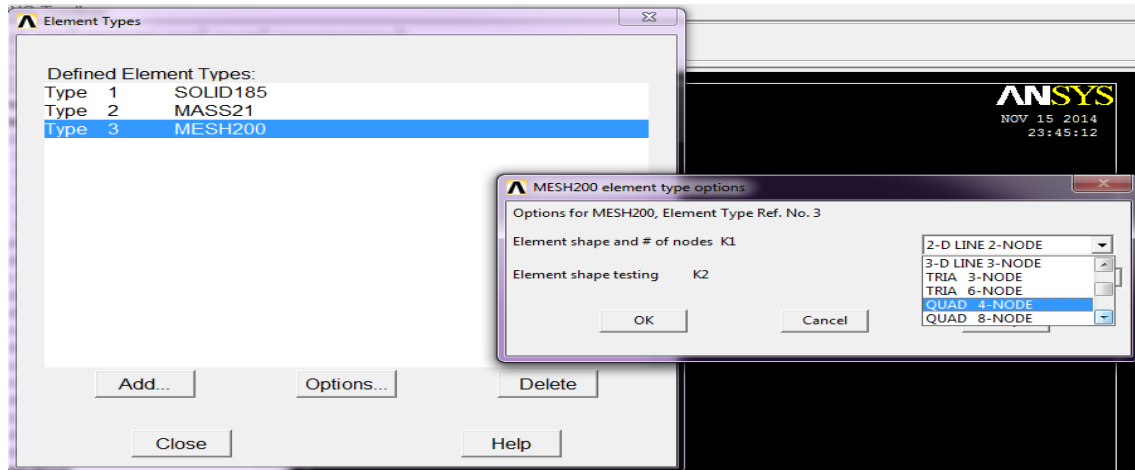
Main Menu>Preprocessor>Element Type>Add/Edit/Delete //select the element//

Add>solid>brick 8 node 185

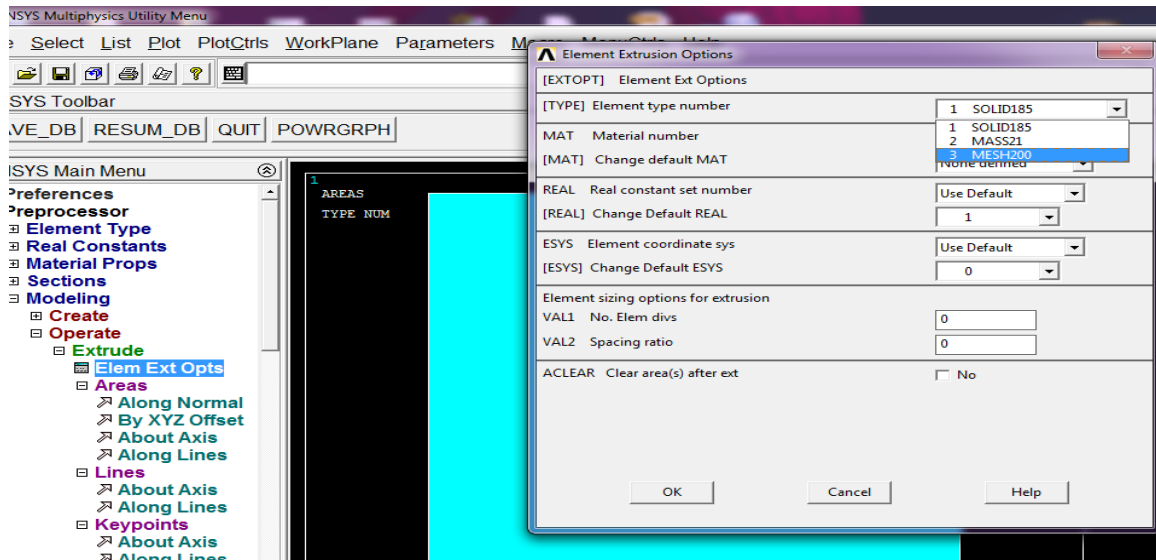
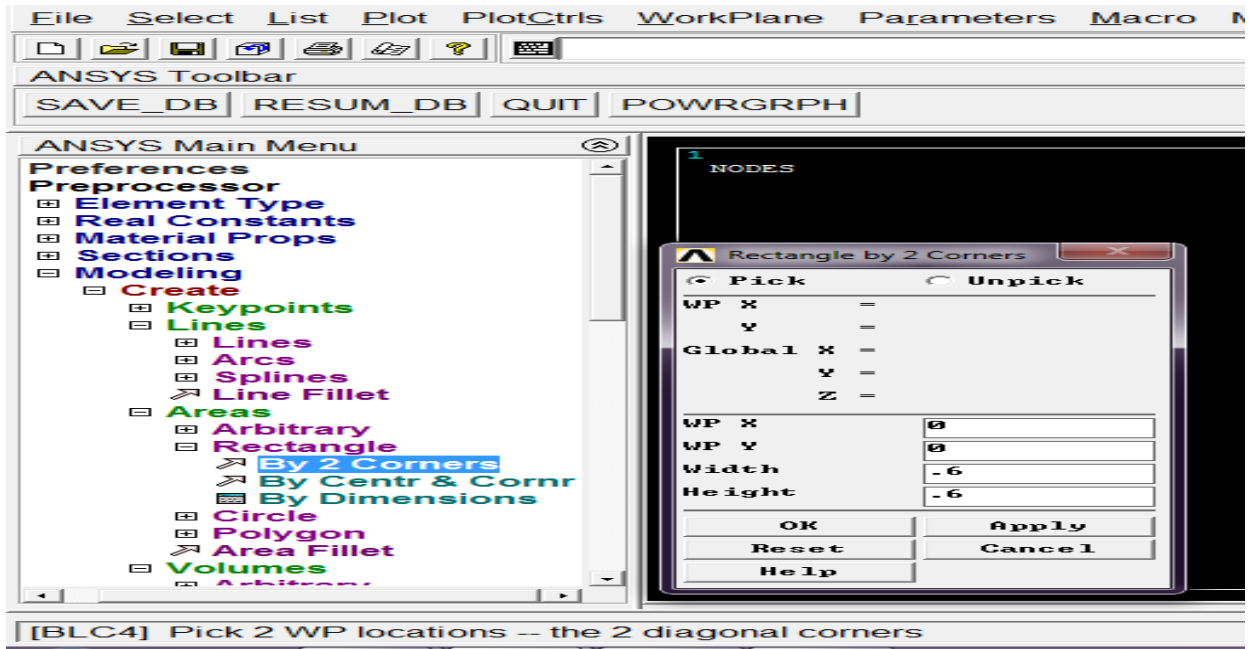
Add>structural mass>3d mass21



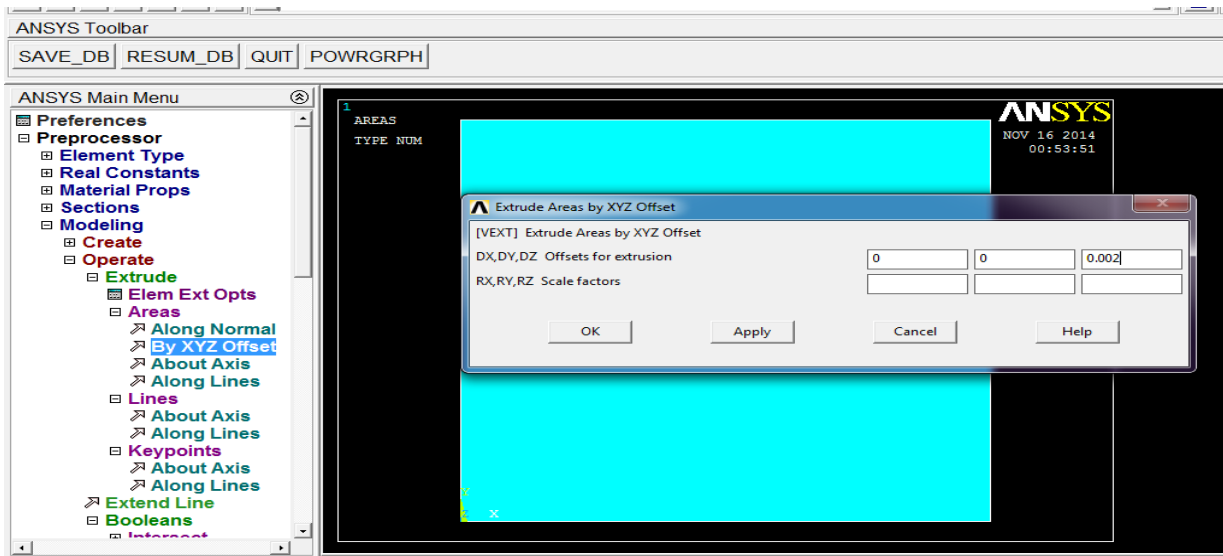
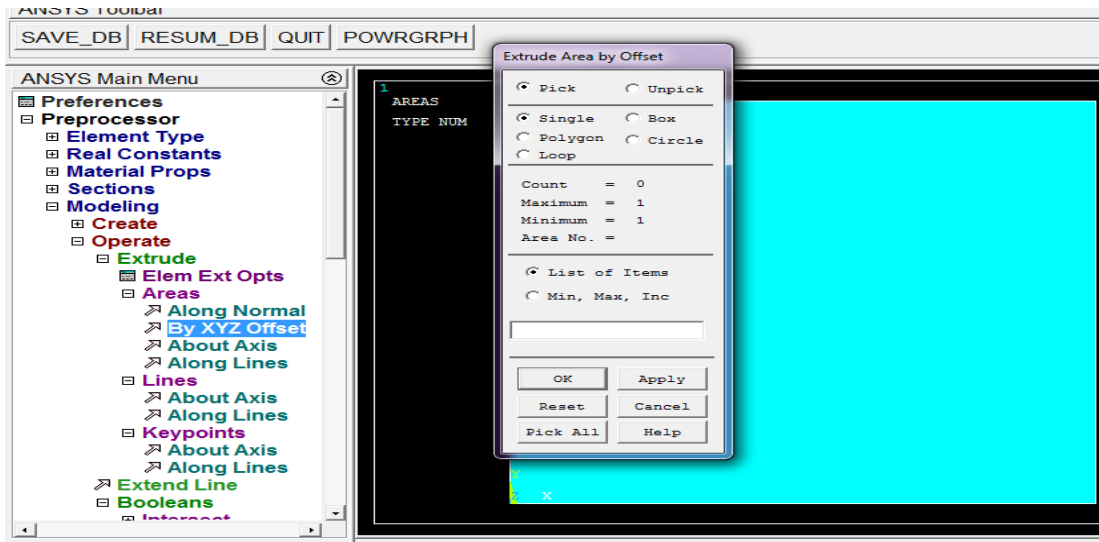


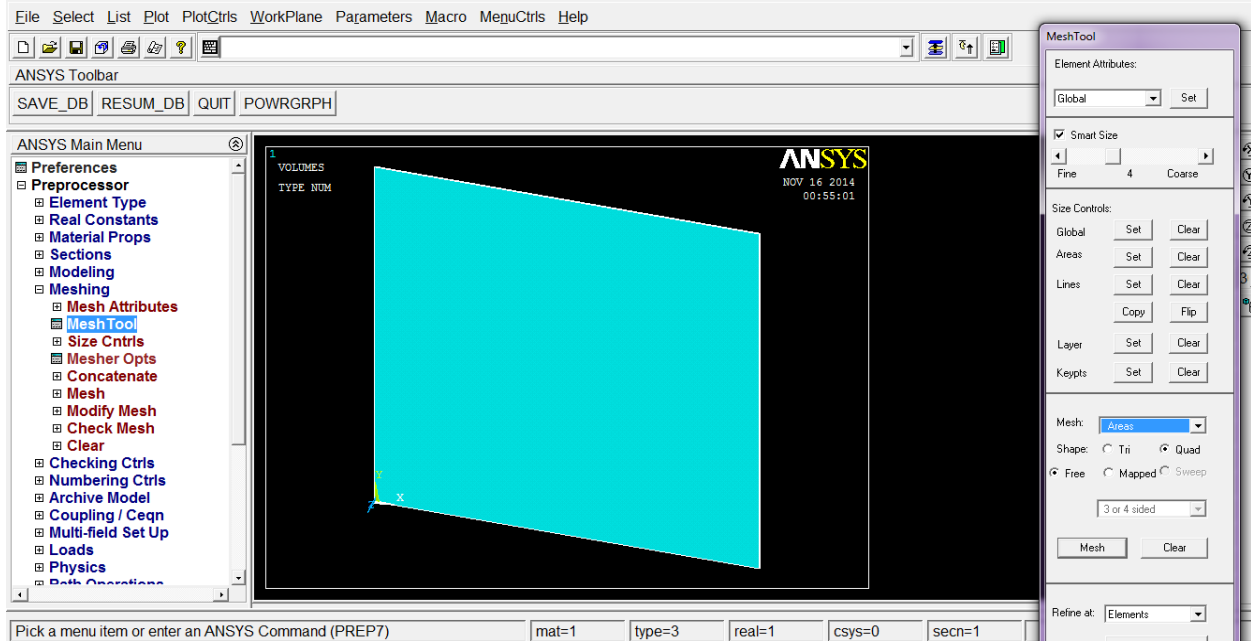
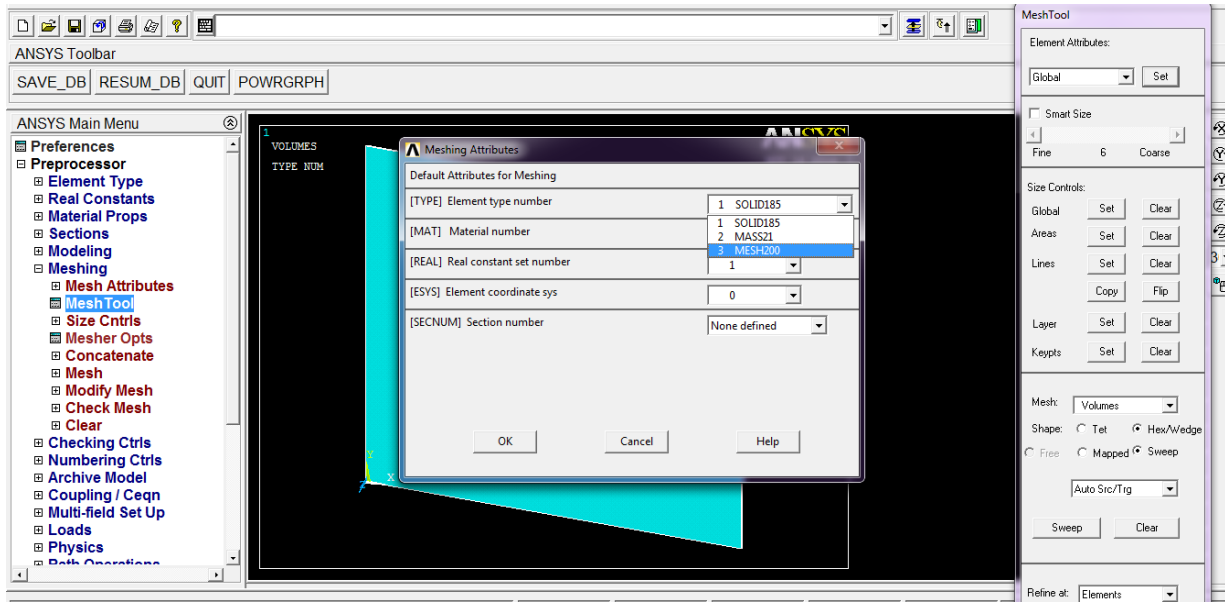


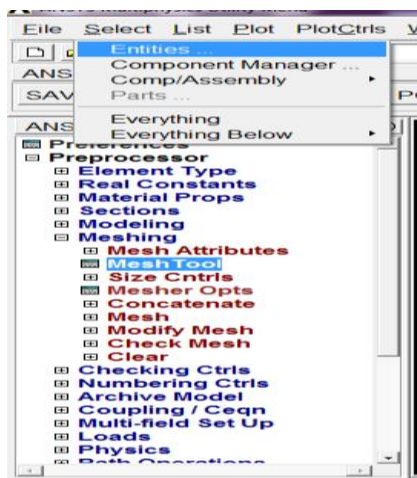
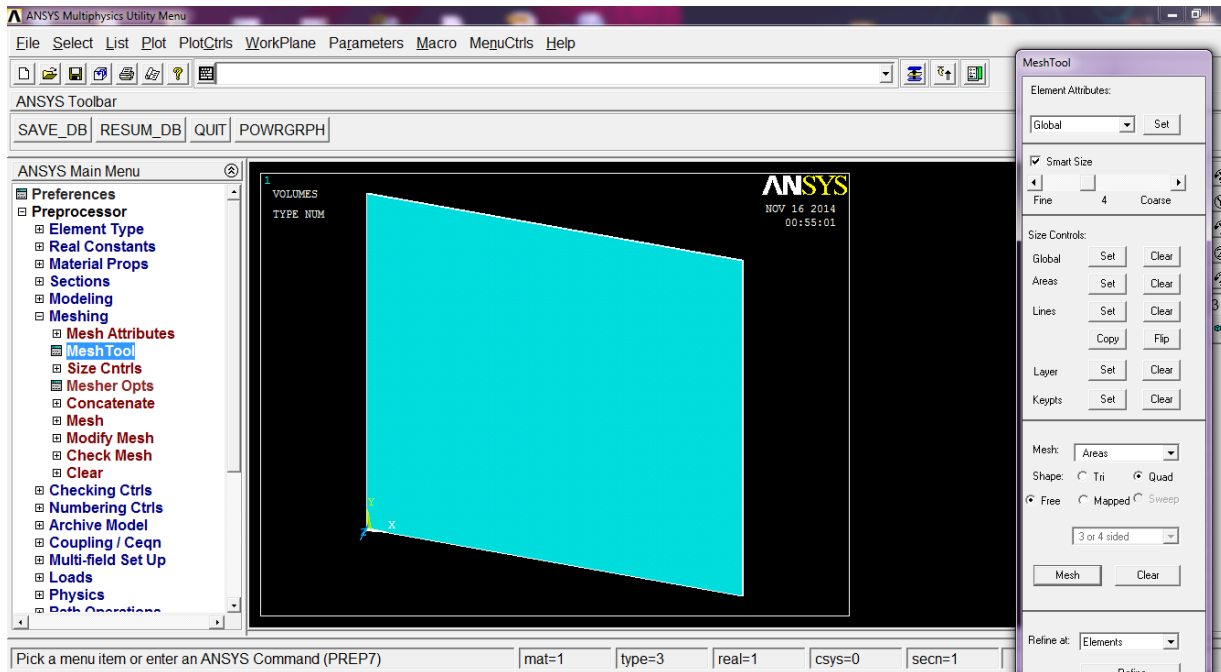
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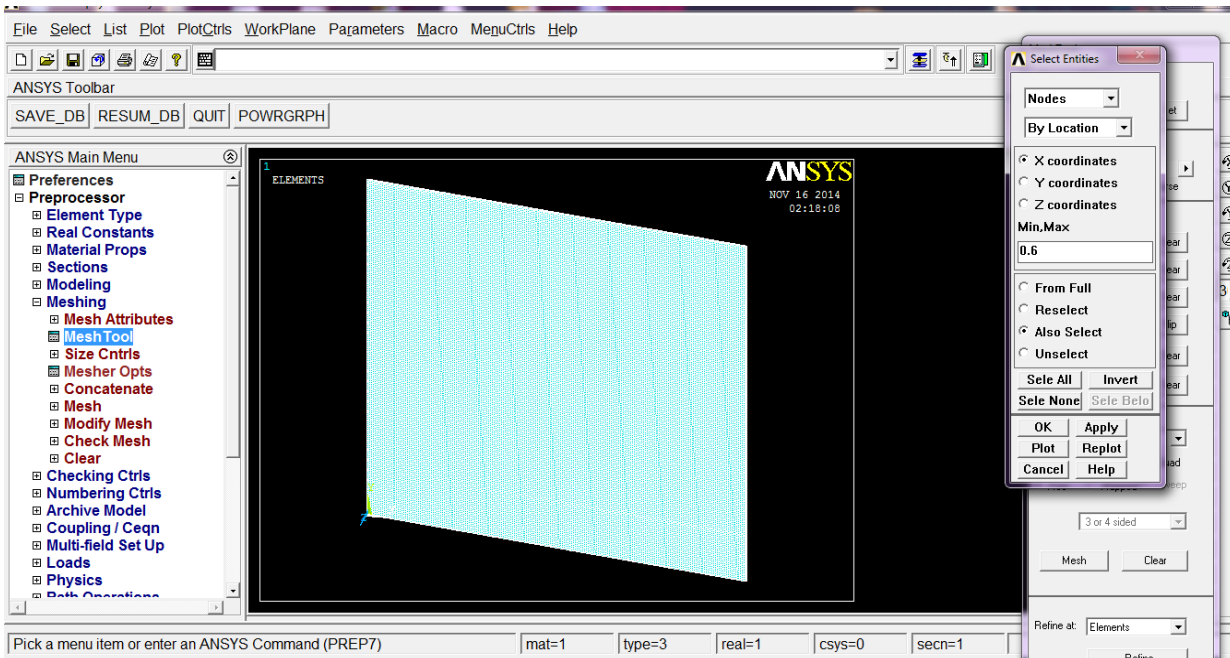
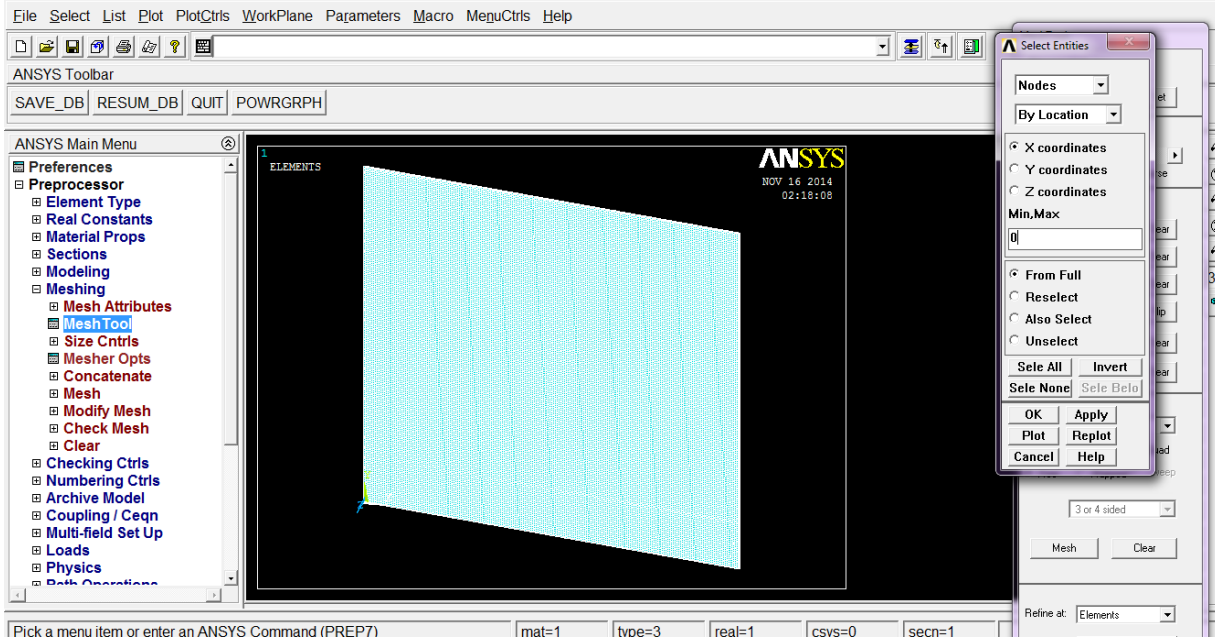


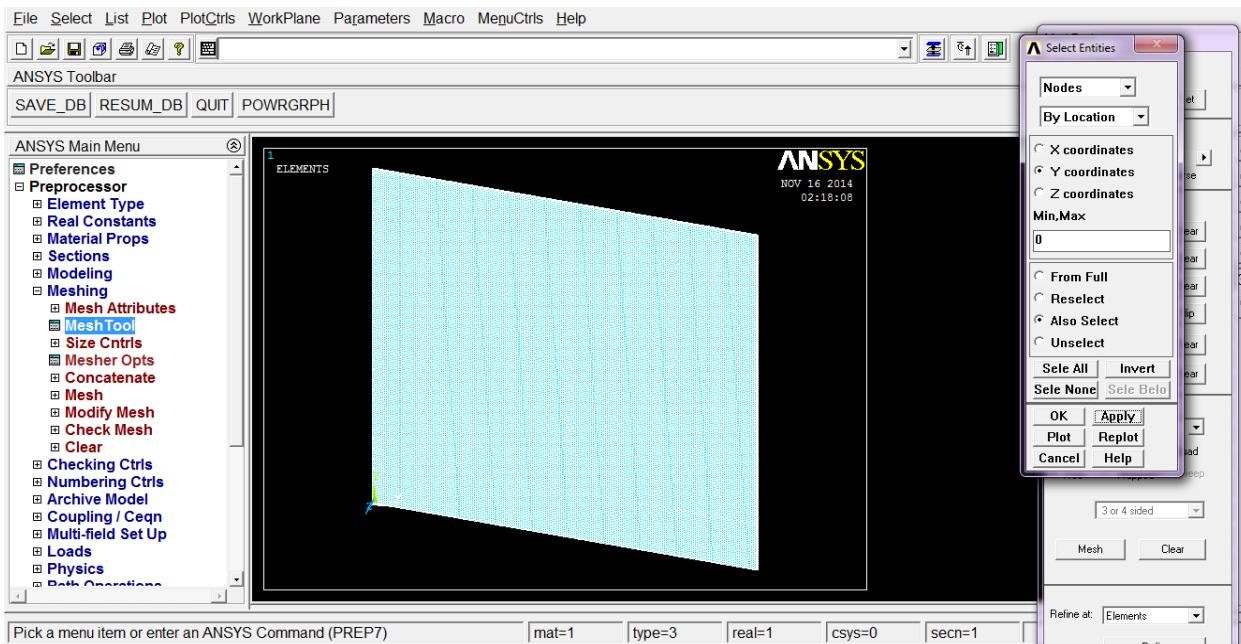
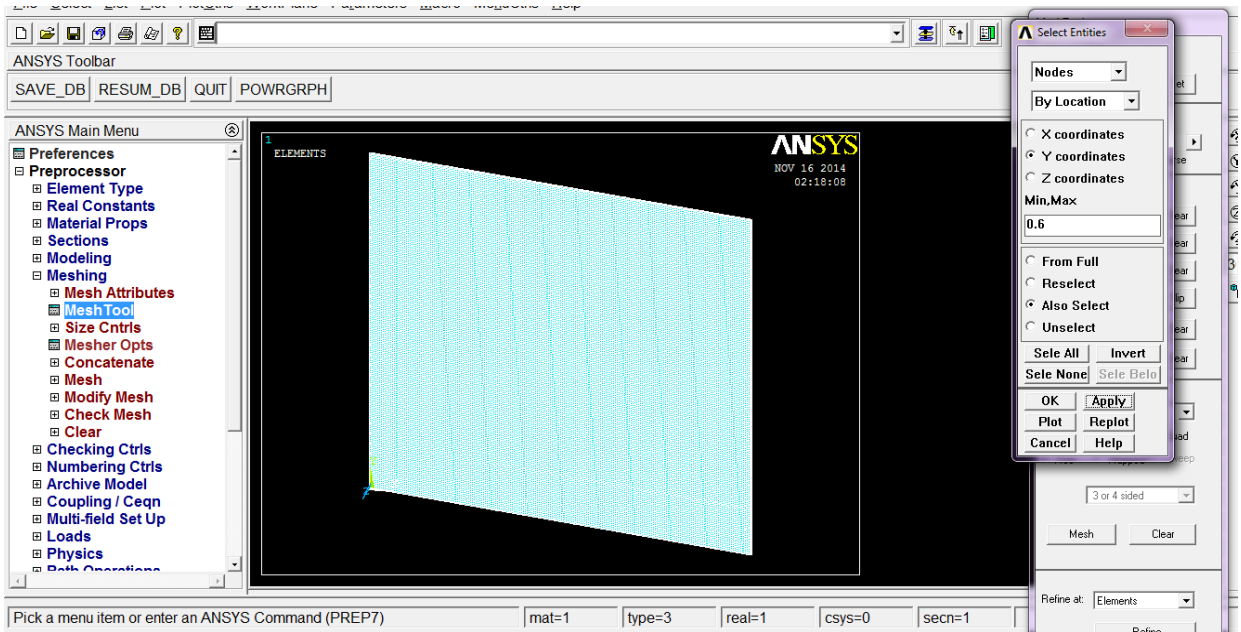


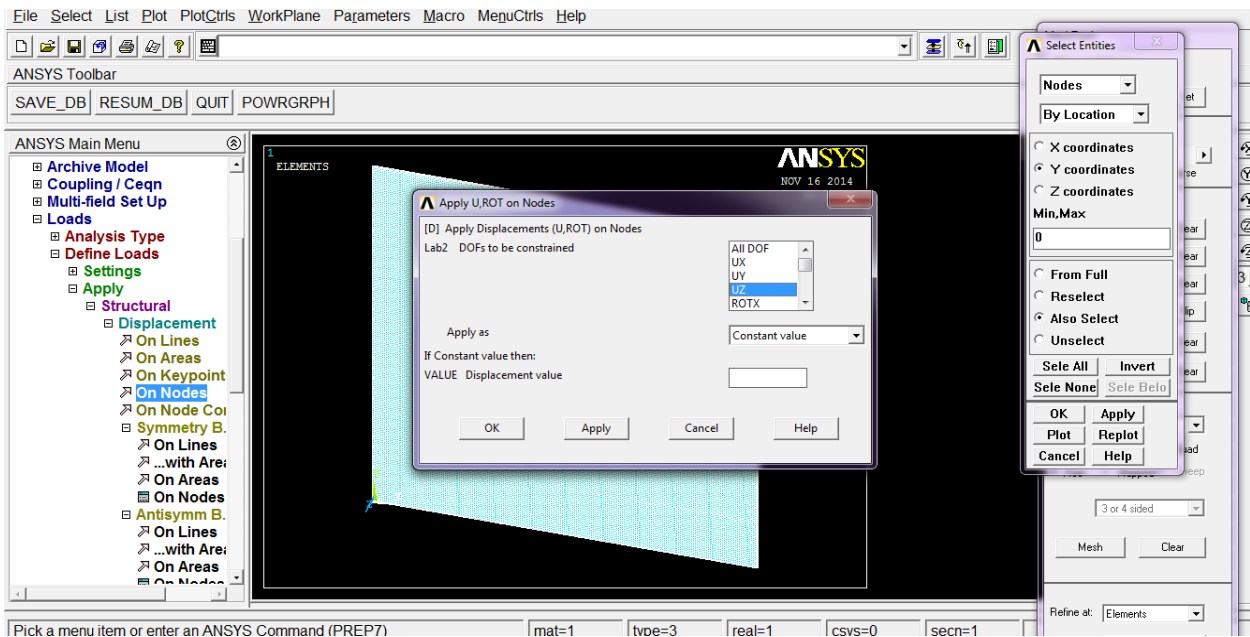
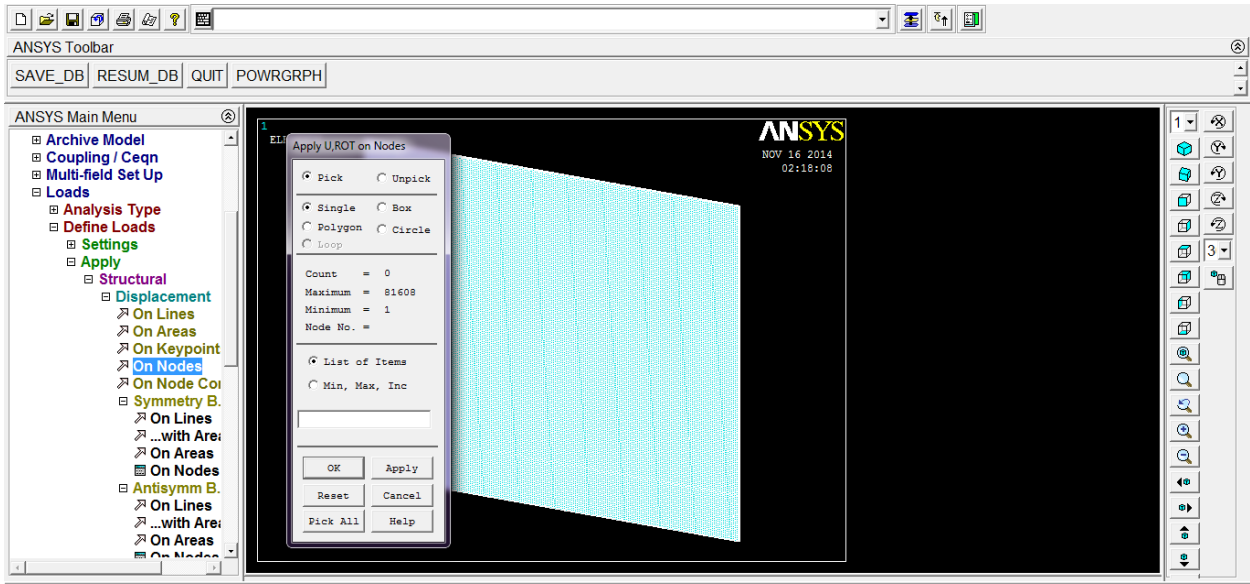




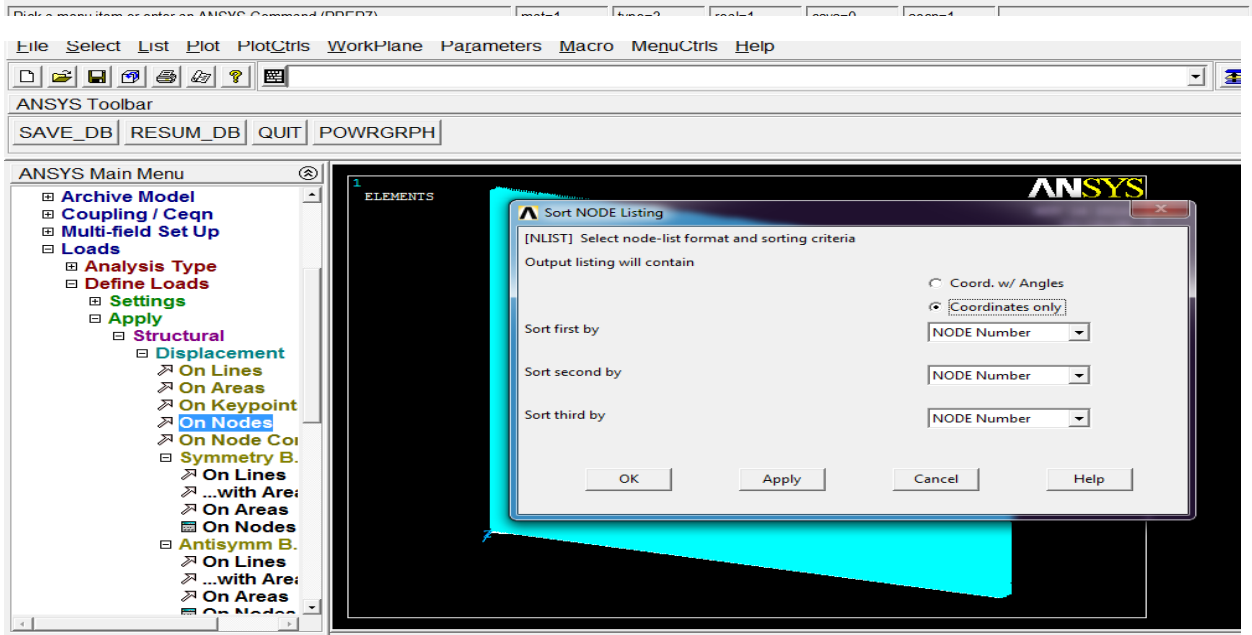
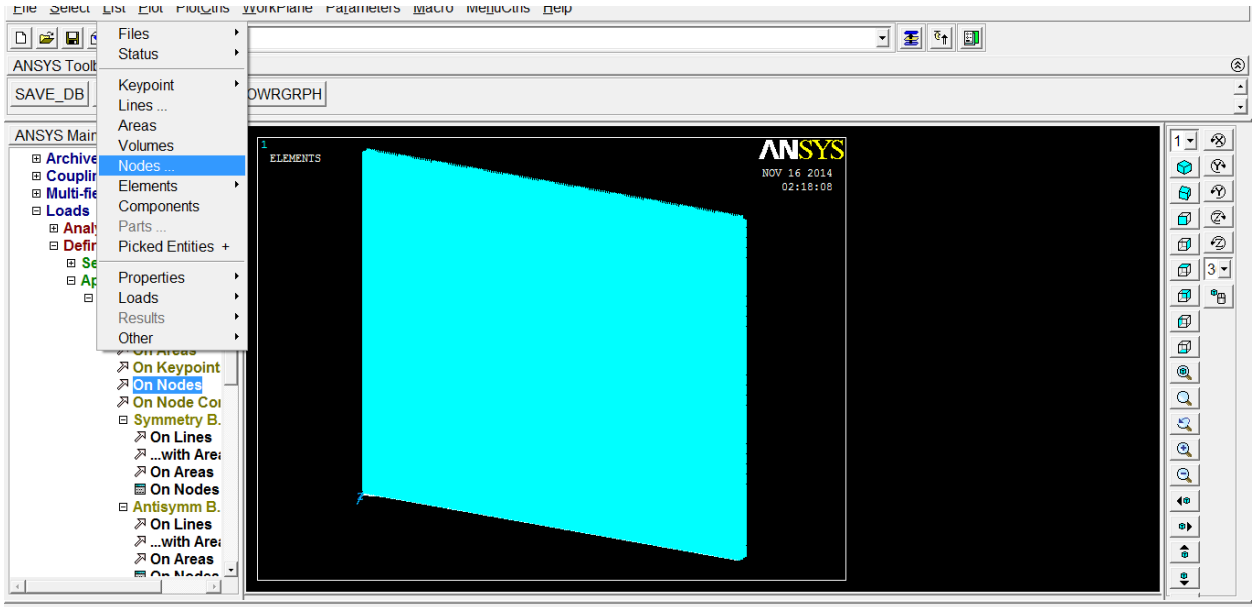
Apply constraints to the model. Constraints will be applied to all nodes according to the end condition. Select all nodes at  $x = 0$ , then apply the displacement constraints. In the same way the nodes at the other end are also selected and constraints are applied.

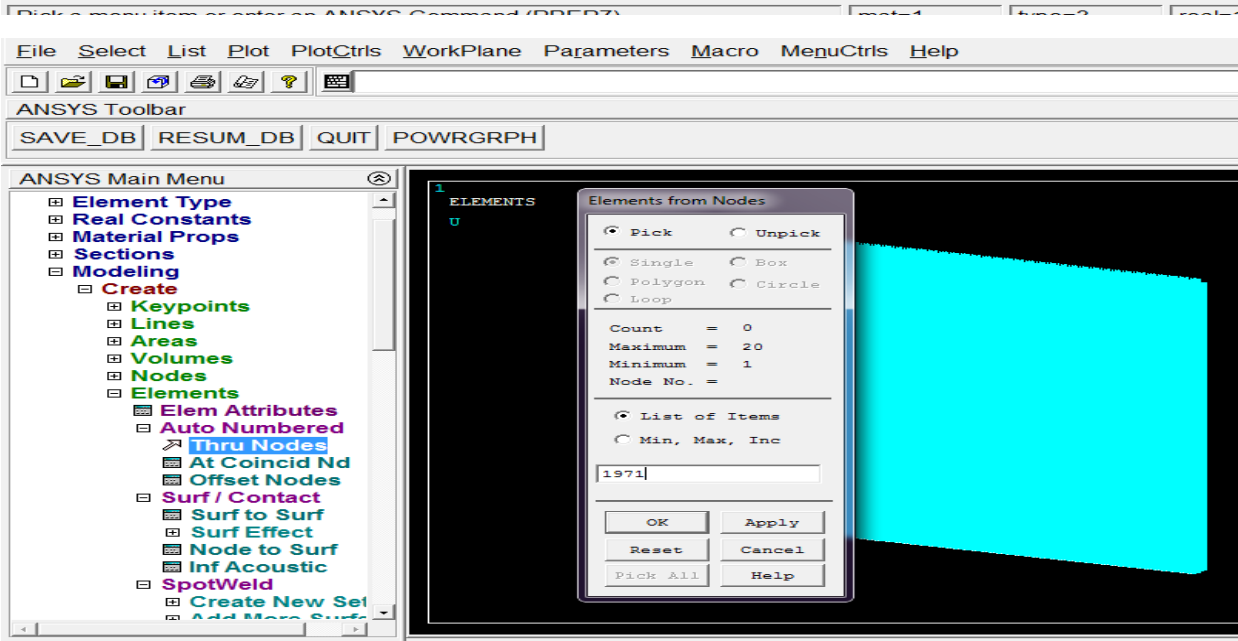
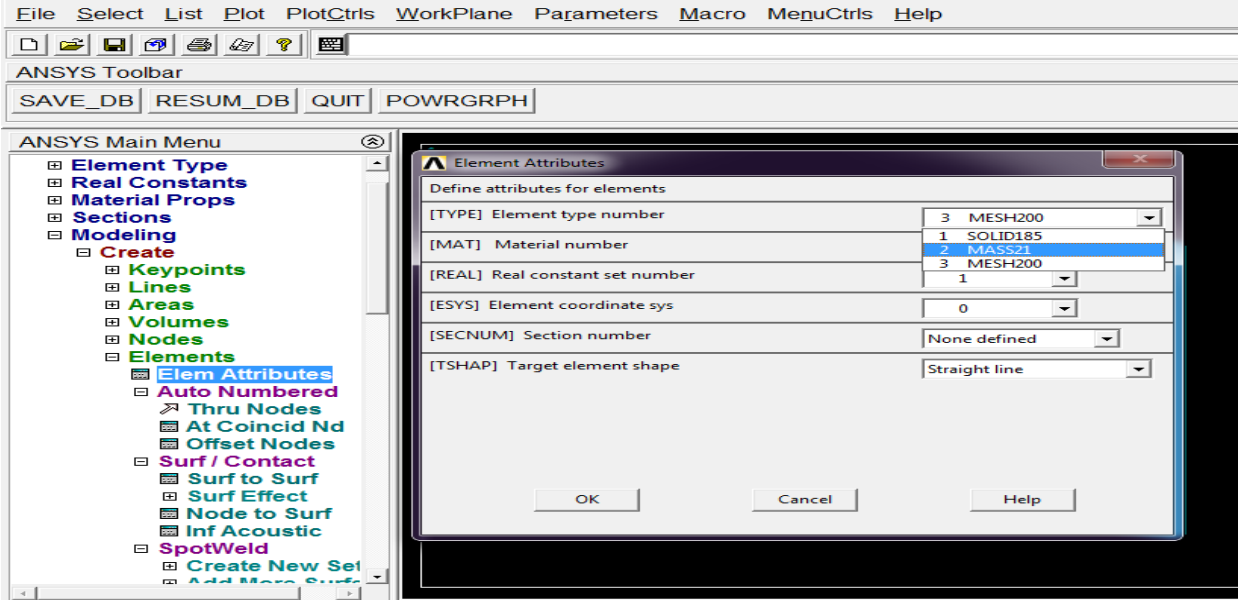




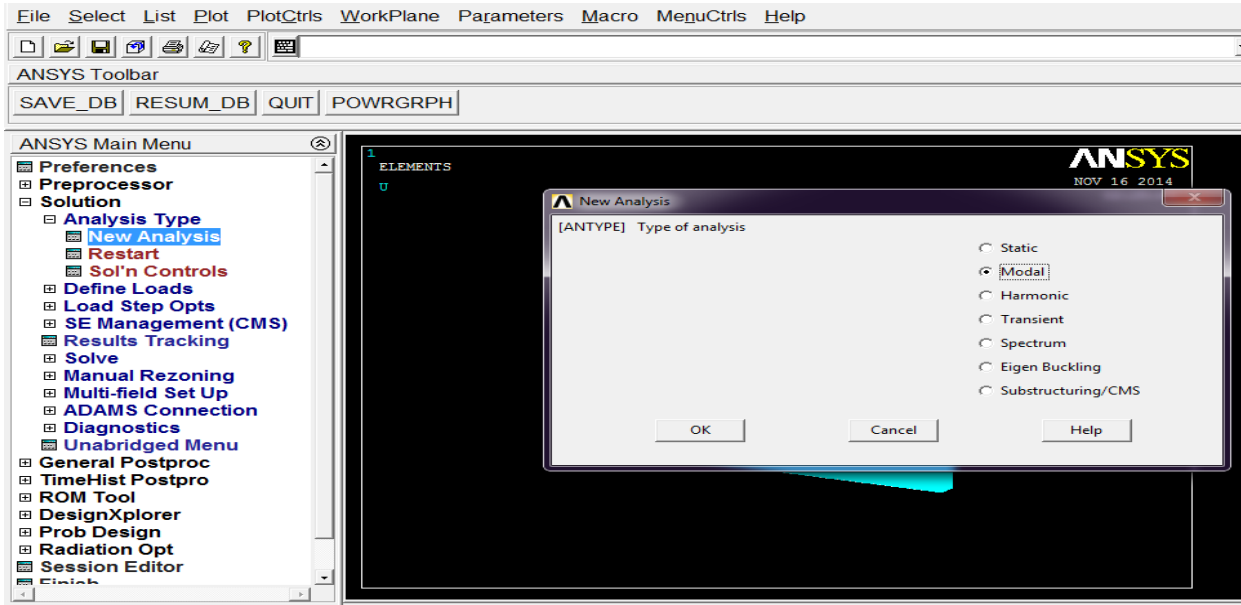
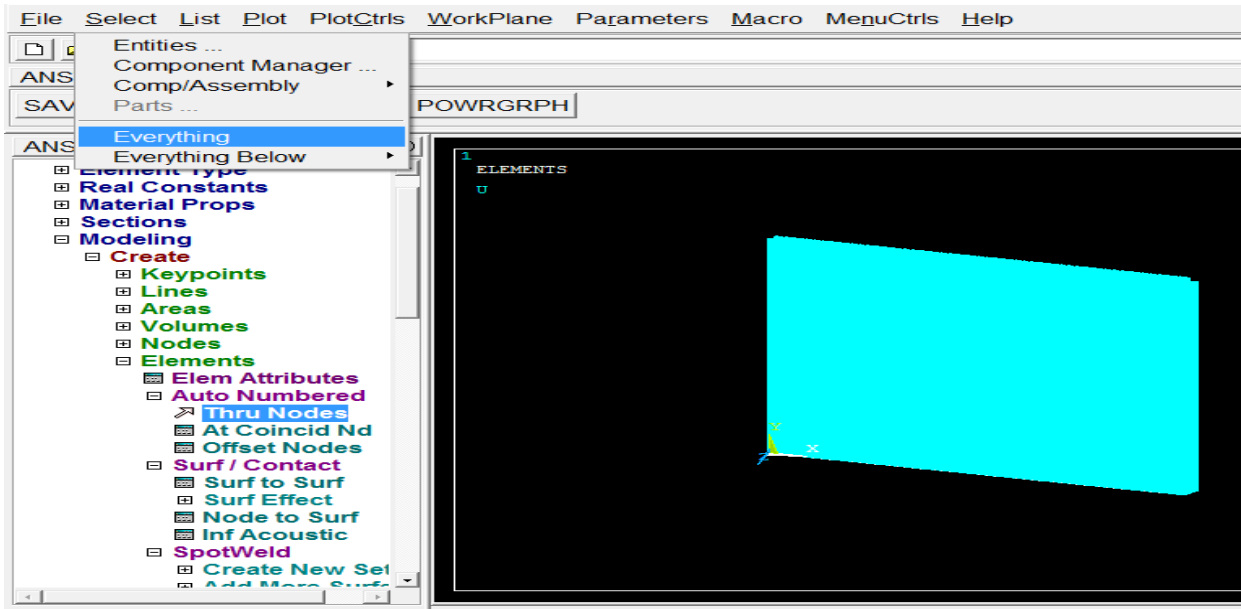


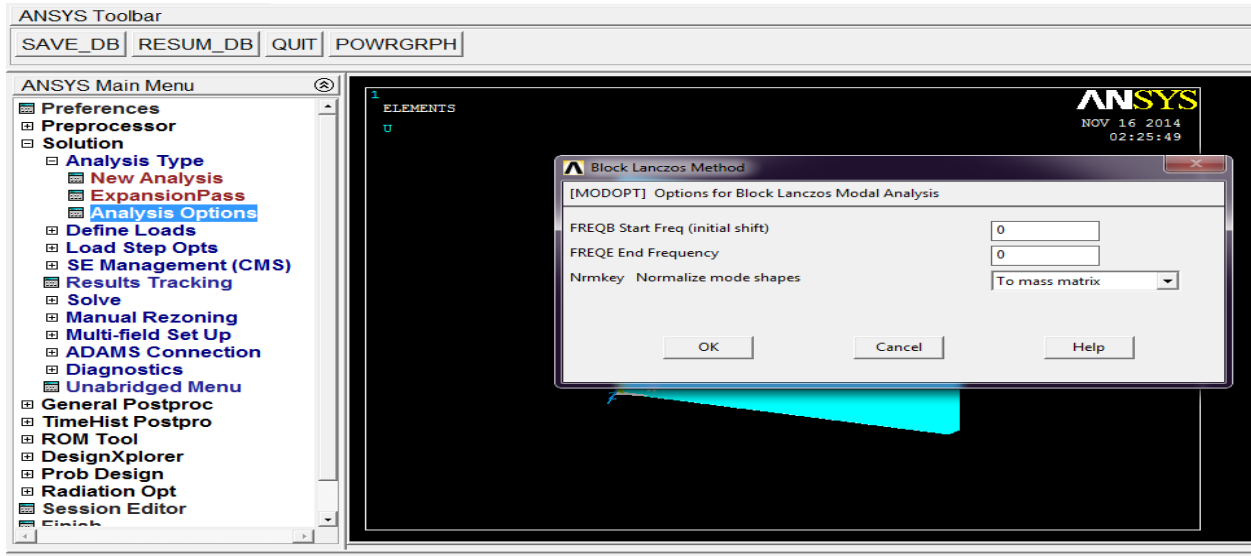
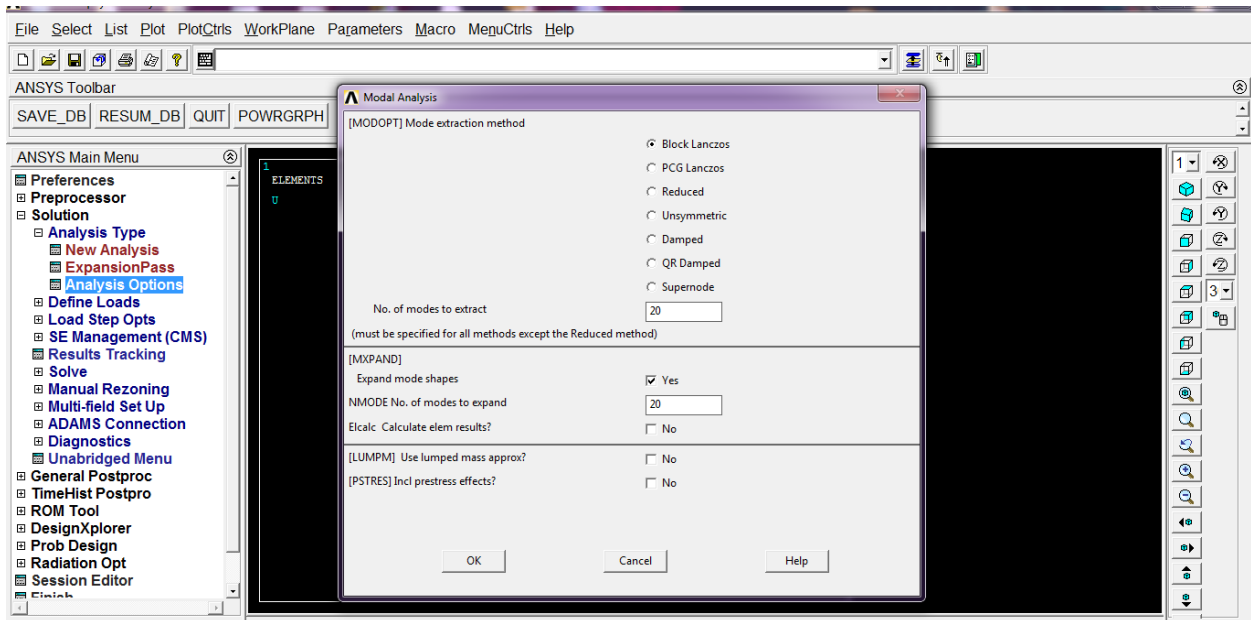


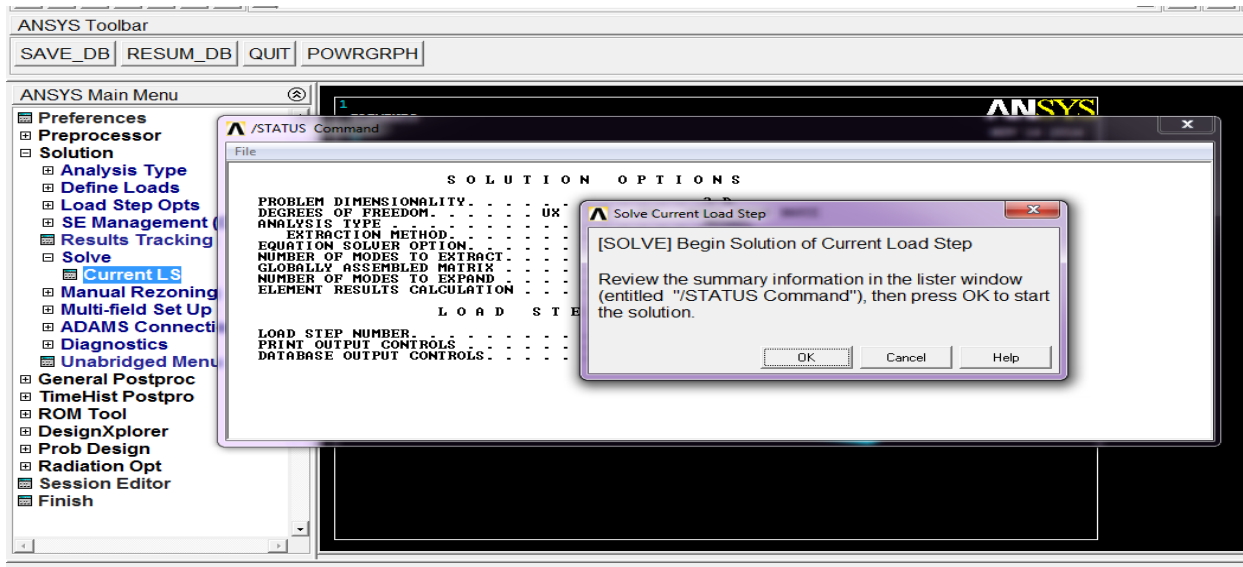




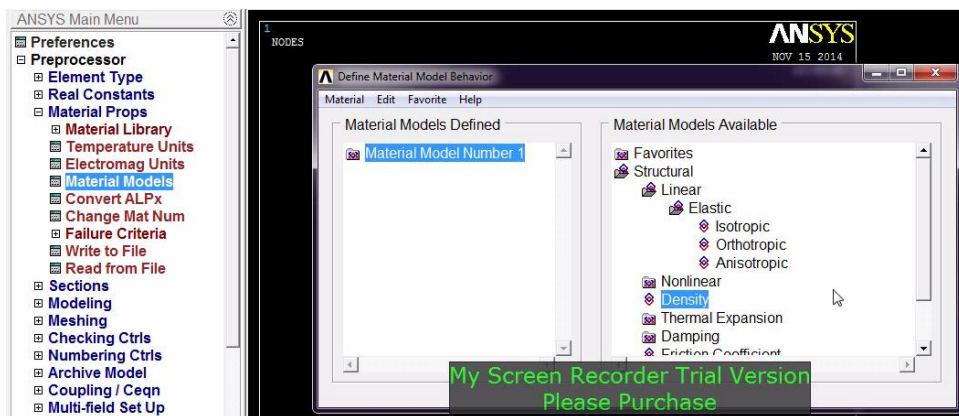


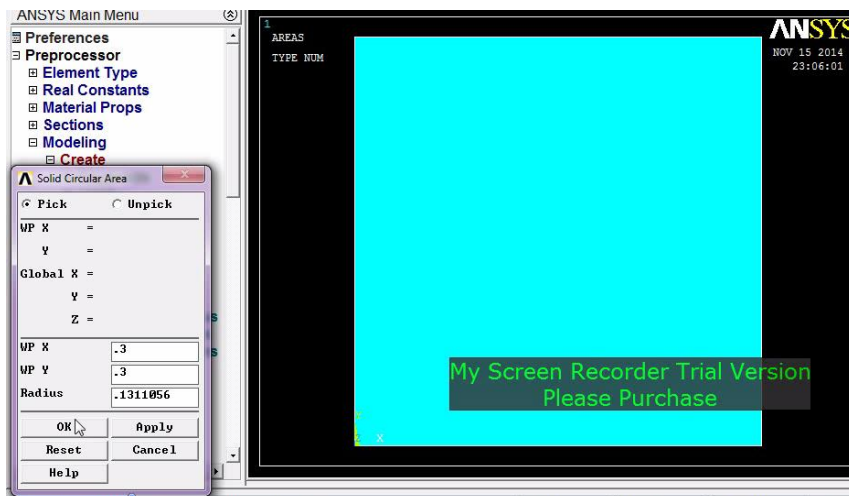
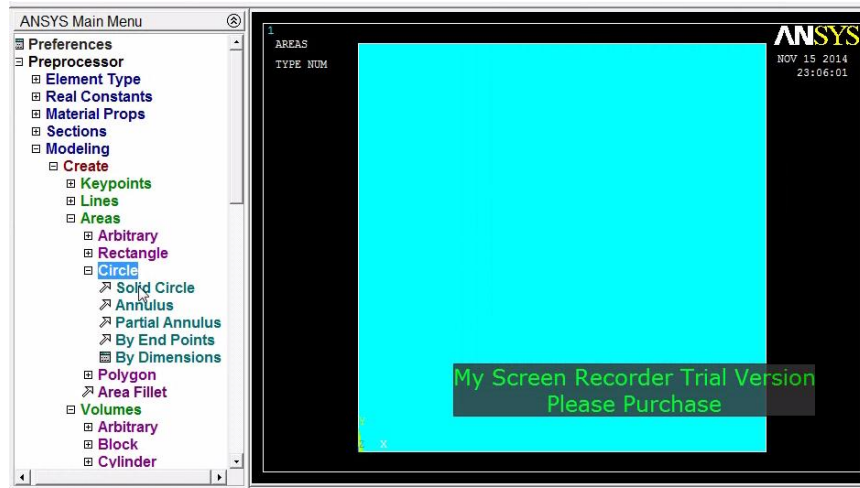
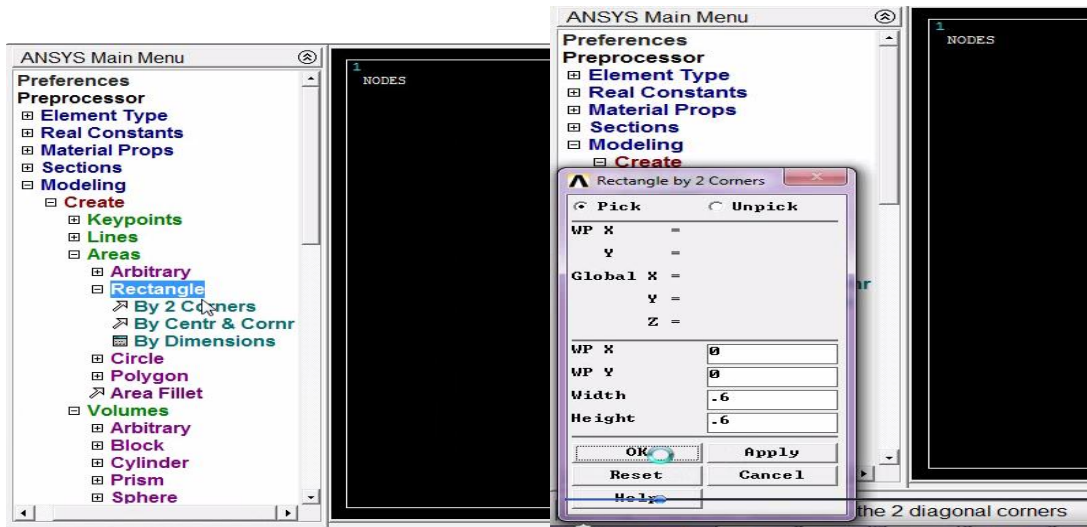


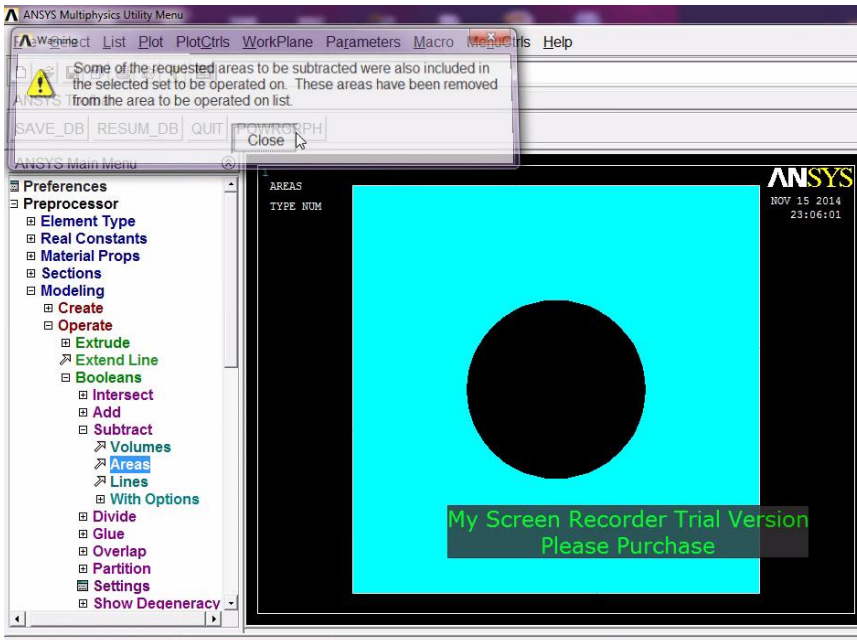
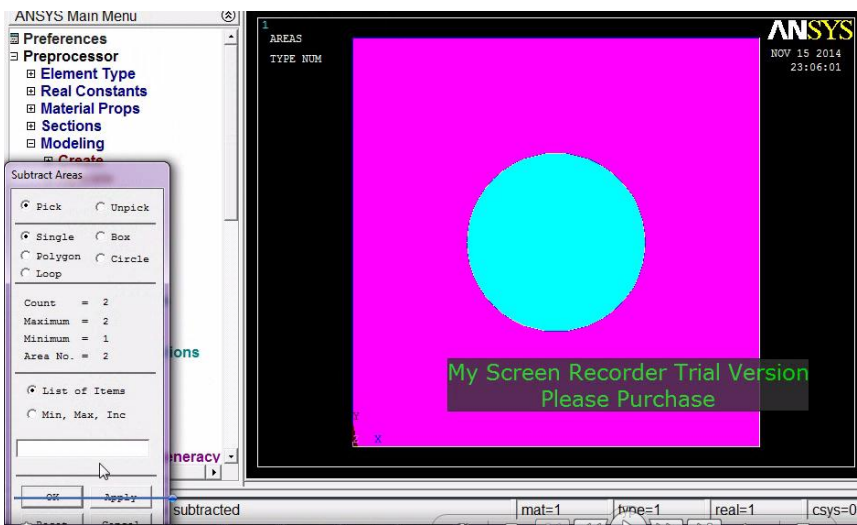
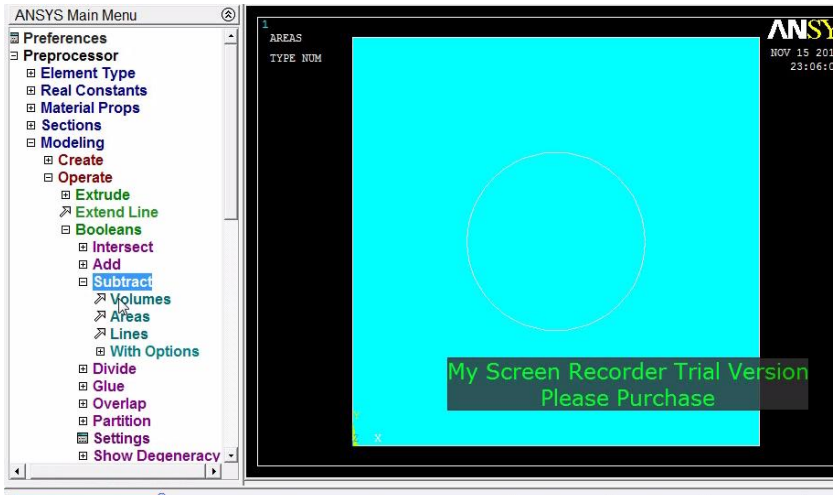


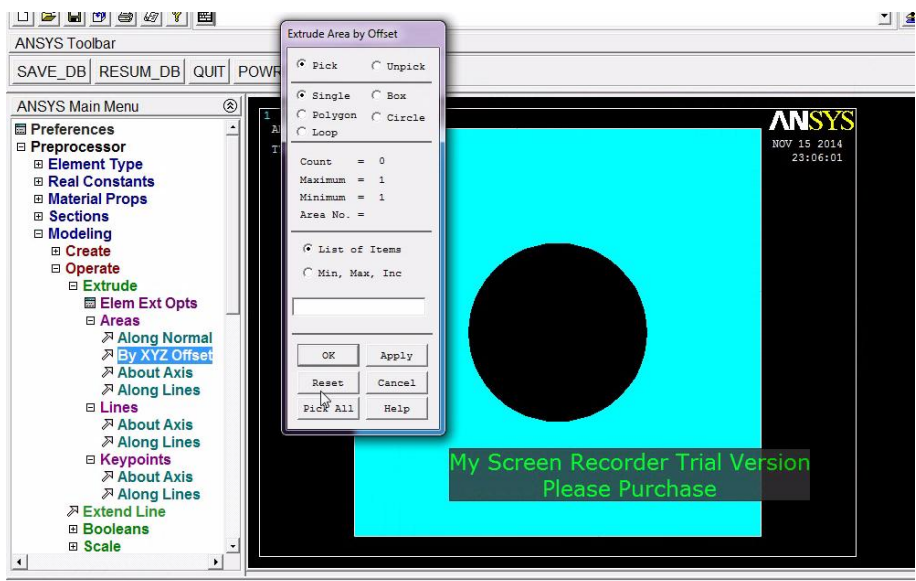
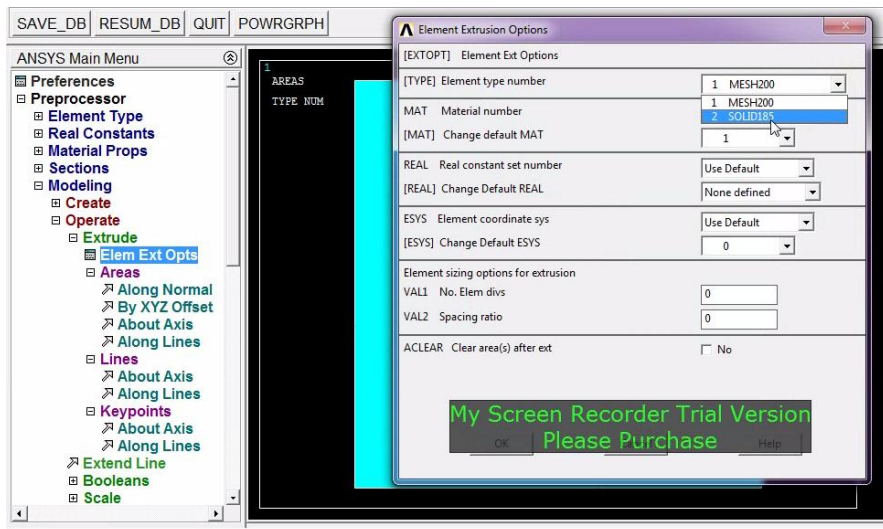
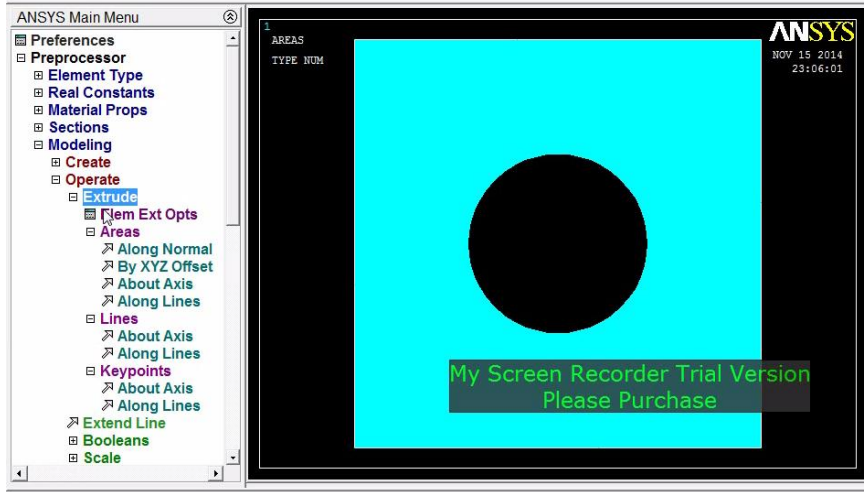


For coutout



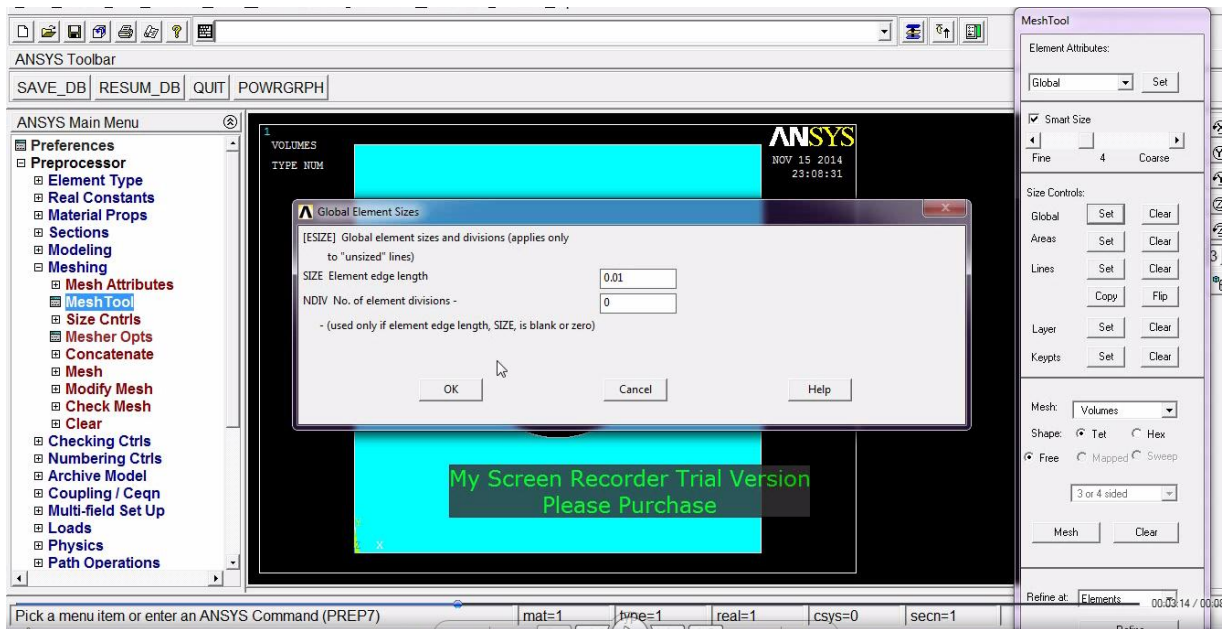
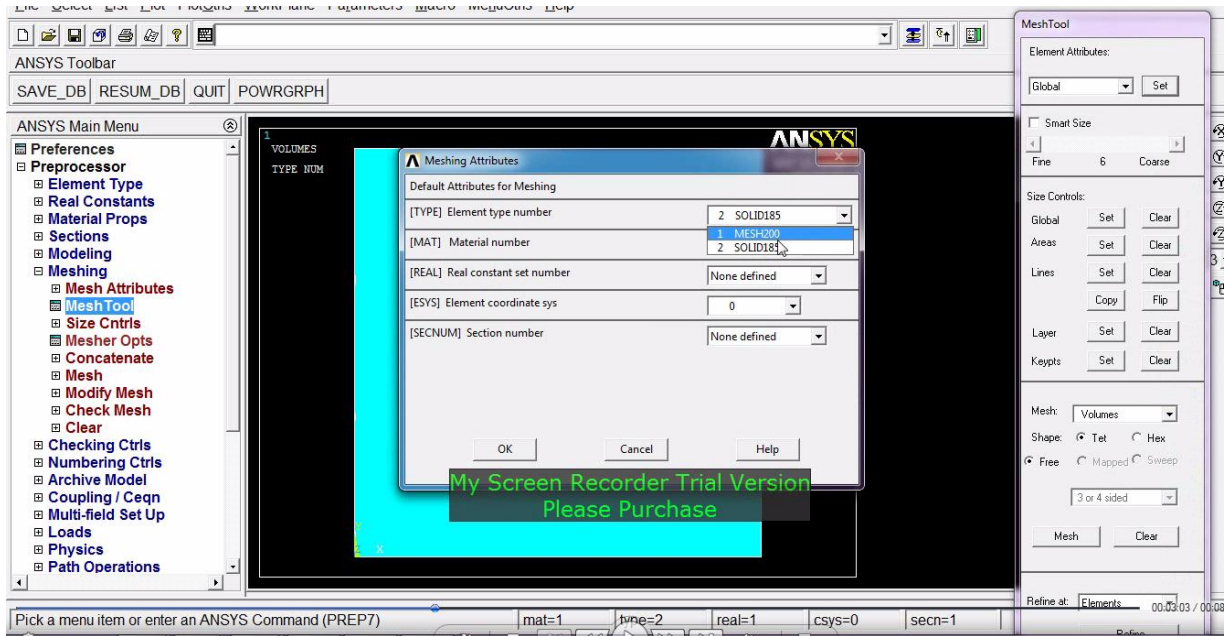




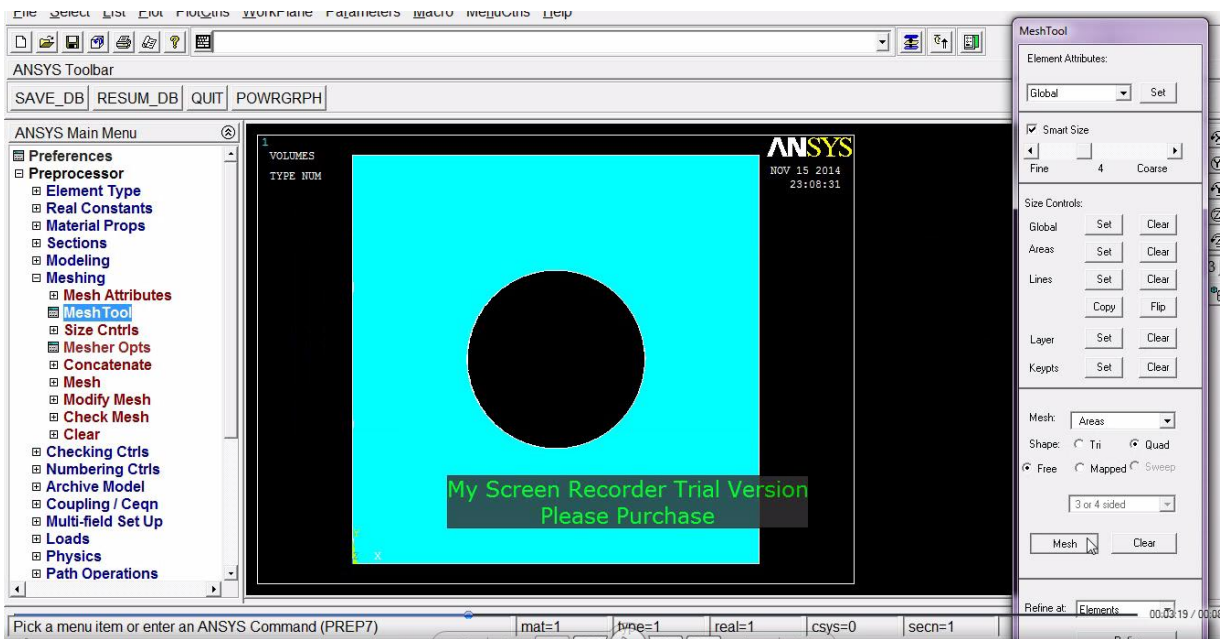
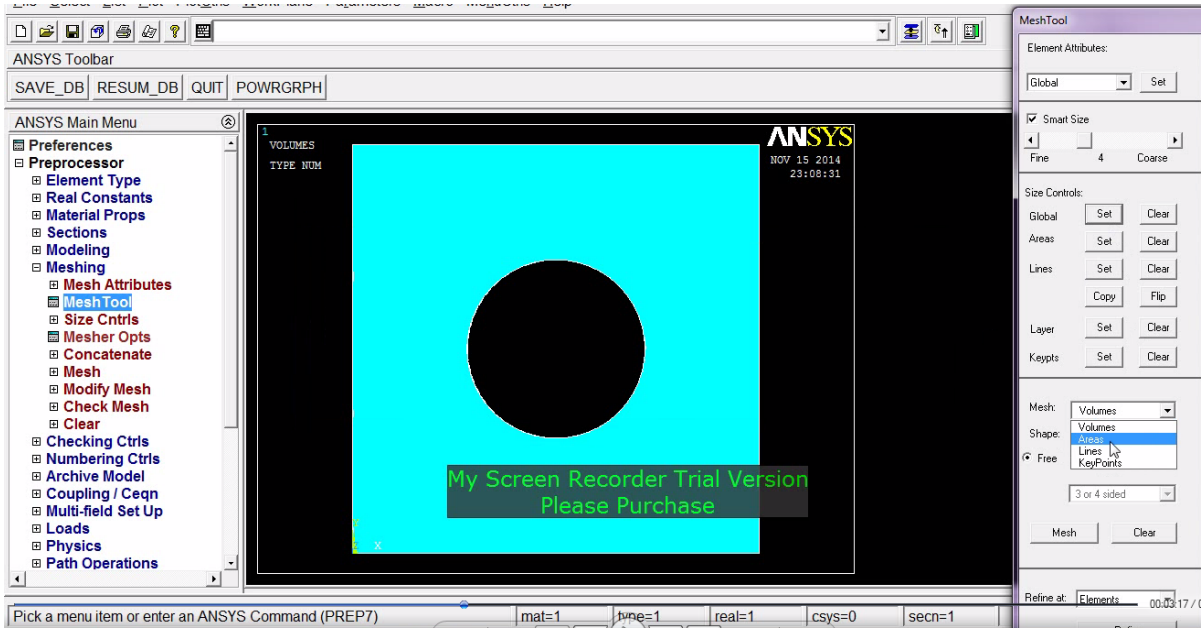


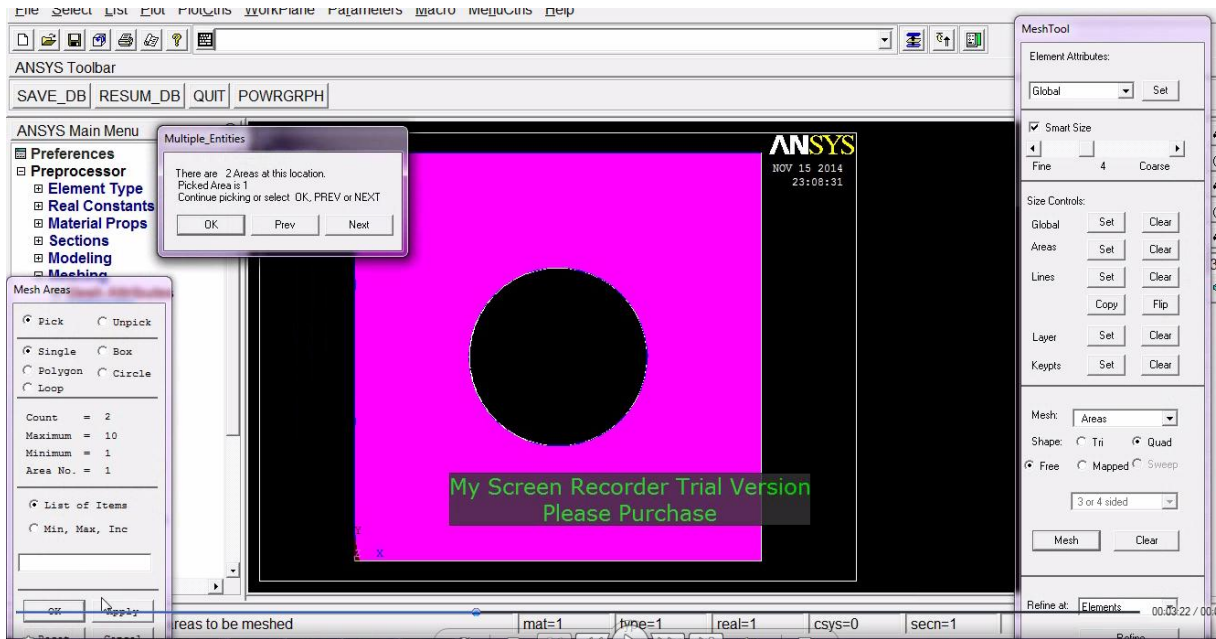
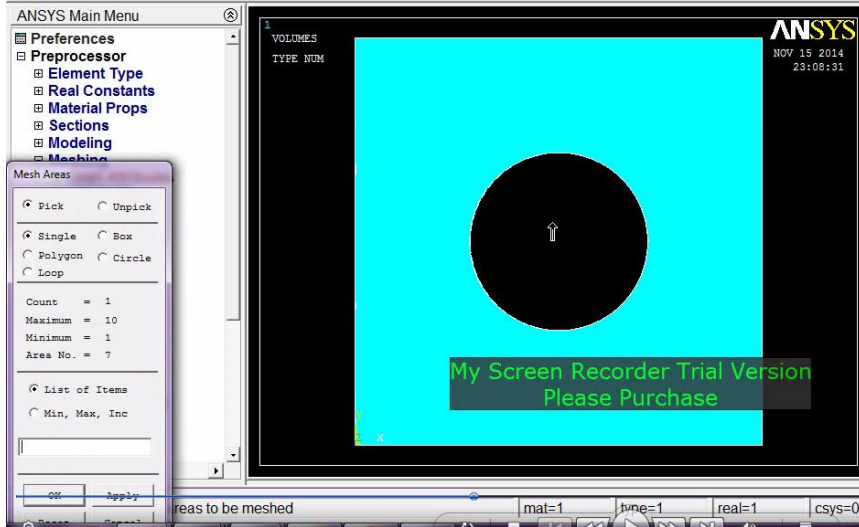


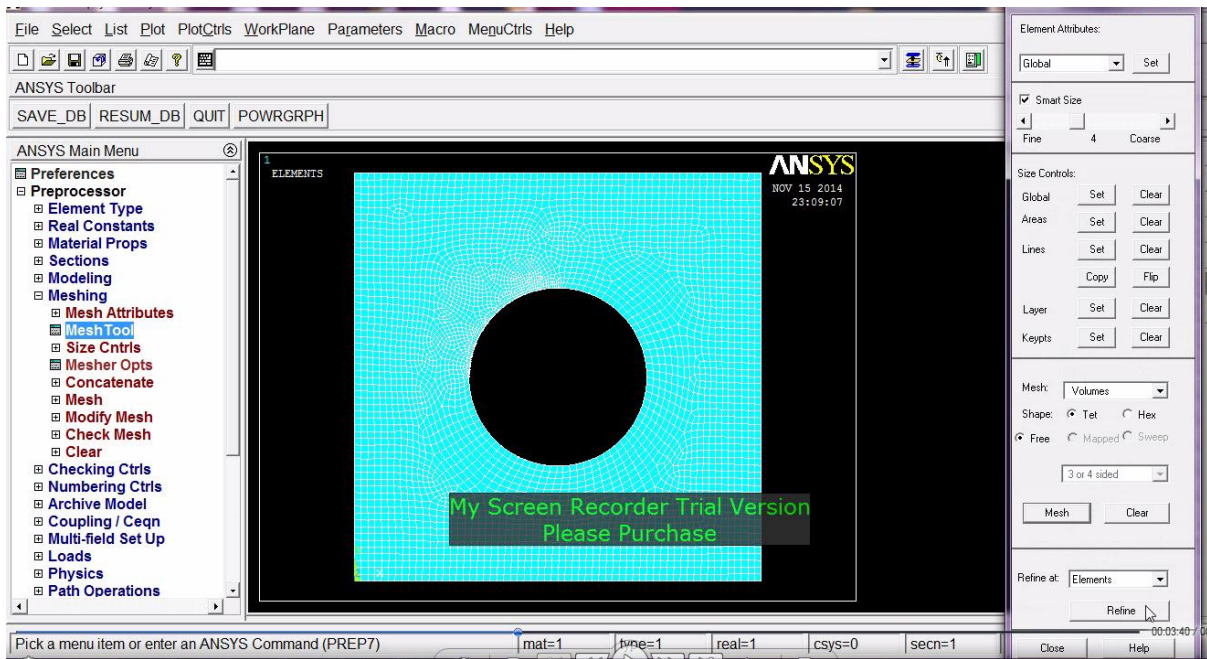
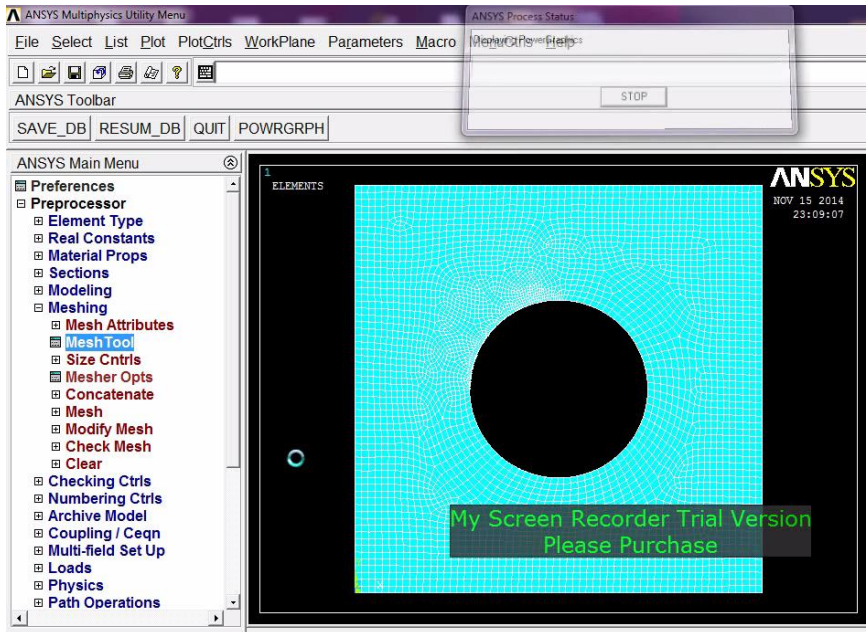


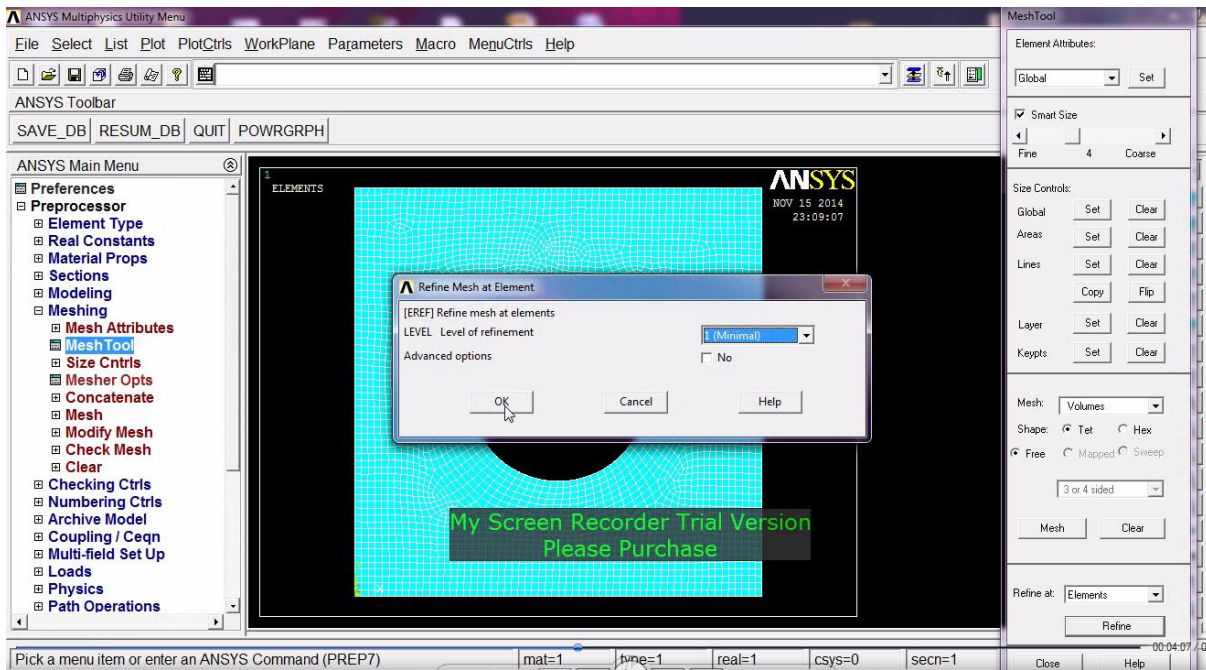
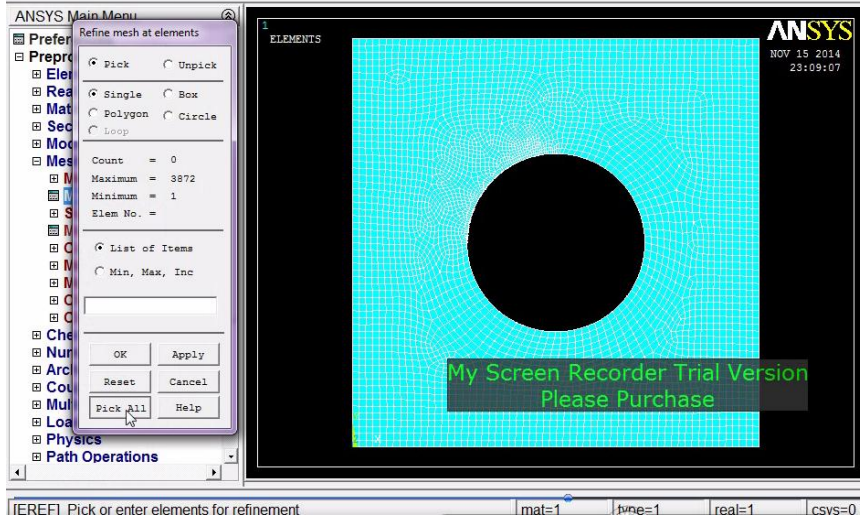




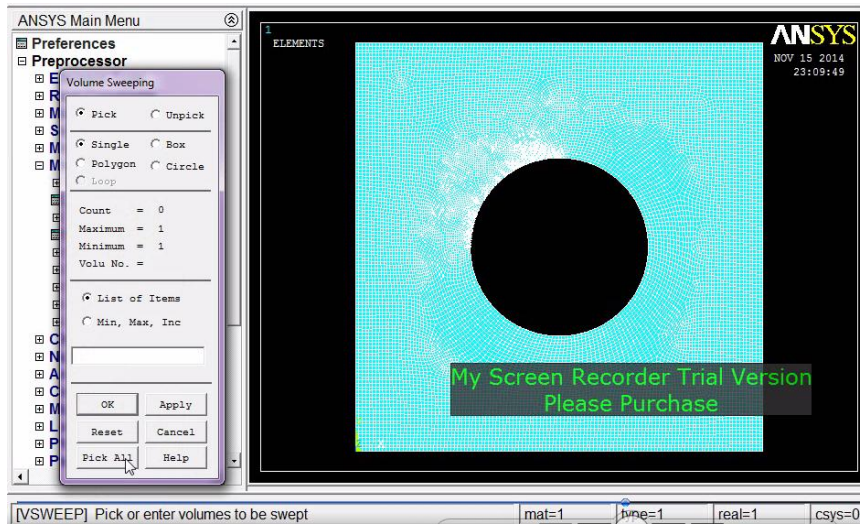
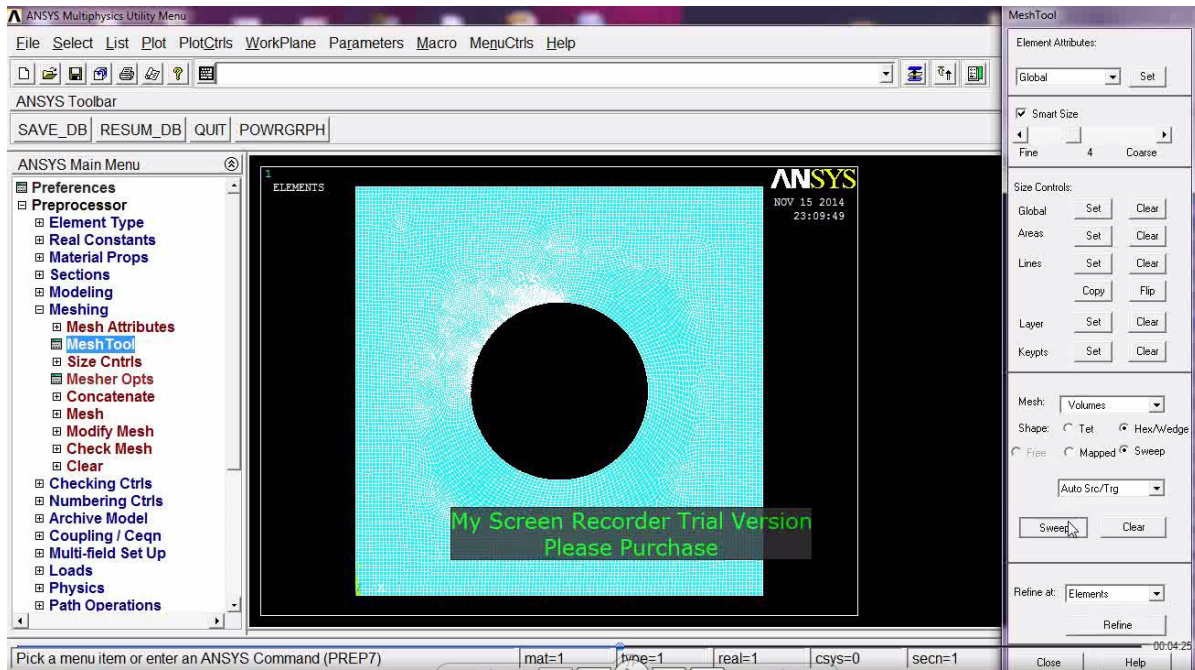


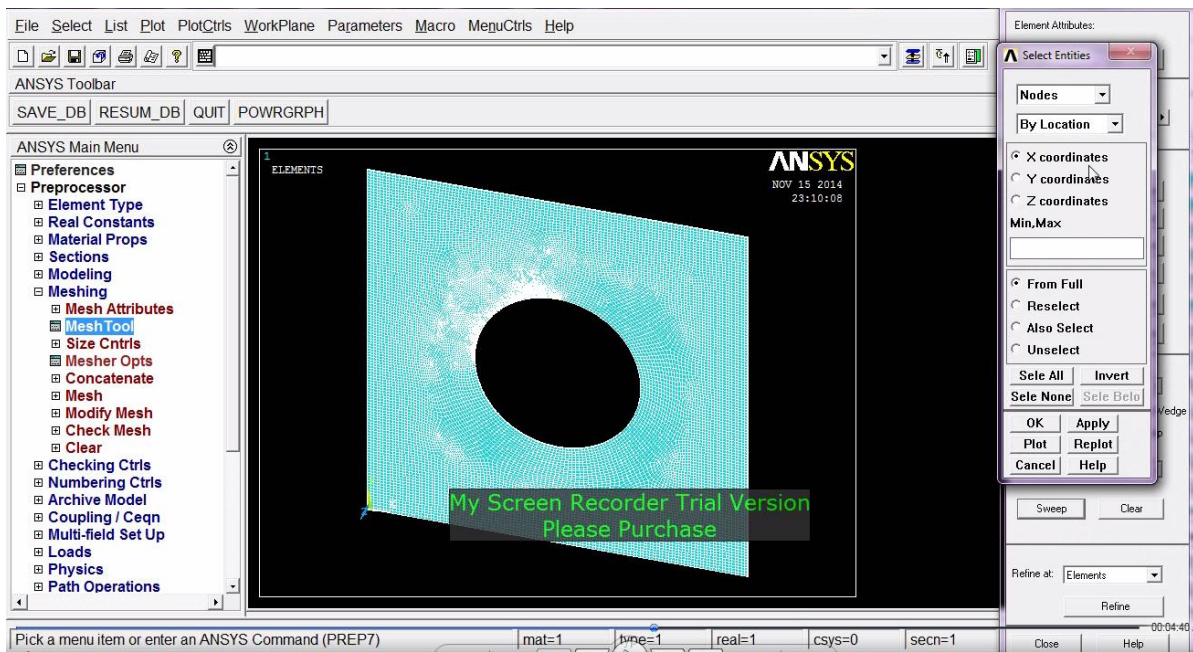
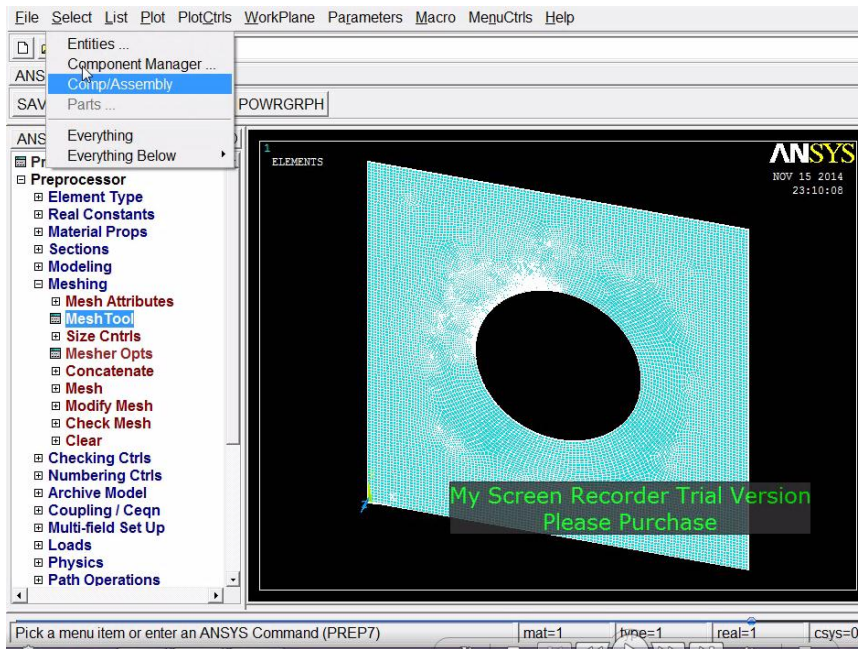


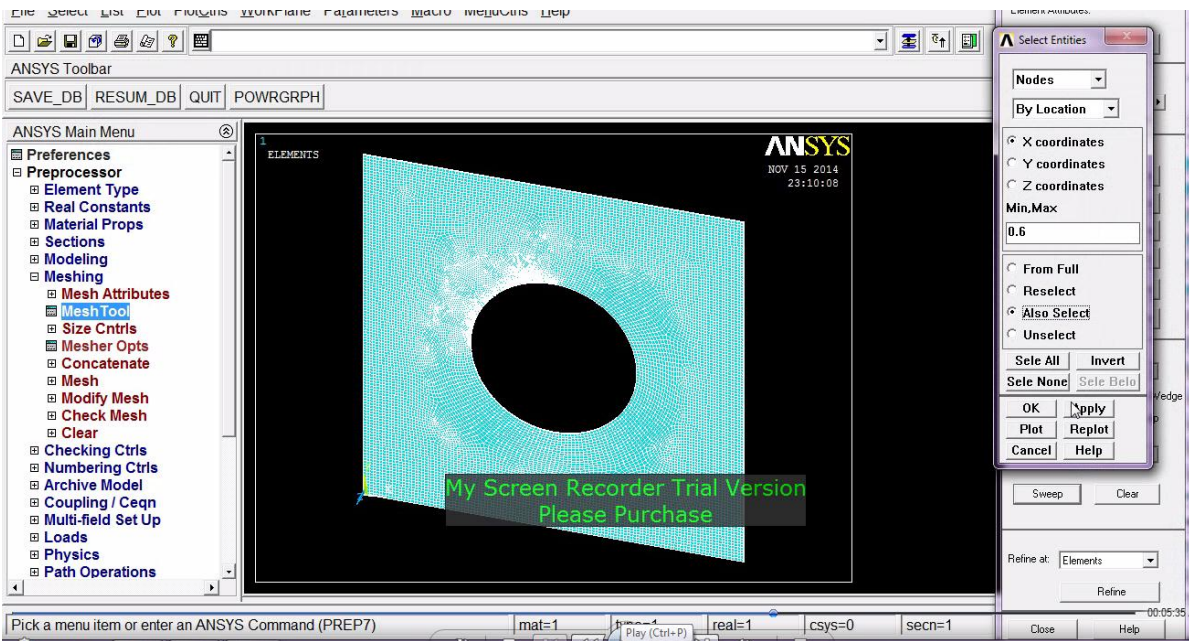
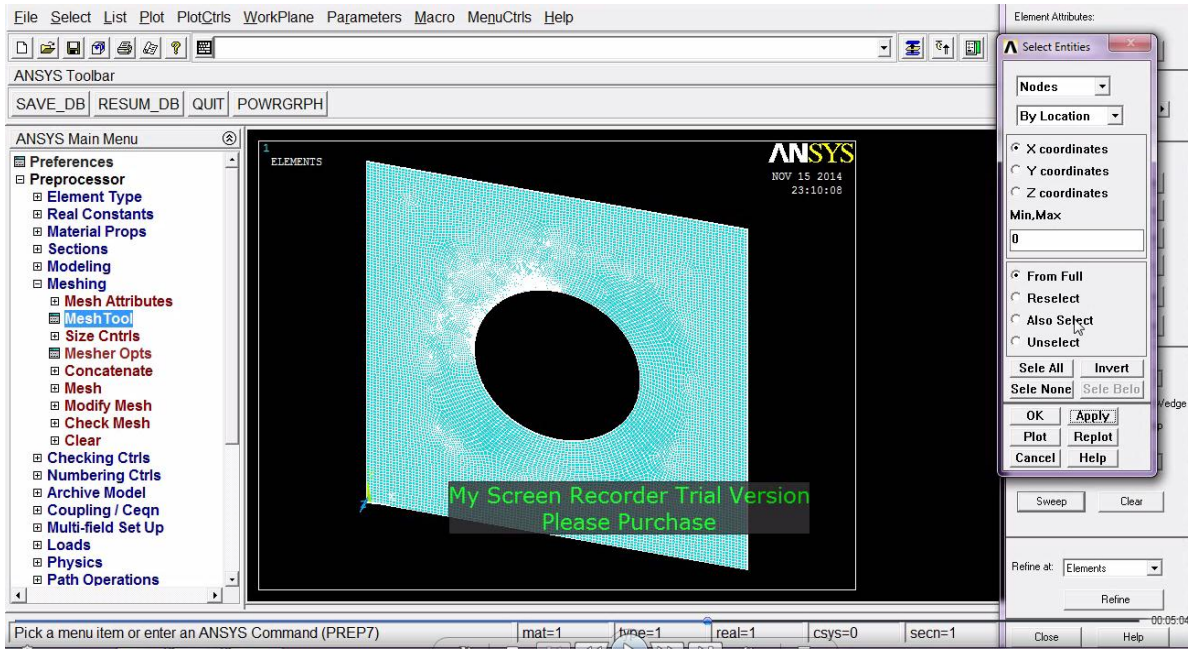




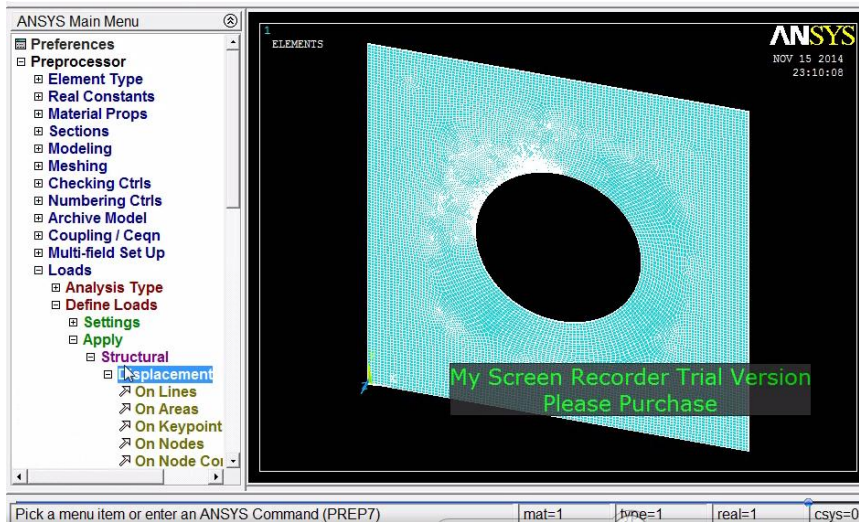
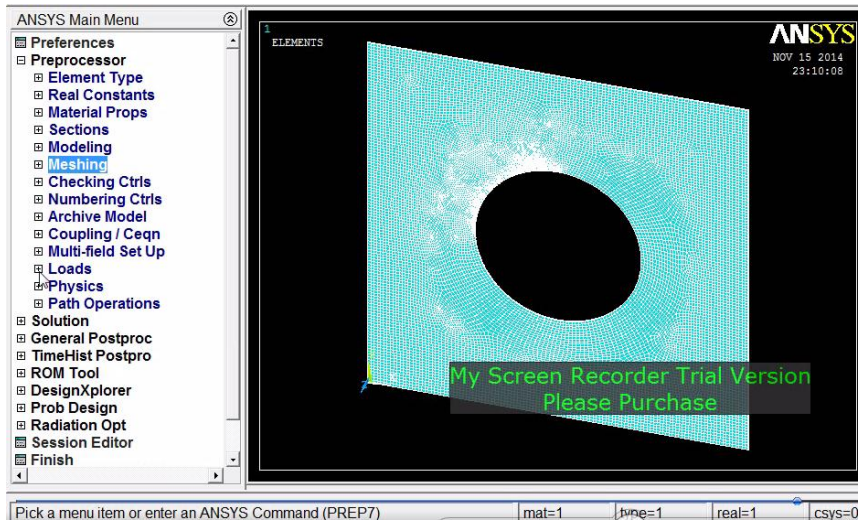
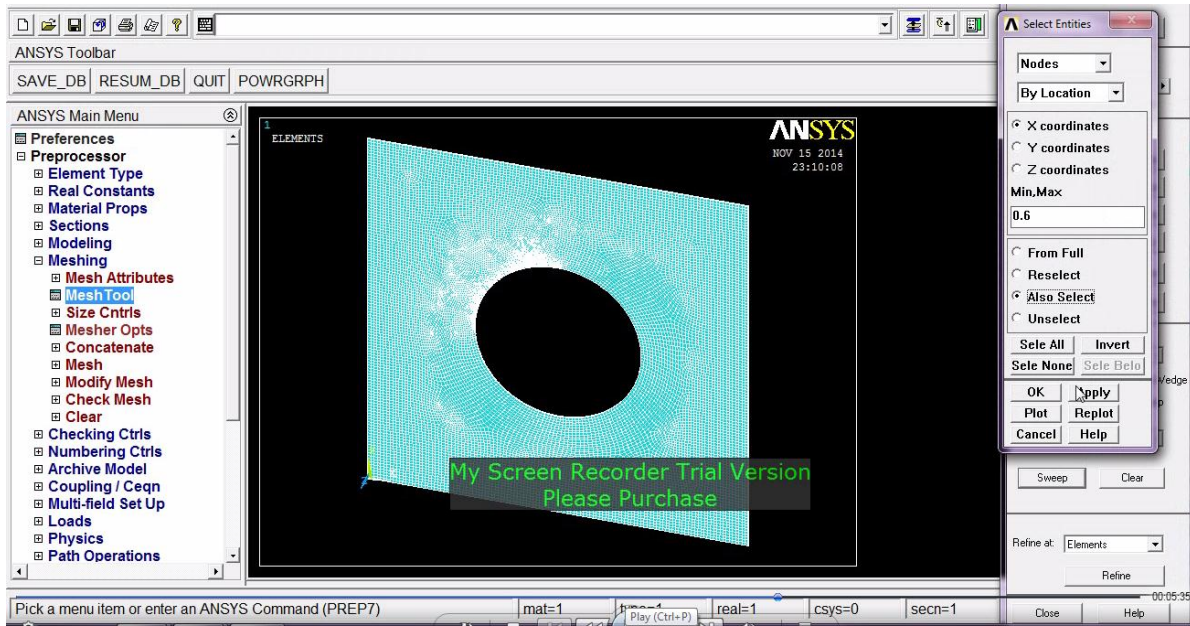




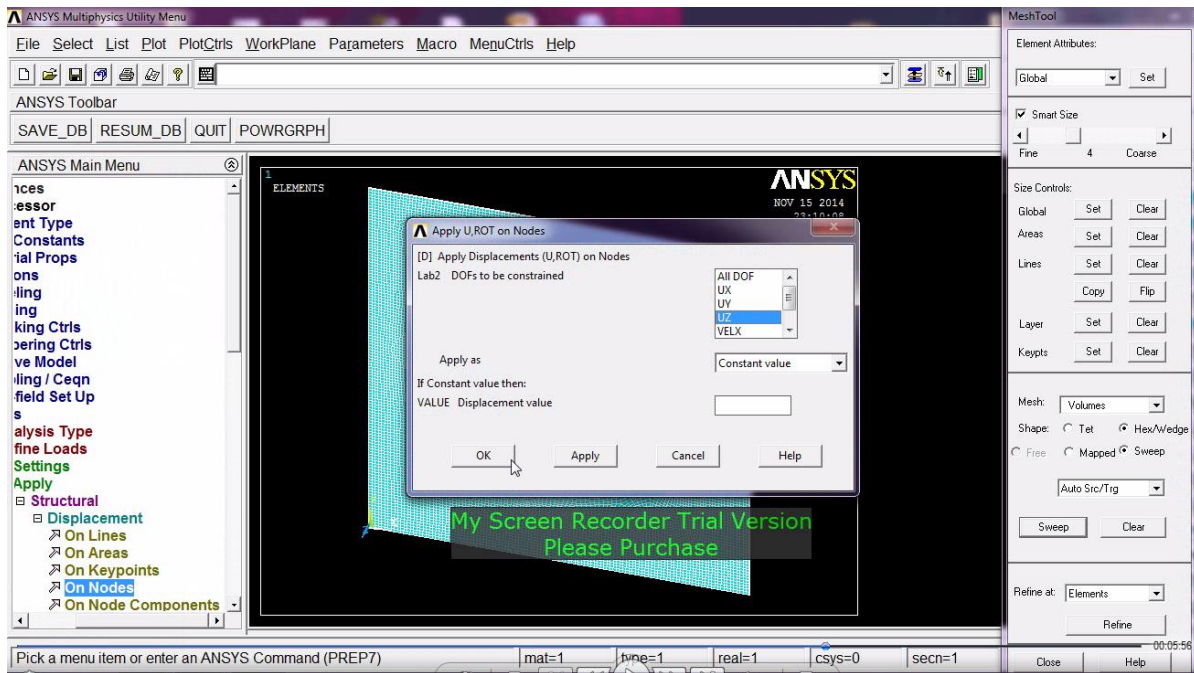
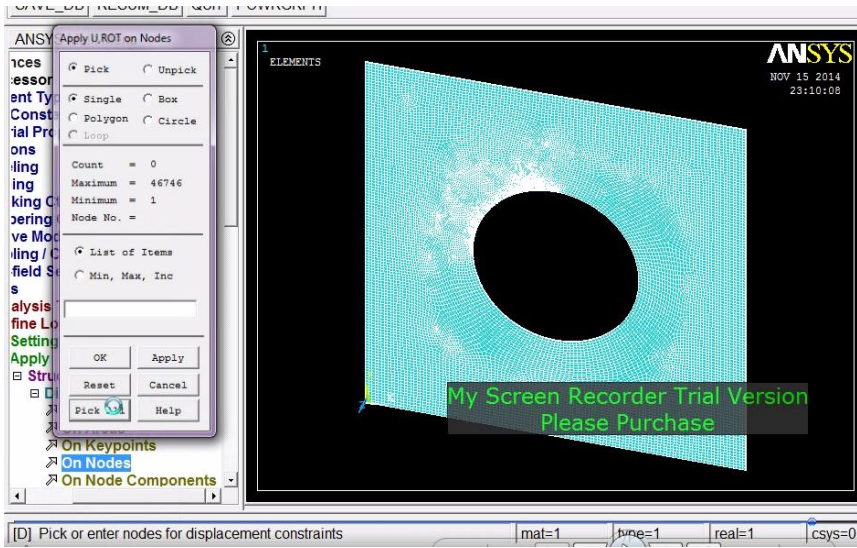




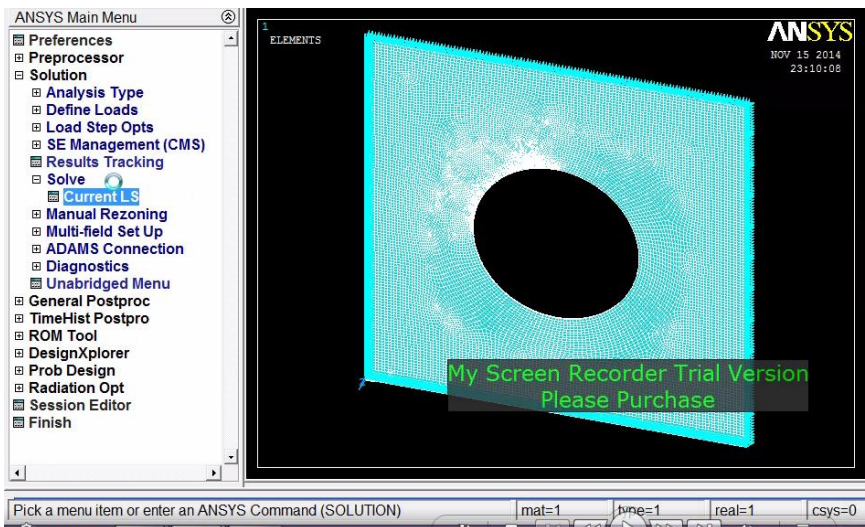
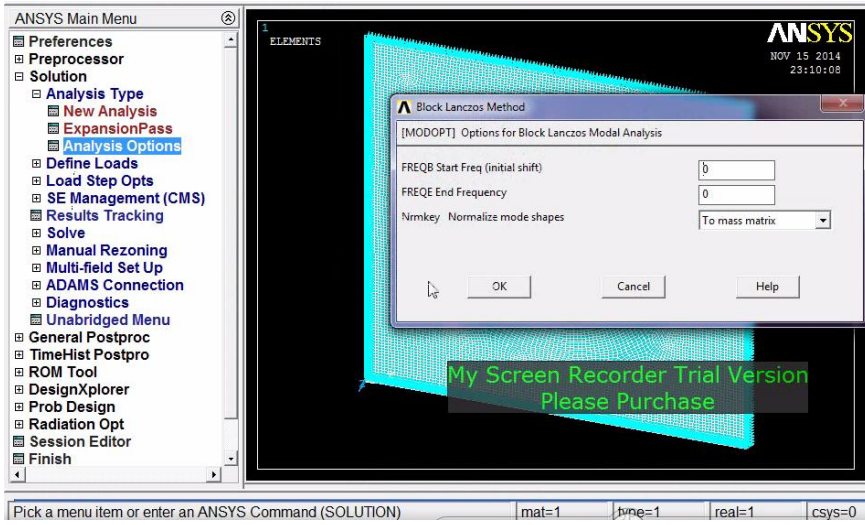
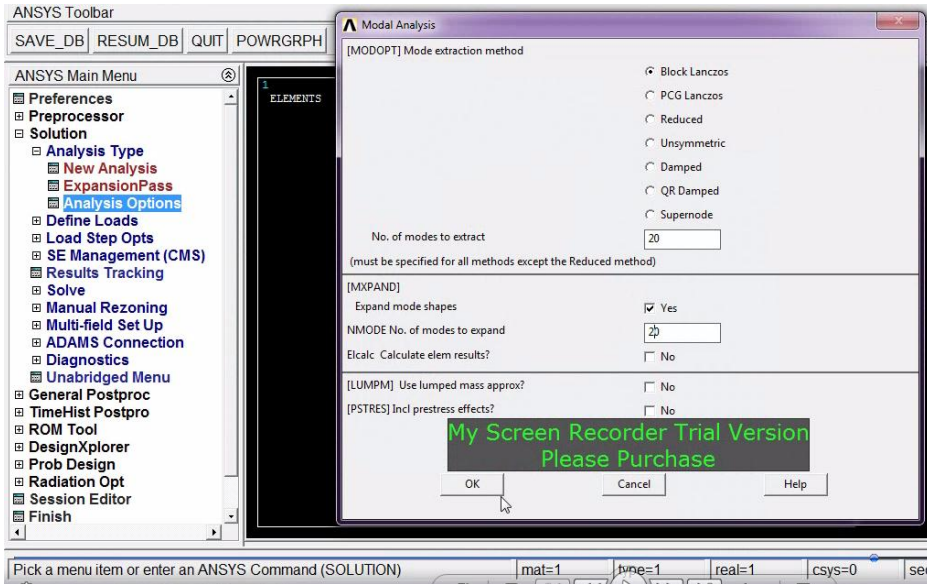




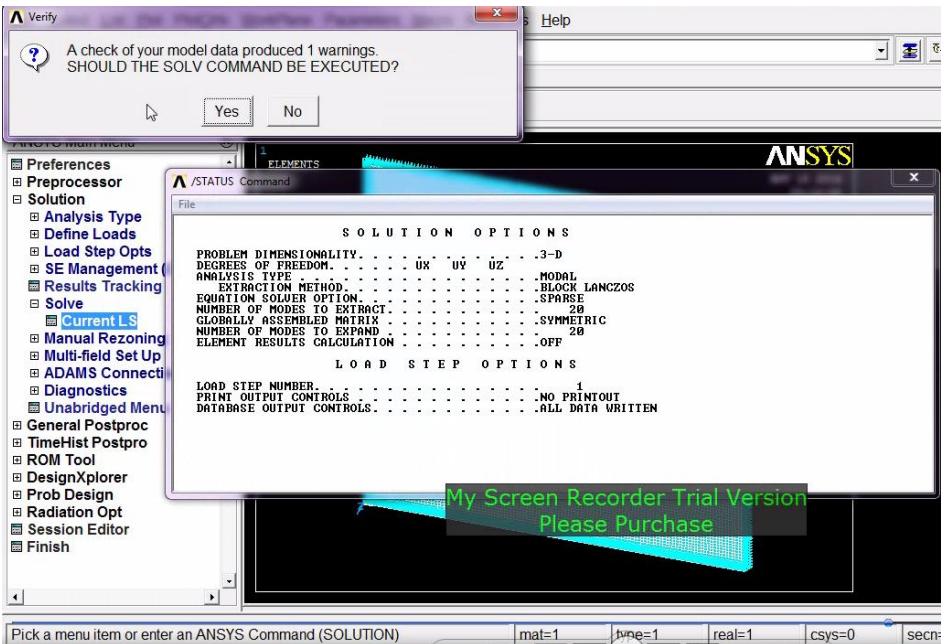
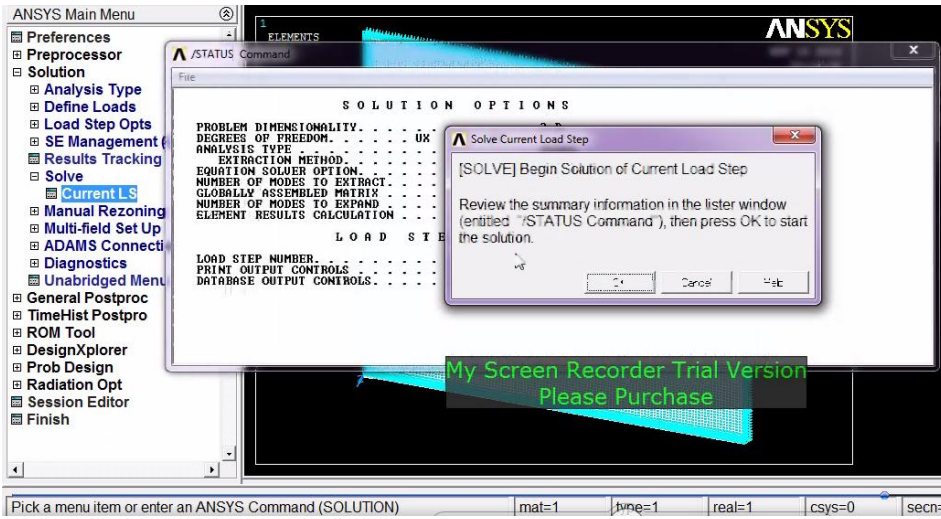


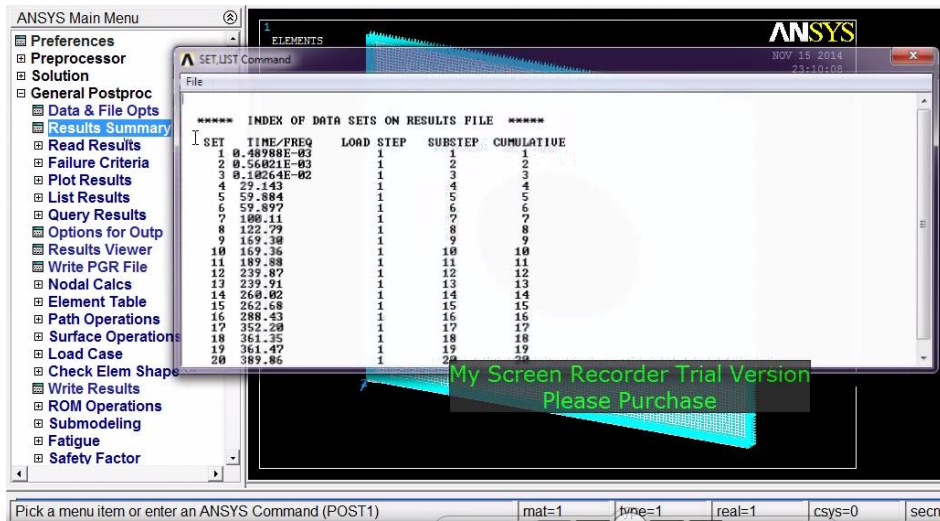
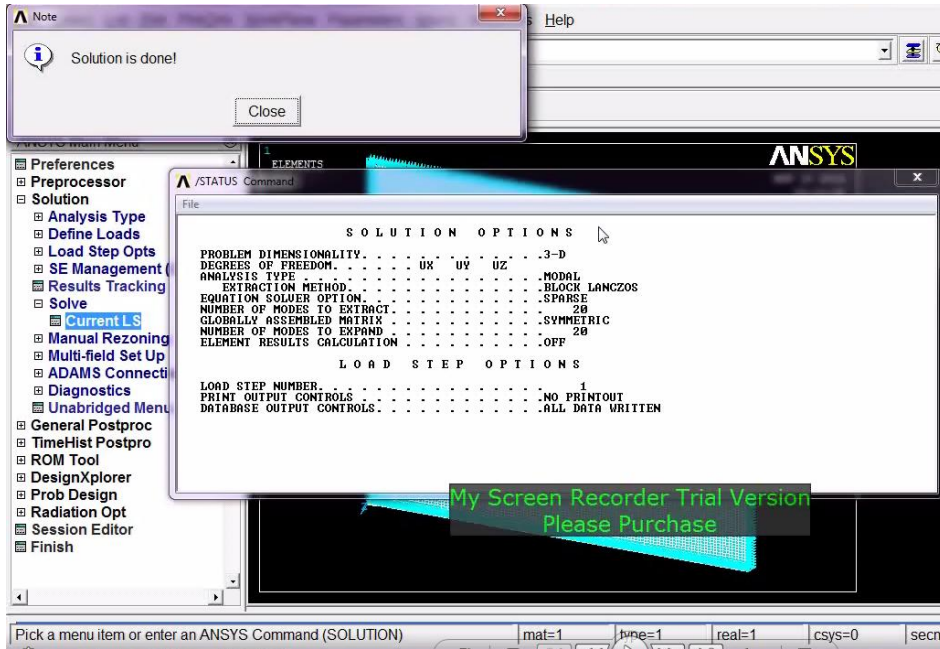










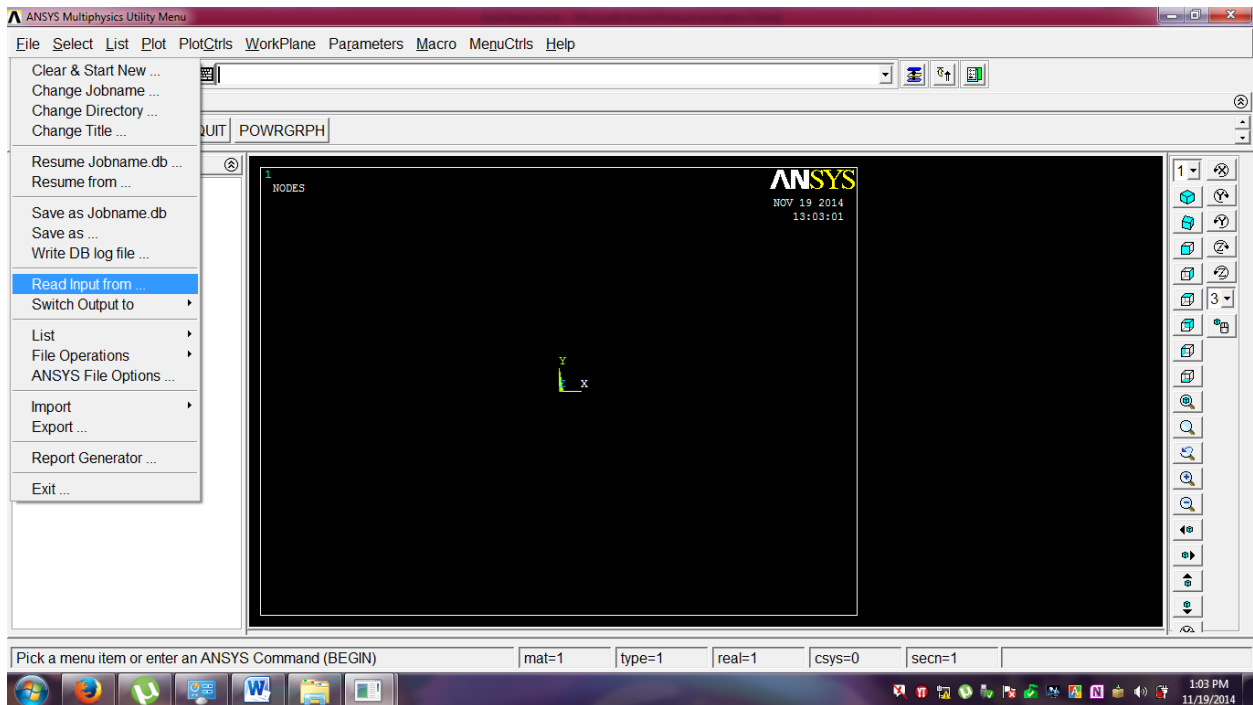


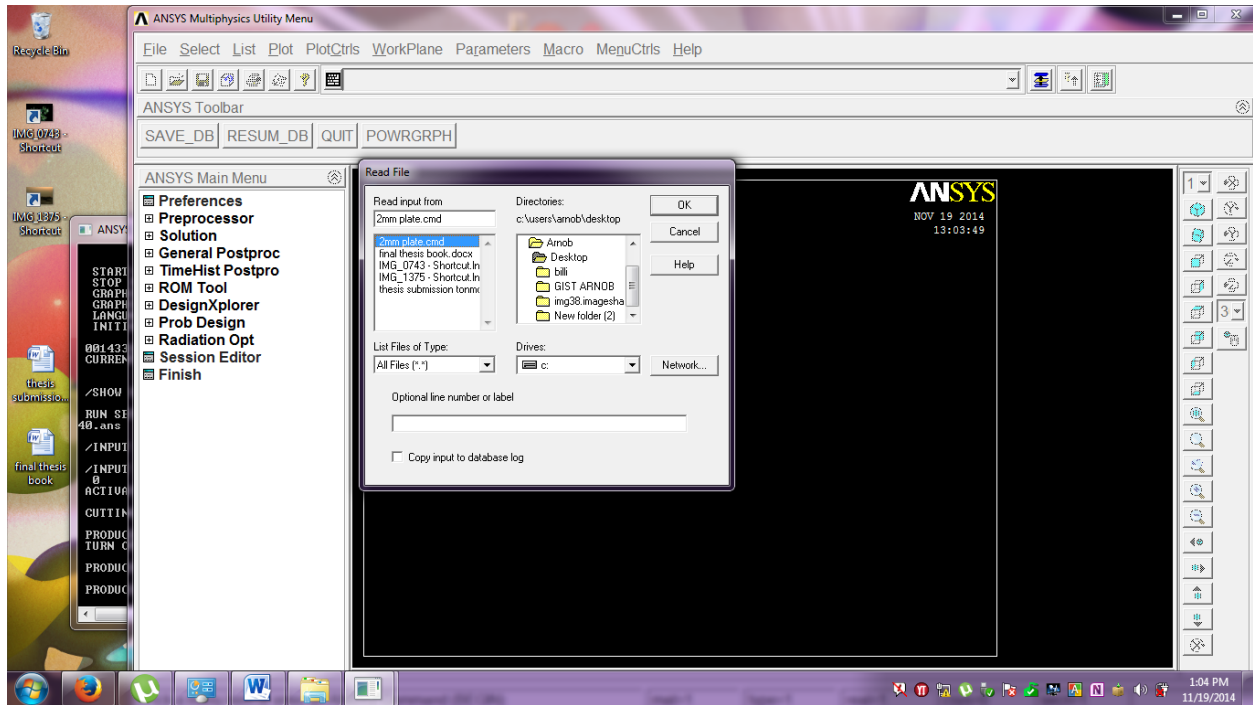
## CODING FOR ANSYS INPUT:

We can input coding in lieu of drawing the geometry.

For 2mm bare simply supported square plate the codings are given below:

Write the codings in notepad and save it in cmd format (eg:NAME.cmd).





CODING:

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For/prep7
keyw,PR_SET,1
keyw,PR_STRUC,1
rho=7850
nu=0.3
modulas=210e09
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et,2,mesh200,6
mp,dens,1,rho
mp,ex,1,modulas
mp,prxy,1,nu
length=0.6
width=0.6
thickness=0.002
rectng,0,0.6,0,0.6
/vup,all,z
type,1
mat,1
extopt,esize,10
vext,all,,,,,0.002
type,2
esize,0.01
amesh,1
mshape,0,3d
mshkey,2
TYPE,    1
MAT,      1
REAL,
ESYS,      0
SECNUM,
!*
SMRT,6

```

```

SMRT,5
SMRT,4
ESIZE,0.01,0,
MSHAPE,0,2D
MSHKEY,0
!*
CM,_Y,AREA
ASEL, , , , 1
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
!*
AMESH,_Y1
!*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!*
CM,_Y,VOLU
VSEL, , , , 1
CM,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,S,_Y
!*
VSWEEP,_Y1
!*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
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NSEL,A,LOC,Y,0.6
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FINISH

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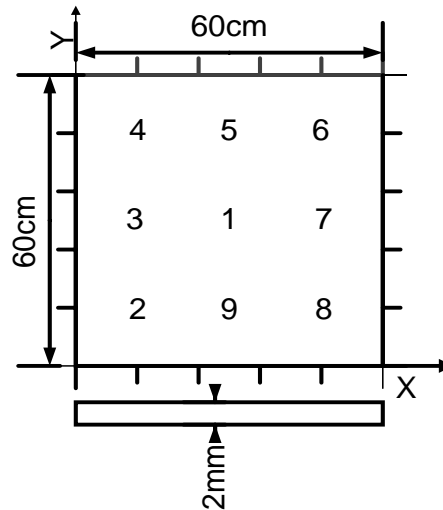


## CHAPTER 4

### RESULTS AND DISCUSSION

#### FOR CUTOUT

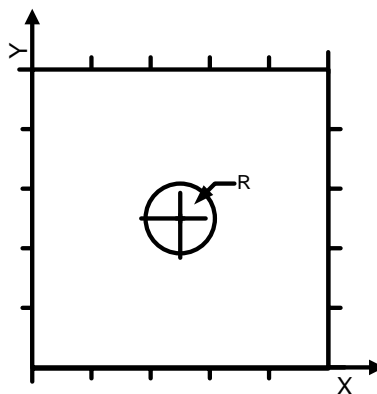
A simply supported square plate made of mild steel was considered with dimensions of  $60\text{cm} \times 60\text{cm} \times 0.2\text{cm}$ . The geometry of the bare plate structure under this analysis is shown in Figure 1 indicating different positions of the plate. The thickness of the plate is constant in the  $z$  direction. The material of the plate was chosen to have the mechanical properties defined by Young's modulus  $210\text{ GPa}$ , density  $7850\text{ Kg/m}^3$  and Poisson's ratio  $0.3$ . A Finite Element Method by ANSYS APDL has been carried out to investigate the natural frequency of bare plate and plate with different sizes and orientations of circular cutouts.



**Figure 3: A simply supported square plate with uniform thickness**

#### **For Case 1:**

A circular cutout is considered at the middle of the plate and the percentage of mass reduction is changed by changing the size of the cutout (cutout circle radius). The natural frequency is compared with the bare plate of different sizes of cutout. (Figure 2).



**Figure 4: Circular Cutout of Various Radius (position 1)**

## Results:

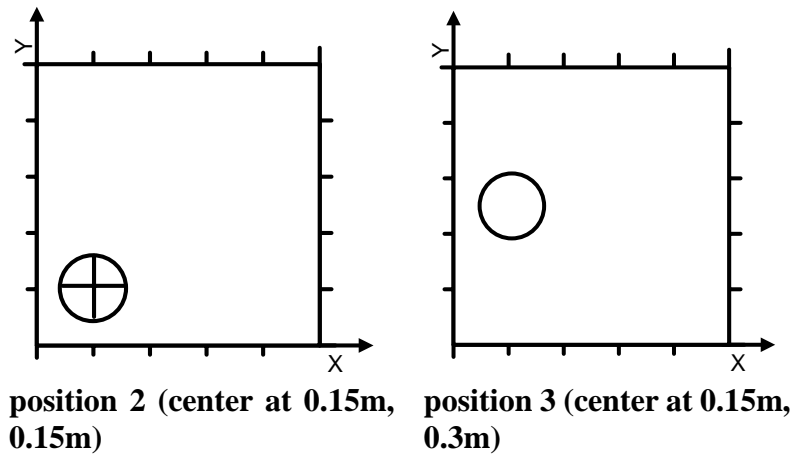
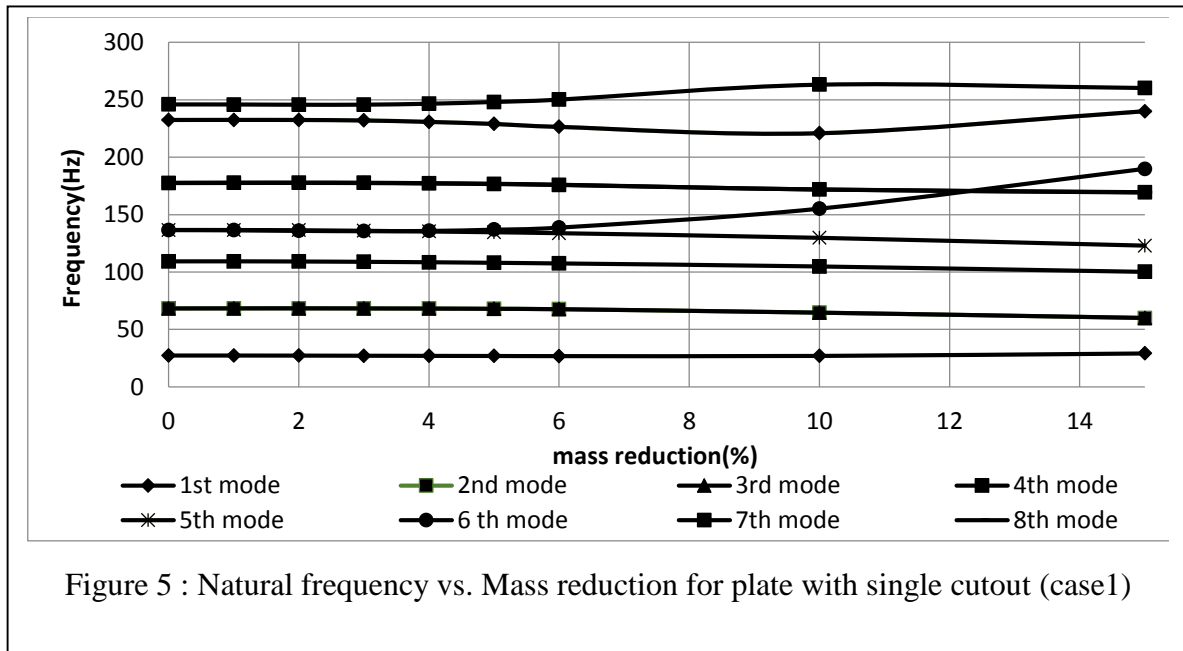
The natural frequency of the plate with single cutout for different percentage of mass reduction is compared with the natural frequency of the bare plate for 11 modes is shown in Table 1 and Figure 7. In this case, mass reduction of the plate by single cutout is considered from 0 to 15% of the bare plate mass, where 0% indicates the bare plate. If the reduction of mass is increased as shown in Figure 7, the natural frequency of the plate deviates from the natural frequency of the bare plate shows visible amount of deviation after 5% of the total mass reduction. For 10% mass reduction, the frequency deviation from the bare plate ranges 1-13.6% where the maximum deviation occurs at 6<sup>th</sup> mode. Again for 15% mass reduction, the frequency deviation from the bare plate ranges 3-38.9% where the maximum deviation occurs also at 6<sup>th</sup> mode.

**Table 1: Natural frequency change due to size of a single cutout at the middle (case1).**

Mode No	Bare Plate	3% mass reduction	% of deviation from bare plate	6% mass reduction	% of deviation from bare plate	10% mass reduction	% of deviation from bare plate	15% mass reduction	% of deviation from bare plate
1 <sup>st</sup>	27.306	27.127	-0.655	26.787	-1.900	26.963	-1.2561	29.153	6.764
2 <sup>nd</sup>	68.288	68.291	0.0043	67.609	-0.994	64.596	-5.4065	59.975	<b>-12.173</b>
3 <sup>rd</sup>	68.288	68.298	0.0146	67.619	-0.979	64.612	-5.3830	59.958	<b>-12.198</b>
4 <sup>th</sup>	109.25	108.85	-0.366	107.45	-1.647	104.84	-4.036	100.15	<b>-8.329</b>
5 <sup>th</sup>	136.64	135.94	-0.512	133.81	-2.068	129.75	-5.042	122.89	<b>-10.062</b>
6 <sup>th</sup>	136.64	135.65	-0.724	138.75	1.547	155.23	<b>13.605</b>	189.83	<b>38.927</b>
7 <sup>th</sup>	177.59	177.62	0.0168	175.90	-0.947	171.82	-3.249	169.40	-4.608
8 <sup>th</sup>	177.59	177.64	0.0281	175.94	-0.926	171.89	-3.209	169.35	-4.636
9 <sup>th</sup>	232.42	232.03	-0.167	226.32	-2.623	220.87	-4.969	239.96	3.244
10 <sup>th</sup>	232.42	231.98	-0.189	226.42	-2.580	220.89	-4.960	240.02	3.269
11 <sup>th</sup>	245.9	245.69	-0.085	250.07	1.697	263.2	<b>7.035</b>	260.26	5.839

**For Case 2:**

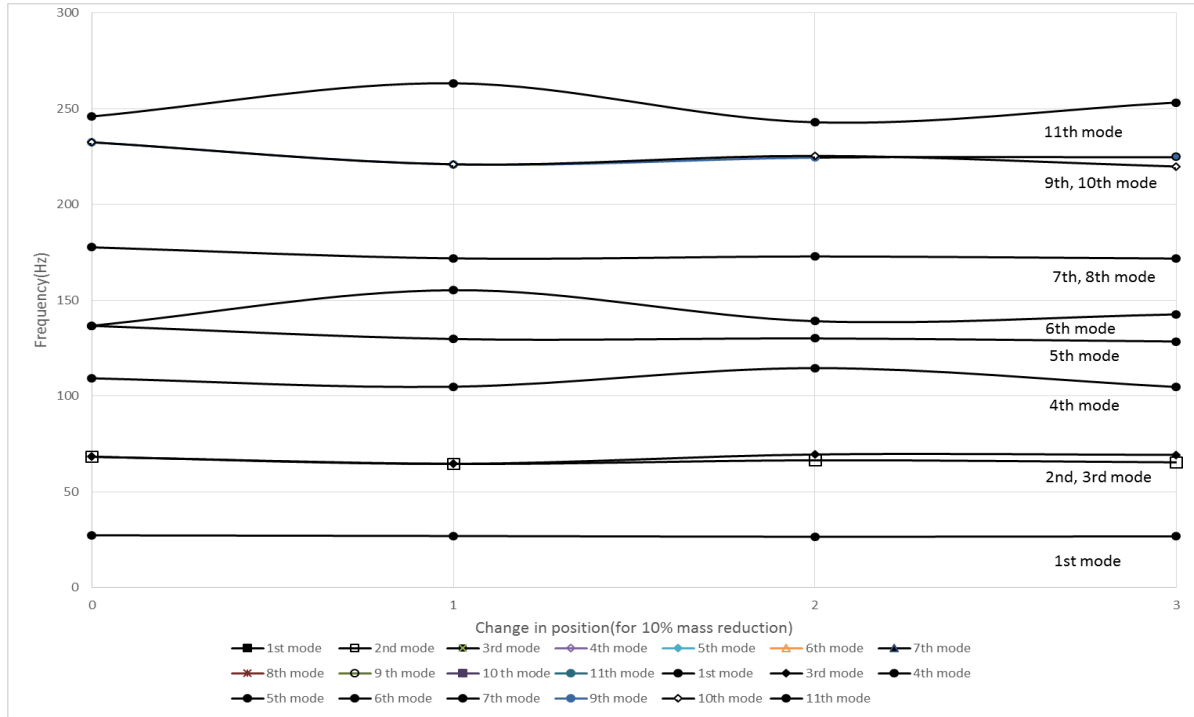
The single circular cutout of constant radius is placed into several positions on the plate as shown in Figure 3.



**Figure 6: Circular Cutout at Various Positions**

## Results:

Cutout at position 1 is now changes to position 2 and position 3 (Figure 3) to observe the change of natural frequency compared to bare plate and case1. Mass reduction is considered in this case only for 10% & 15 %.



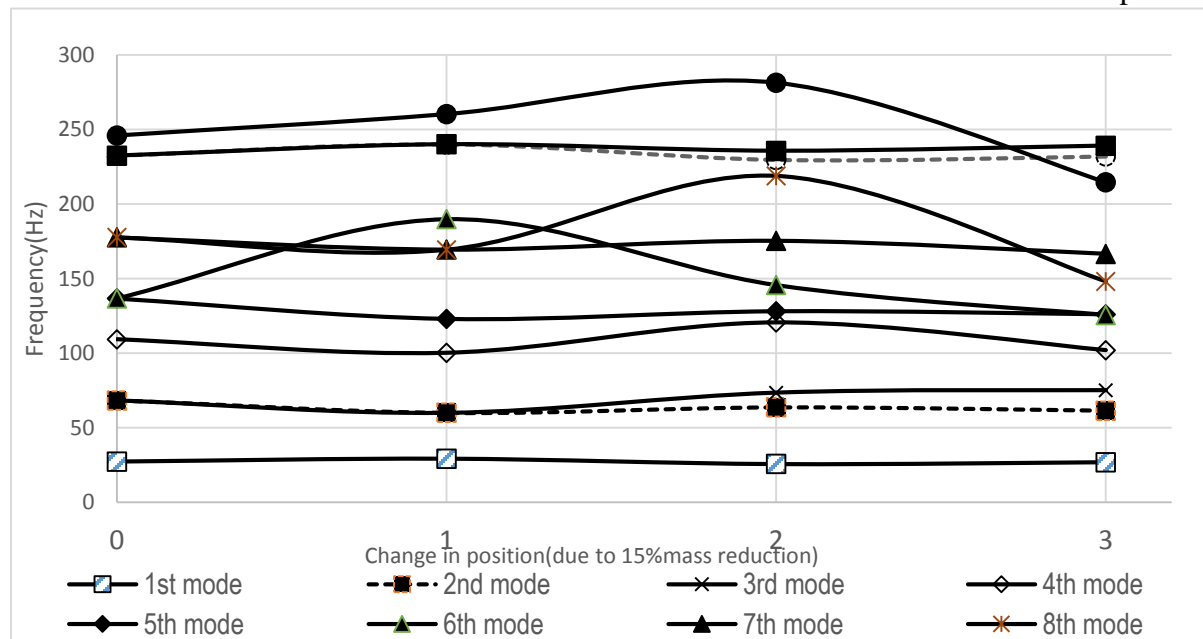
**Figure 7:** Natural Frequency vs. Position of Cutout (for 10% mass reduction)

**Table 1: Natural Frequency of 10% Mass Reduction Due to Change in Position**

For cutout with 10% reduction of the total mass, the deviation is between 1-13.6% when it is situated at position 1 (Table 3). The % of deviation shows between 1-5.68 for position 2 which is much less than position 1(case 1). The maximum deviation which was at 6<sup>th</sup> mode in position 1 is now shifted to 8<sup>th</sup> mode. The % of deviation shows between 1-8.1% for position 3 which is much less than position 1 (case 1). The maximum deviation which was at 6<sup>th</sup> mode in position 1 is now

Mode No.	Natural Frequency (bare plate)	Natural Frequency for cutout at Position 1	% of deviation from bare plate	Natural Frequency for cutout at Position 2	% of deviation from bare plate	Natural Frequency for cutout at Position 3	% of deviation from bare plate
1 <sup>st</sup>	27.306	26.963	-1.2561	26.545	-2.7869	26.792	-1.8827
2 <sup>nd</sup>	68.288	64.596	-5.4065	66.464	-2.6710	65.386	-4.2496
3 <sup>rd</sup>	68.288	64.612	-5.3830	69.485	1.7528	69.271	1.4394
4 <sup>th</sup>	109.25	104.84	-4.0366	114.56	4.8604	104.7	-4.1647
5 <sup>th</sup>	136.64	129.75	-5.0424	130.05	-4.8228	128.46	-5.9865
6 <sup>th</sup>	136.64	155.23	<b>13.6050</b>	139.03	1.7491	142.59	4.3545
7 <sup>th</sup>	177.59	171.82	-3.2490	172.82	-2.6859	171.73	-3.2997
8 <sup>th</sup>	177.59	171.89	-3.2096	187.68	<b>5.6816</b>	192.01	<b>8.1198</b>
9 <sup>th</sup>	232.42	220.87	-4.9694	224.42	-3.4420	224.58	-3.3732
10 <sup>th</sup>	232.42	220.89	-4.9608	225.31	-3.0591	219.75	-5.4513
11 <sup>th</sup>	245.9	263.2	<b>7.0353</b>	242.82	-1.2525	253.16	2.9524

shifted to 8<sup>th</sup> mode. The data shows for that the maximum deviation increases than position 2.



**Figure 8:** Natural Frequency VS Change of Position of Cutout (for 15% mass reduction)

**Table 2: Natural Frequency of 15% Mass Reduction Due to Change in Position**

Mode No	Natural Frequency (bare plate)	Natural Frequency for cutout at Position 1	% of deviation	Natural Frequency for cutout at Position 2	% of deviation	Natural Frequency for cutout at Position 3	% of deviation
1st	27.306	29.153	6.7640	25.656	-6.042	26.86	-1.633
2nd	68.288	59.975	-12.173	63.616	-6.841	61.459	-10.000
3rd	68.288	59.958	-12.198	73.452	7.562	75.085	9.953

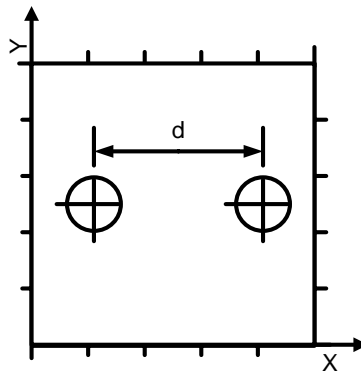
4th	109.25	100.15	-8.329	120.53	10.325	101.92	-6.709
5th	136.64	122.89	-10.062	128.06	-6.2792	126	-7.786
6th	136.64	189.83	<b>38.927</b>	145.62	6.572	125.61	-8.072
7th	177.59	169.406	-4.608	175.35	-1.261	166.6	-6.188
8th	177.59	169.356	-4.636	218.89	<b>23.255</b>	148.15	<b>-16.577</b>
9th	232.42	239.96	3.244	229.62	-1.204	231.91	-0.219
10th	232.42	240.02	3.2699	235.67	1.398	239.11	2.878
11th	245.9	260.26	5.839	281.3	14.396	214.63	-12.716

For cutout with 15% reduction, the deviation is between 3.24-38.92% when it is situated at position 1 (Table 4). The % of deviation shows between 1.2-23.25 for position 2 which is much less than position 1(case 1). The maximum deviation which was at 6<sup>th</sup> mode in position 1 is now shifted to 8<sup>th</sup> mode.

The % of deviation shows between 1-8.1 for position 3 which is much less than position 1(case 1). The maximum deviation which was at 6<sup>th</sup> mode in position 1 is now shifted to 8<sup>th</sup> mode. The data shows for that the maximum deviation increases than position 2.

### For Case 3:

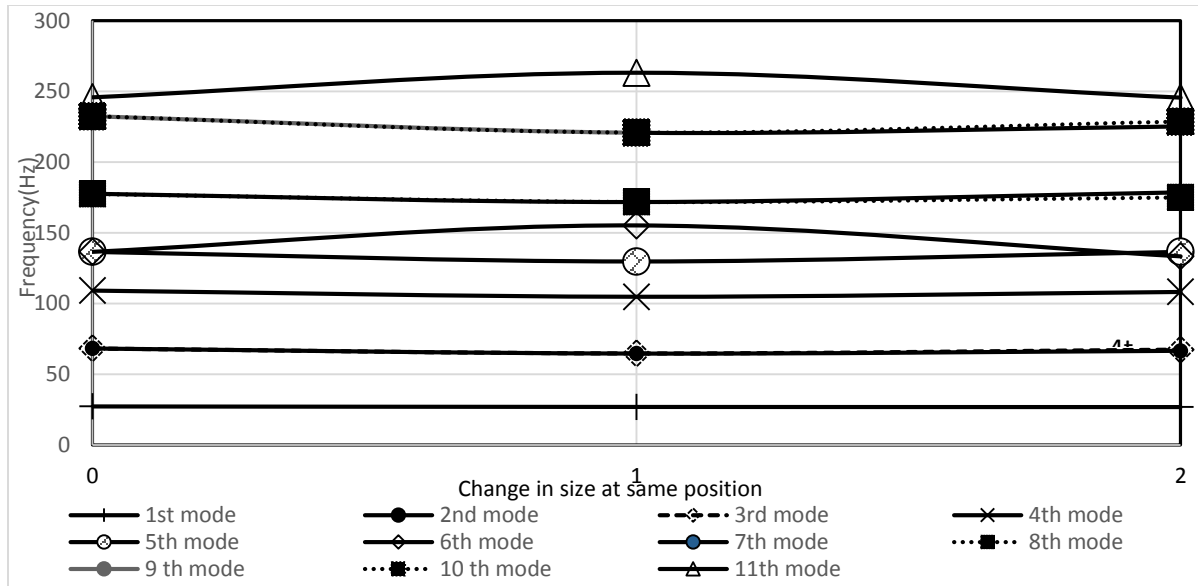
Same amount of cutout of case 1 is equally distributed into two cutouts on the plate with a distance  $d$ . Then the natural frequency is compared with the bare plate's natural frequency and with the natural frequency of plate with single cutout (Figure 4).



**Figure 4: Circular cutout distributed equally at two positions.**

### Results:

Cutout at position 1 (case 1) is now divided into two cutouts to observe the change of natural frequency compared to bare plate. Mass reduction is considered in this case only for 10% & 15%, which show significant deviation of natural frequency from bare plate.



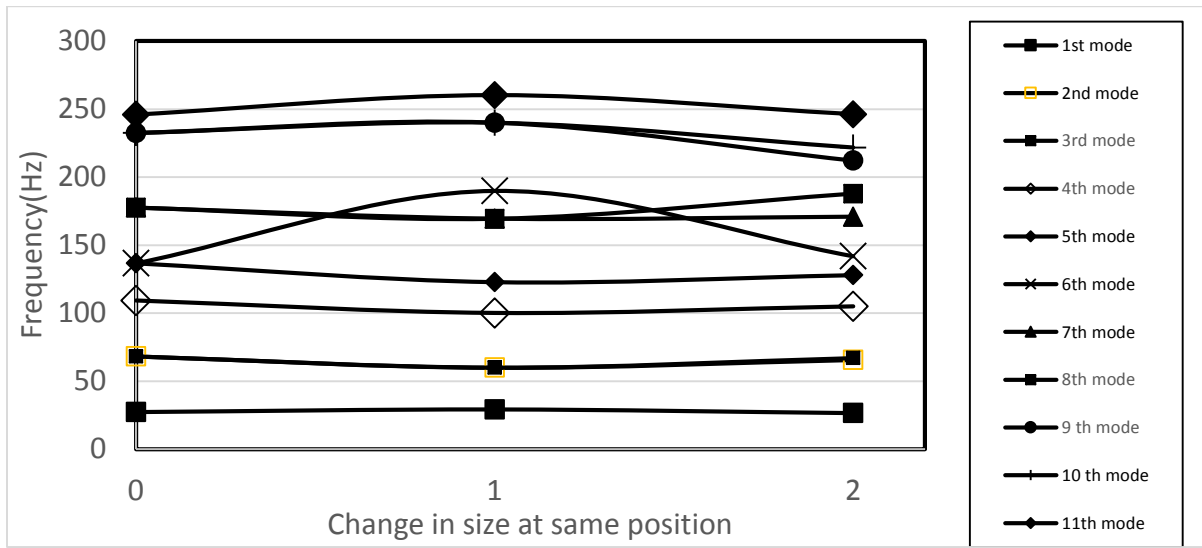
**Figure 9: Natural Frequency VS Distribution of Mass Reduction (for 10% mass reduction)**

For single cutout with 10% reduction, the deviation is between 1.2-13.6% when it is situated at position 1 (Figure 3). The % of deviation shows between 0.1-3.04 for 2

Mode no.	Natural frequency for bare plate	Natural frequency for single cutout (case 1)	% of deviation	Natural frequency for two cutout	% of deviation
1 <sup>st</sup>	27.306	26.963	-1.25613	26.866	-1.61137
2 <sup>nd</sup>	68.288	64.596	-5.40651	66.611	-2.45578
3 <sup>rd</sup>	68.288	64.612	-5.38308	67.467	-1.20226
4 <sup>th</sup>	109.25	104.84	-4.03661	108.27	-0.89703
5 <sup>th</sup>	136.64	129.75	-5.04245	136.66	0.014637
6 <sup>th</sup>	136.64	155.23	<b>13.60509</b>	133.47	-2.31996
7 <sup>th</sup>	177.59	171.82	-3.24906	178.55	0.540571
8 <sup>th</sup>	177.59	171.89	-3.20964	175.1	-1.40211
9 <sup>th</sup>	232.42	220.87	-4.96945	225.34	<b>-3.04621</b>
10 <sup>th</sup>	232.42	220.89	-4.96085	228.63	-1.63067
11 <sup>th</sup>	245.9	263.2	7.03538	245.69	-0.0854

**Table 3: Natural frequency Due to Distribution of cutout (10%)**

Cutout which is much less than single cutout (case 1). The maximum deviation which was at 6<sup>th</sup> mode at the middle is now shifted to 9<sup>th</sup> mode for two cutout. In this case the data shows that the deviation is less than the shifting the position of the cutout. For single cutout with 15% reduction, the deviation is between 3.2-38.9% when it is situated at position 1 (Figure 3). The % of deviation shows between 0.13-8.71% for 2 cutout which is much less than single cutout (case 1). The maximum deviation which was at 6<sup>th</sup> mode at the middle is now shifted to 9<sup>th</sup> mode for two cutout. In this case the data shows that the deviation is less than the shifting the position of the cutout.



**Figure 10: Natural Frequency VS Distribution of Mass Reduction (15% mass reduction)**

**Table 4: Natural frequency Due to Distribution of cutout (15%)**

Mode no.	Natural frequency for bare plate	Natural frequency for single cutout	% deviation from bare plate	Natural frequency for double	% deviation from bare plate
1	27.306	29.153	6.764081	26.567	-2.70636
2	68.288	59.975	-12.1734	65.663	-3.84401
3	68.288	59.958	-12.1983	67.052	-1.80998
4	109.25	100.15	-8.32952	104.97	-3.91762
5	136.64	122.89	-10.0629	128	-6.32319
6	136.64	189.83	<b>38.92711</b>	141.83	3.798302
7	177.59	169.406	-4.60837	170.87	-3.784
8	177.59	169.356	-4.63652	187.87	5.788614
9	232.42	239.96	3.244127	212.17	<b>-8.71268</b>
10	232.42	240.02	3.269942	221.74	-4.59513
11	245.9	260.26	5.839772	246.22	0.130134

**For Case 4:**

The single circular cutout of case 1 is distributed equally into three, four and five different positions of the plate (Figure 5).



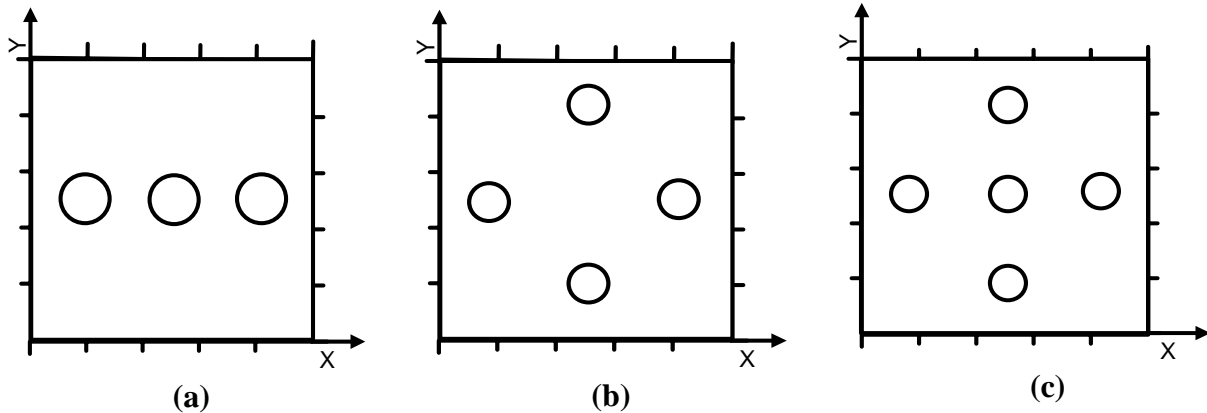


Figure 5: Circular cutout distributed equally at different positions.

Cutout at position (case 1) is now divided into 2,3 and 4 circular cutouts to observe the change of natural frequency compared to bare plate of case1. Mass reduction are considered in this case only for 10%. For single cutout the deviation is between 1.2-13.6% when it is situated at position 1 (Table 7). The % of deviation shows between 0.1-3.04 for 2 cutout which is much less than single cutout (case 1). The % of deviation shows between 0.03-1.2 for 4 cutouts which is much less than 3 cutouts. It shows that the more the no of cut the deviation becomes lower. The maximum deviation which was at 6<sup>th</sup> mode for single cutout is now shifted to 5<sup>th</sup> mode for four cutouts.

**Table 5: Natural frequency Due to Distribution of cutout (10%)**

Mode no	Bare plate	1 cutout (case 1)	% of deviation from bare	2 cutout	% of deviation from bare	3 cutout	% of deviation from bare plate	4 cutout	% of deviation from	5 cutout	% of deviation from bare
1	27.3	26.96	-1.25	26.86	1.61	26.85	1.66	27.6	-1.07	27.05	0.926
2	68.28	64.59	-5.41	66.6	2.46	67.1	1.78	69.23	-1.388	67.74	0.789
3	68.29	64.61	-5.38	67.46	1.202	67.96	0.47	69.016	-1.06	67.77	0.758
4	109.23	104.8	-4.036	108.2	0.897	108.5	0.68	111.6	-2.15	109.2	0.045
5	136.6	129.7	-5.042	136.66	-0.01	134.67	1.441	138.31	-1.22	134.89	<b>1.280</b>

6	136.6	155.2	<b>13.60</b>	133.4	2.319	134.1	<b>1.836</b>	139.12	-1.81	135.3	0.966
7	177.59	171.8	-3.25	178.6	-0.54	176.3	0.755	180.3	-1.52	176.5	0.613
8	177.59	171.8	-3.21	175.1	1.402	176.5	0.591	180.75	-1.78	176.54	0.591
9	232.4	220.8	-4.96	225.3	<b>3.046</b>	229.8	1.118	236.88	-1.92	232.2	0.086
10	232.4	220.9	-4.96	228.6	1.630	230.5	0.8174	238.35	- <b>2.55</b>	232.33	0.038
11	245.9	263.2	7.035	245.69	0.085	244.9	0.378	251.03	-2.08	244.57	0.54

### For Case 5:

The distributed three and five cutouts of case 4 are relocated into different orientation of the plate

### Results:

Previously we saw, for three circular cutout as shown in figure 5(a), the maximum frequency deviation was 1.83% (6<sup>th</sup> mode). At the time we change the orientation of these cutout as shown in figure 6(a) the maximum frequency deviation reduces to 1.59%. So the frequency become closer to the natural frequency of the bare plate. But the maximum deviation changes its mode. Now it occurs at first mode.

Again when the orientation of five circular cutout is changed as shown in figure 6(b). We can see that the maximum frequency deviation does not change more. It just changes its mode. Previously the maximum deviation occurs at fifth mode now it shifts to fourth mode.

**Table 6: Natural Frequency Due to Orientation of cutout (10%)**

Mode NO	Bare plate	3 cutout	% of deviation from bare plate	5 cutout	% of deviation from bare plate
1	27.306	26.871	<b>-1.59306</b>	27.095	-0.77272
2	68.288	67.33	-1.40288	67.832	-0.66776

3	68.288	67.993	-0.43199	67.882	-0.59454
4	109.25	107.57	-1.53776	107.85	<b>-1.28146</b>
5	136.64	134.99	-1.20755	136.13	-0.37324
6	136.64	134.67	-1.44174	135.16	-1.08314
7	177.59	176.16	-0.80523	176.72	-0.48989
8	177.59	176.89	-0.39417	176.76	-0.46737
9	232.42	230.22	-0.94656	232.28	-0.06024
10	232.42	230.78	-0.70562	232.31	-0.04733
11	245.9	244.51	-0.56527	244.69	-0.49207

## Conclusion:

Investigation has been done for single cutout, change of position of single cutout and double (distributed) cutout. The findings can be summarized as bellow:

- 1) Mass reduction (cutouts) affects the natural frequency of plate mainly after a certain value and significant above the 5%.
- 2) No single position shows the minimum division of natural frequency from the bare plate for all modes. No of cutout also affects the natural frequency of the plate, although the amount of mass reduction is same for single cut at the middle.
- 3) Increases in the no of cutout with this specified orientation (Figure 5) decreases the deviation of the natural frequency.
- 4) Validation has been made for this simulation for single cut and has been found in good agreement.
- 5) Limitation: Different orientation of circular cutout change the natural frequency of the plate. In this literature we have analyzed only for two different orientation. Other orientations show the significant change of natural frequency.

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