

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



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PEAK TO AVERAGE POWER RATIO REDUCTION IN MULTICARRIER TRANSMISSION

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ABSTRACT

Orthogonal Frequency Division Multiplex (OFDM) is a very suitable technique for achieving high-rate wireless data transmission. High spectral efficiency, robustness to channel fading, immunity to impulse interference and less nonlinear distortion are among the favorite properties of OFDM.

One major drawback of OFDM is high Peak to Average Power Ratio (PAPR) of the transmitted signal. The high peaks of an OFDM signal are distorted nonlinearly by the high power amplifier (HPA) because the HPA heavily distorts all signal parts that come close to or exceed saturation. The distortion causes inter carrier interference (ICI) and out-of-band (OOB) radiation. While ICI disturbs the transmitted signal, it degrades the bit error rate (BER). OOB radiation disturbs signals on adjacent frequency bands, so it should be avoided.

A PAPR reduction scheme based on joint Hadamard transform and Discrete Cosine Transform (DCT) with selected mapping (SLM) is proposed. The PAPR reduction performance and BER performance are evaluated by computer simulation. Simulation results state that the PAPR reduction performance is improved compared with selected mapping used only. On the other hand, the BER of system using proposed PAPR reduction scheme is not degraded.

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LIST of ACRONYMS

ACE	Active Constellation Extension
ACF	Additive Corrective Function
ACI	Adjacent Channel Interference
ADSL	Asymmetric Digital Subscriber Lines
ADC	Analogue to Digital Converter
ALU	Arithmetic Logical Unit
ASIC	Application Specific Integrated Circuit
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
BSLM	Blind Selected Mapping
CDF	Cumulative Distribution Function
CCDF	Complementary Cumulative Distribution Function
CAM	Content Addressable Memory
CI	Cyclic Interferometry
CI-OFDM	Carrier Interferometry Orthogonal Frequency Division Multiplexing
CDMA	Code Division Multiple Access
CF	Crest Factor
CV	Code Vectors
DAC	Digital to Analogue Converter
DAB	Digital Audio Broadcasting
DCT	Discrete Cosine Transform
DSI	Dummy Sequence Insertion
DVB	Digital Video Broadcasting
DVQ	Dynamic Vector Quantization
FFT	Fast Fourier transforms
FPGA	Field Programmable Gate Arrays
FMPP	Functional Memory Type Parallel Processor
HDTV	High-Definition Television
HPA	High Power Amplifier
HDSL	High-Bit-Rate Digital Subscriber Lines
IBO	Input Back Off
ISI	Inter Symbol Interference
ISE	Integrated Software Environment
IFFT	Inverse Fast Fourier transforms
IOB	Input Output Blocks
JPEG	Joints Photographic Experts Groups
LPF	Low Pass Filter
LUT	Look up Tables

LVQ	Learning Vector Quantization
MAC	Medium Access Control
MAE	Mean Absolute Error
MSE	Mean Squared Error
MC	Multi Carrier
MIMD	Multiple Instruction Multiple Data Stream
NNS	Nearest Neighbor Search
NN	Neural Network
OBO	Output Back Off
OOB	Out Of Band
OFDM	Orthogonal Frequency Division Multiplexing
PAR	Peak to Average Ratio
PAPR	Peak to Average Power Ratio Reduction
PA	Power Amplifier
PE	Processing Element
PEP	Peak Envelope Power
PMEPR	Peak to Mean Envelop Power Ratio
PRC	Peak Reducing Carriers
PTS	Partial Transmit Sequence
QAM	Quadrature Amplitude Multiplexing
QPSK	Quadrature Phase Shift Keying
RAM	Random Access Memory
SBC	sub-Block Coding
SNR	Signal to Noise Ratio
SLM	Selected Mapping
SIMD	Single Instruction Multiple Data Stream
TI	Tone Injection
TR	Tone Reservation
TSVQ	Tree Structured Vector Quantization
VQ	Vector Quantization
VLSI	Very Large Scale Integration
VHDSL	Very High-Speed Digital Subscriber Lines
W-CDMA	Wideband Code Division Multiple Access
WLAN	Wireless Local Area Network
XOR	Exclusive OR
ZFP	Zero Forcing Peaks

DEDICATION

To
Our Beloved Parents
And
Our Whole Family

Chapter 1

Introduction to 3 GPP LTE Evolution Systems

1.1. Introduction

From humble beginnings in the 1980s with bulky 1G analog handsets, the wireless cellular communications industry now enjoys a major presence in most countries, with nearly 5 billion subscribers worldwide and revenues in the one trillion USD range. Consumer demand for services is expected to grow rapidly in the next few years, fuelled by new applications such as mobile web-browsing, video downloading/streaming, online gaming, and social networking. The commercial deployment of 3G networks began with 3GPP UMTS/WCDMA in 2001 and has evolved into current UMTS/HSPA networks. To maintain a competitive edge, 3GPP UMTS networks need to support higher bit rates, improved spectrum efficiencies, and lower delays, all at a reduced cost. A well-planned and natural evolution to 4G networks is considered essential. Long-term evolution (LTE) and LTE-Advanced are important steps in this transition. LTE technology demonstrations began as early as 2006 and commercial LTE networks are starting to be deployed by wireless carriers worldwide.

The demand for cellular communication services is expected to continue its rapid growth in the next decade, fuelled by new applications such as mobile web-browsing, video downloading, on-line gaming, and social networking. The commercial deployment of 3G. A cellular network technology began with 3GPP UMTS/WCDMA in 2001 and has evolved into current UMTS/HSPA networks. To maintain the competitiveness of 3GPP UMTS networks, a well-planned and graceful evolution to 4G networks [1] is considered essential. LTE is an important step in this evolution, with technology demonstrations beginning in 2006. Commercial LTE network services started in Scandinavia in December 2009 and it is expected that carriers worldwide will shortly be starting their upgrades.

The main design goals behind LTE are higher user bit rates, lower delays, increased spectrum efficiency, reduced cost, and operational simplicity. The first version of LTE, 3GPP Release 8, lists the following requirements [2].

- (1) peak rates of 100 Mbps (downlink) and 50Mbps (uplink); increased cell-edge bit rates;
- (2) a radio-access network latency of less than 10ms;
- (3) two to four times the spectrum efficiency of 3GPP Release 6 (WCDMA/HSPA);
- (4) support of scalable bandwidths, 1.25, 2.5, 5, 10,15, and 20MHz; support for FDD and TDD modes; smooth operation with and economically viable transition from existing networks.

The key features and the performance requirements of LTE system defined by 3GPP standards [3] are as follows:

- Higher data rate than that of other 2G and 3G networks, 100 Mbps in downlink and 50 Mbps in uplink.
- High spectral efficiency, with 2~4 times than that of 3GPP Release 6 standards; 5 bps/Hz in downlink and 2.5 bps/Hz in uplink, assuming two receive antennas and single transmit antenna at terminal.
- Very low latency, the Round Trip Time (RTT) is even less than 10 m sec.
- Flexible bandwidth allocation of 1.4, 3, 5, 10, 15 and 20 MHz.
- Wide range of frequency spectrum available.
- Support for both paired and unpaired spectrum, FDD and TDD modes possible.
- In downlink transmission it implements OFDM technique which support for wide carriers and high peak rates.
- In uplink it makes use of SC-FDMA technique which provides lower PAPR than OFDM.
- All-IP based network architecture.
- Exploit Multiple Input Multiple Output (MIMO) system for enhancing capacity and coverage.
- Optimized for low-speed (0 to 15 km/hr) mobile terminals, and also supporting very high-speed mobile terminals from 120 km/hr to 350 km/hr (or even 500 km/hr depending on the frequency band).
- Support for co-existence and interoperability with existing 2G or 3G systems and non-3GPP technologies.
- Provides cost effective and smooth migration path from earlier 3GPP releases radio interface and architecture.
- Support for enhancement mode broadcast and multicast services.
- Reasonable system and terminal complexity, cost, and power consumption.

1.2. LTE Evolution

LTE can be considered as an emerging and evolutionary technology of its predecessor like GSM/EDGE, UMTS, and HSPA. Although LTE uses different radio access technology it can readily coexist and interwork with other 2G and 3G systems with just some changes in overall network. 3GPP has done lots of work on standard development for LTE in Release 8 and with some enhancements update in Release 9. Until the full standardization and operation of 4G systems (LTE Advance), for many years LTE will serve to provide high speed data capability. The Figure 1.1 shows evolution of LTE.

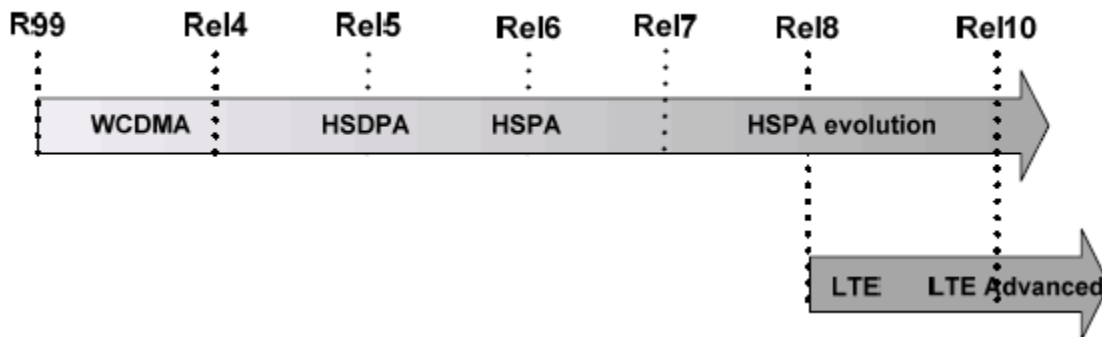


Figure 1.1 LTE and HSPA evolution [4].

1.3. LTE Upgrade Path

LTE provides a smooth evolutionary path for operators deploying all 3GPP and non- 3GPP technologies. It helps operators to enhance their network to LTE even without need to follow any specific path for modification. For example, an operator running GSM or WCDMA can easily enhance their network to LTE without having to employ HSPA [5]. Also LTE can offer smooth migration path towards 4G.

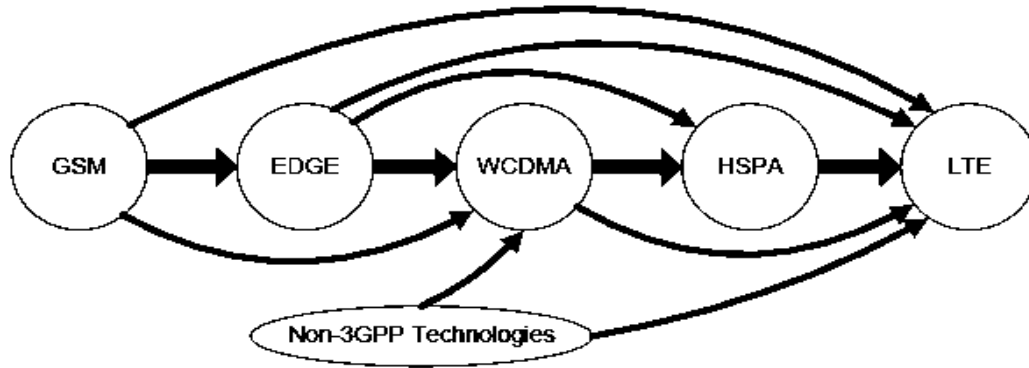


Figure 1.2. Different LTE evolutionary paths [3].

1.4. LTE Network Architecture

The LTE network architecture comprises Evolved UMTS Terrestrial Radio Access Network (E-UTRAN) on the radio access side and Evolved Packet Core (EPC) on the core side. The overall architecture of a LTE network is as shown in Figure 2.4. The EUTRAN consists of eNBs which are interconnected with each other with X2 interface, while they are connected to EPC (mainly MME/S-GW) with S1 interface.

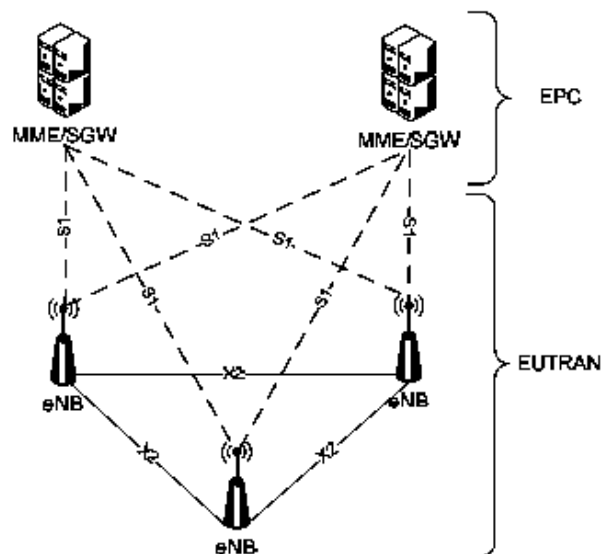


Figure 1.3. E-UTRAN overall architecture

1.5. LTE Radio Interface

LTE radio interface supports three fundamental technologies: multicarrier technology, multiple antenna technology, and the application of packet-switching to the radio interface [6]. This chapter mainly discusses about radio interface technologies implemented in both uplink and downlink direction of a LTE system. The physical channels and signals used in LTE system are also briefly discussed.

1.5.1 OFDMA

Orthogonal Frequency Division Multiplexing (OFDM) principle is a multi-carrier transmission technique. In a single carrier transmission high-rate stream of data are transmitted with a single carrier; while in OFDM the available spectrum is divided into multiple numbers of closely spaced orthogonal carriers, called subcarriers, which are transmitted simultaneously in parallel. Each subcarrier is modulated with a conventional modulation scheme at a low symbol rate, and can be used in both FDD and TDD formats – which is one of the key requirements of LTE. In E-UTRAN, downlink modulation schemes QPSK, 16QAM, and 64QAM can be used. In OFDM, in time domain, a guard interval (also called Cyclic Prefix) is inserted prior to each symbol in order to mitigate Inter Symbol Interference (ISI) which may arise due to channel delay spread.

Orthogonal Frequency Division Multiple Access (OFDMA) is a form of OFDM using narrow and mutually orthogonal subcarriers. In OFDMA, subcarriers are allocated dynamically among different users at the same time which enables distribution of available resources to multiple users. OFDMA supports for MIMO technique and is very robust against multipath fading. However, OFDMA also has drawback as it suffers from high Peak-to-Average Power Ratio (PAPR) problem. The high PAPR leads to inefficient amplification and expensive transmitters. High PAPR is not a problem in power amplifiers of eNBs, but for UE terminals it will lead to higher power consumption and shorter battery life, and makes a unit expensive. The allocation of subcarriers to different users in OFDM and OFDMA can be visualized from the Figure 2.1 and which we will be discussed later in the preceding chapters.

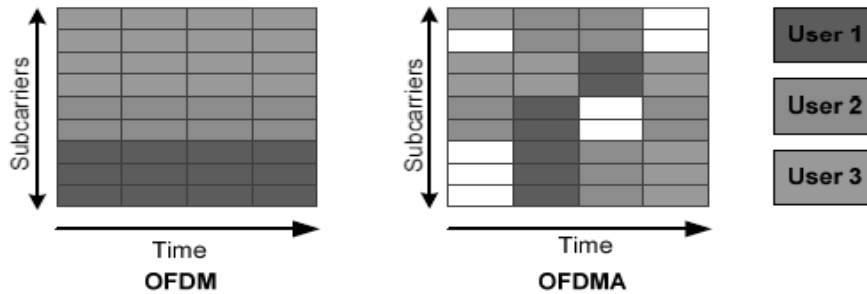


Figure 2.1. OFDM and OFDMA subcarrier allocation

1.5.2 SC-FDMA

Though OFDMA is a favorable option for LTE uplink transmission, it is not considered a good choice for LTE downlink transmission as it suffers from high PAPR. This is because high PAPR properties demand for large back-off requirement and high cost design of UE power amplifiers, which in turn impact the uplink coverage. Single Carrier-Frequency Division Multiple Access (SC-FDMA) is an OFDMA variant technology and is also called DFT-spread OFDM. SC-FDMA offers similar performance and complexity as OFDMA.

The basic transmitter and receiver architecture is also very similar to OFDMA. In OFDM the data symbols directly modulate subcarrier independently, while in SC-FDMA the signal modulated onto a given subcarrier is a linear combination of all the data symbols transmitted at the same time instant. In each symbol period, all the transmitted subcarriers of an SC-FDMA signal carry a component of each modulated data symbol, which gives SC-FDMA its crucial single-carrier property[6]. SC-FDMA transmits symbols sequentially so that the PAPR is reduced by spreading a symbol power over subcarriers, which in turn helps to enhance the efficiency of uplink power amplifier and reduce power consumption. Hence, these properties of SC-FDMA make it an optimum choice for LTE uplink transmission.

Chapter 02

Fundamental of OFDM technique & Peak to Average Power Ratio (PAPR)

2.1. Background

There has been momentous progress in the field of wireless communication during last twenty years. The internet and digital communication evolution has resulted in enormous increase in methods of personal communication as well as commercial applications. The new paradigm of information access to everybody everywhere all the time is in making.

To achieve the ever increasing demands of higher data transfer rates for new multimedia applications, the physical wireless link of wireless communication networks is constantly under trial. The phenomenon of multipath fading, mobility and the limited availability of bandwidth are major precincts. Lately, there have been many breakthroughs to triumph over these limitations. Many modulation techniques compete for new solutions and future applications. Modulation schemes can be broadly categorized in to single carrier and multi carrier. Wideband code division multiple access (W-CDMA) is single carrier modulation scheme. While OFDM, Multi Carrier Code Division Multiple Access (MC-CDMA) and multi carrier (MC-IDMA) are multi carrier schemes. W-CDMA which has been adopted by UMTS and it supports up to 2 [Mbit/s] at high mobility and large range. OFDM which is one of the multi-carrier techniques and digital broadcast systems like digital audio/video broadcasting (DAB/DVB) are already using this technique. It is also being used in wireless local area network (WLAN).

OFDM is popular for high data rate (broadband) communications because it is immune to multipath delay spread, it results in low complexity equalizer, and it has high spectral efficiency. It is also immune to channel behavior and uses efficient modulation/demodulation.

Amongst several disadvantages, high PAPR makes OFDM very sensitive to nonlinear components like HPA in the transmission path. The HPA causes additional in-band distortion and adjacent channel interference leading to low power efficiency which is the main impediment to execution of OFDM especially in low-cost applications.

2.2. OFDM System

OFDM, first introduced in 1966 [7] and patented few years later [8], is known for high-speed data transmission. Early on OFDM gained attention because data was sent in parallel on different sub carriers, hence high speed equalization was no longer required. Other incentives of OFDM were high spectral efficiency and immunity to the effects of multipath fading.

In 1971 the idea of using the discrete Fourier transform in the modulation/demodulation process was introduced [9]. Prior to this breakthrough, OFDM systems were prohibitively complex because arrays of sinusoidal generators and coherent demodulators were necessary in

the implementation. With special-purpose Fast Fourier transform (FFT) chips, now it became possible to implement the entire OFDM system digitally and efficiently.

Lately, OFDM has been put into practice in DAB, digital television and high definition television (HDTV)[10], high-bit-rate digital subscriber lines (HDSL), very high-speed digital subscriber lines (VHDSL), asymmetric digital subscriber lines (ADSL), and mobile wideband data transmission (IEEE 802.11a, Hiperlan II). It is also used in the IEEE 802.16 WiMAX [11] standard. Some applications along with carrier frequency and data rates are tabulated here under in Table 2.1.

Table 2.1 OFDM Systems & Applications

Standard	Meaning	Carrier frequency	Rate (Mbps)	Applications
DAB	Digital audio broadcasting	FM Radio	0.008-0.384	Audio Broadcasting
DVB-T	Digital video broadcasting	UHF	3.7-32	Digital TV Broadcasting
DVB-H	Digital video broadcasting	UHF	13.7	Digital TV Broadcasting to handheld
IEEE.11a	Wireless LAN/ WiFi	5.2GHz	6-54	Wireless internet
IEEE.11g	Wireless LAN/ WiFi	2.4GHz	6-54	Wireless internet
IEEE.11n	Wireless LAN (high speed)	2.4GHz - ??	6-100	Wireless internet
IEEE.16	Broadband & wireless access	2.1GHz & others	0.5-20	Fixed / Mobile Wireless internet
IEEE.20	Mobile and wireless Access	3.5GHz	~1	Mobile internet / voice

In spite of all the advantages, OFDM still has its shortcomings. One problem is vulnerability to carrier frequency estimation errors. Error rate can increase drastically due to small frequency offset because it leads to loss of orthogonality between the sub carriers. [12] [13] [14] [15]. The second problem is that OFDM signals suffer from high PAPR. High PAPR requires a system to accommodate an instantaneous signal power that is much larger than the signal average power, necessitating low operating power efficiency.

2.3. Mathematical Model of OFDM System

In an OFDM system, data is modulated in the frequency domain to N adjacent sub carriers. These N sub carriers span a bandwidth of B Hz and are separated by a spacing of $\Delta f = B/N$. The continuous-time base band representation of this is

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi\Delta f k t / T} \quad , t \in [0, T] \quad (2.1)$$

Where $T = 1/\Delta f$ is the symbol period and $\{X[k]\}_{k=0}^{N-1}$ are the data symbols drawn from a finite constellation.

In practice, the base band modulation is done in the digital domain using an over sampled version of $x(t)$ given by

$$x(n/L) = \frac{1}{\sqrt{LN}} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{NL}} \quad n \in [0, NL-1] \quad (2.2)$$

Where 'L' is the over sampling factor. Notice that when $L = 1$, $x[n]$ is the Nyquist sampled version of $x(t)$. For notational convenience, we define column vectors

$$\mathbf{x}_L = \left\{ x\left[\frac{n}{L}\right] \right\}_{n=0}^{NL-1} \quad \text{and} \quad \mathbf{X} = \left\{ X[k] \right\}_{k=0}^{N-1}$$

In order to avoid inter-symbol interference (ISI), OFDM systems append a cyclic prefix (CP) to the time domain signal. The length of the CP is set to be at least the maximum delay spread, $N\tau$, of the channels response $\{h[n]\}_{n=0}^{N-1}$ where $h[N\tau-1] \neq 0$. The CP can be thought of as a buffer that shields the symbol of interest from being corrupted by reflected versions of the previous symbol. A Nyquist-sampled OFDM symbol with a CP is defined by $\{x_{cp}[n]\}_{n=-N_t}^{N-1}$

Where

$$x_{cp}[n] = x[N+n] \quad n \in [-N_t, -1] \quad (2.3)$$

$$x_{cp}[n] = x[n] \quad (2.4)$$

Using a CP in OFDM considerably simplifies equalization operation at the receiver end. First of all, it allows OFDM symbols to be equalized one symbol at a time. Secondly, if the delay profile is constant over the entire symbol period, then

$$y[n] = x_{cp}[n] * h[n] \quad n \in [-N_t, N+N_t-2] \quad (2.5)$$

$$y_{nocp}[n] = y[n] \quad n \in [0, N-1] \quad (2.6)$$

$$y_{nocp}[n] = x[n] (*)_N h[n] \quad (2.7)$$

Where $(*)_N$ is the N length circular convolution. It is well know from DSP theory that,

$$\text{FFT}\{x[n](*)_N h[n]\} = H[k]X[k] \quad (2.8)$$

Thus, if the channel response is known to recover the data, the receiver can generate

$$X[k] = \text{FFT}\{y_{nocp}[n]\} / H[k] \quad (2.9)$$

Figure 2.1 is a block diagram of an OFDM system. The serial input bit stream is sent to a constellation encoder which outputs N parallel constellations point representing the data. The IFFT of these N samples is used to create the OFDM discrete-time symbol. The parallel time-domain samples are then converted to a serial stream and the CP is added. Next the sequence is up sampled, filtered, and converted to an analog signal, up converted to the carrier frequency, amplified and transmitted. The received chain is a mirror image of the transmit chain except that the received frequency domain data is equalized according to equation (2.9).

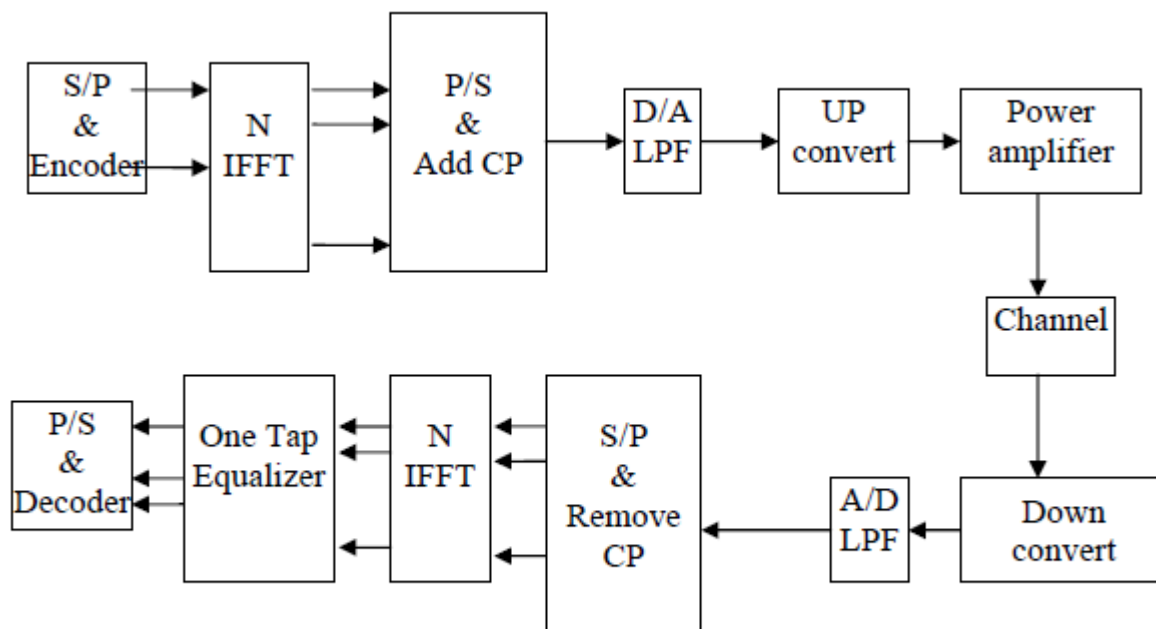


Figure 2.1 OFDM Block Diagram.

2.4. Cause of large Envelop variations

The OFDM is a multi-carrier scheme. It uses IFFT for modulation. In IFFT operation, the sub-carriers are added after being multiplied by sinusoidal functions. Addition of these sub-carriers may result in high peak if all sub carriers add coherently. This coherent addition occurs very rarely. The overall average power of the OFDM signal is quite low. But once large number of sub carriers adds coherently they cause peak which results in high PAPR of the signal to be

transmitted. The ill effects of high PAPR are described below which give justification for its reduction.

2.5. Envelope Variation Metrics

As mentioned in the introduction, the power efficiency of a system suffers when signal that have large envelope variations is transmitted. There have been several metrics proposed that quantify peaky behavior. One such metric is the power variance, defined in [16] by

$$v_e^2 = \frac{E[|x|^4]}{E[|x|^2]^2} \quad (2.10)$$

Where $E[n]$ denotes the expected value. In [16] the author showed that v_e^2 is directly proportional to the normalized mean squared error between a second-order nonlinear function of x and a linear function of x . While power variance v_e^2 is interesting for analysis, it has not proven to be a useful metric for OFDM signals passing through peak-power limited devices.

Lei, Li and Tang [17] pointed out that in practical systems, signals are frequently clipped to some low level even after envelope peaks have been minimized. Accordingly, they propose that the envelope variation of a signal be measured by the clipping noise power generated at some clipping level.

Another metric to quantify the large envelop variation is Peak to Average Power Ratio. Intuitively, using some normalized measurement of the peak of a signal is appealing. This is the idea behind the peak-to-average ratio and is described in detail in the next section.

2.6. PAPR as Metric of envelope variation

The most popular quantification metric of envelope variation is the Peak-To- Average Power Ratio (PAPR). Rightfully so, PAPR captures the most important aspect of a signal that has to pass through a peak-power limited device which is the peak power. The use of PAPR in communications signals is a result of the use of PAR in radar applications. A radar system shares certain similarities with a communications system; namely, they both have to transmit an amplified radio signal of a certain spectrum. For radar, the spectrum shape is often the only signal constraint, which makes waveform shaping that minimizes peaks a relatively straightforward problem. However, in an OFDM communication system there is the additional

constraint that each sub carrier (Fourier coefficient of the spectrum) is modulated with an information bearing complex number. This additional degree of constraint significantly complicates the problem.

The PAPR of an OFDM signal, x is defined as

$$PAPR\{x\} = \frac{\max |x|^2}{E[|x|^2]} \quad (2.11)$$

Where “ x ” be any signal representation (critically sampled base band, over sampled base-band, continuous-time pass band, etc.) defined over one symbol period. Because the denominator of (2.11) is an expected value and, strictly speaking, not an average, it is true that the term PAPR is a bit of a misnomer. Despite this slight technical inaccuracy, PAPR is the most widely used term. Also, it is noted that the ensemble average power and the expectation in the denominator of (2.11) only differ for non-constant modulus constellations.

2.6.1. PAPR Curve by CCDF

OFDM systems combine multiple sub-carriers which causes increase in PAPR. The increase in PAPR is related to the number of sub carriers and their order of modulation. Complementary Cumulative Distribution Function (CCDF) curves present vital information regarding the OFDM signal to be transmitted. The CCDF curves are applied for many other design applications such as to combine several signals through systems components, visualize the effects of modulation formats, evaluate spread spectrum systems, and design and test RF components. These curves also provide the PAPR data needed by component designer. The main use of power CCDF curves is to identify the power characteristic of the signals which are amplified, mixed and decoded [18].

The plot of relative power levels of signal against their probability of occurrence is called CCDF curve. This curve illustrates the amount of time the signal remains at or above a given power level. The ratio between power level and the average power is expressed in dB. Figure 2.2 shows all possible symbols for 8-carriers BPSK OFDM signal. The plot shows power levels of the symbols and the dotted horizontal line depicts the average power. The probability of occurrence for any power level is percentage of time that signals spends at or above that level.

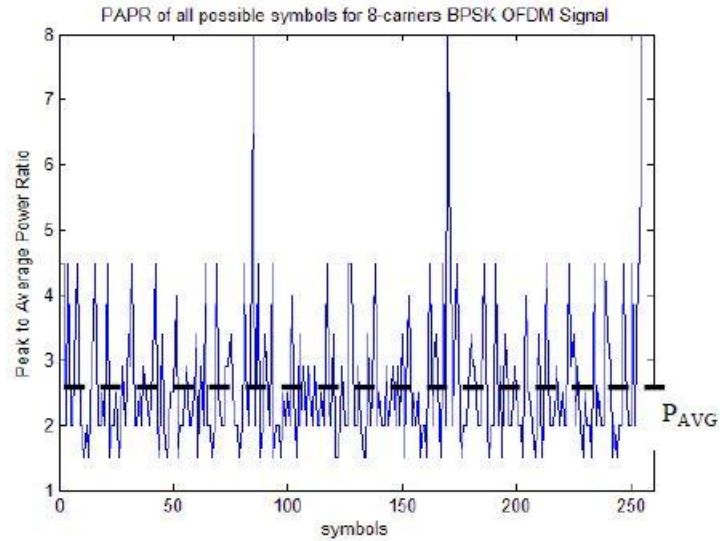


Figure 2.2 A waveform to show a specific power level above the average.

Figure 2.3 is a plot of the PAPR of $x [N]$ for different values of modulation order. It is obvious that at all probability levels the PAPR increases with modulation order.

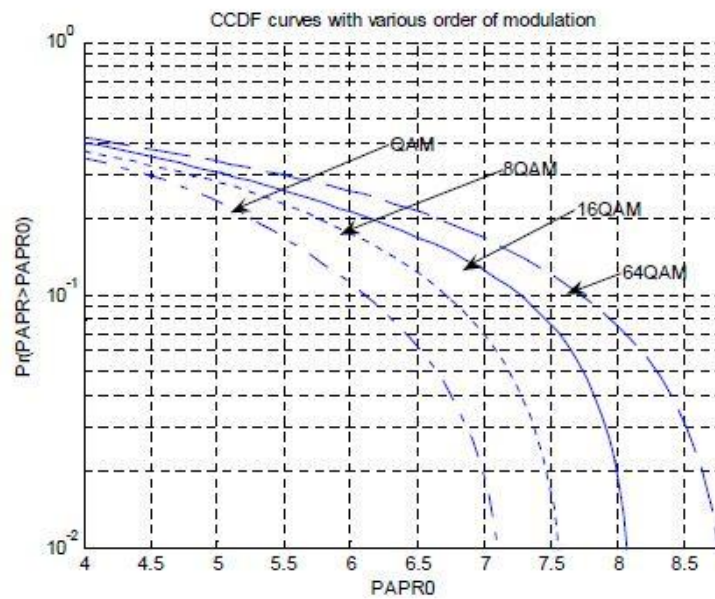


Figure 2.3 CCDF of the discrete-time PAR for various modulation orders

For the derivation of cumulative distribution function (CDF) few assumptions are made considering only Nyquist sampled analog signal in the discrete-time case. Consider that discrete-time domain signal has a complex Gaussian distribution assuming that the number of sub carriers N is large enough [19]. It result in Chi- Squared distributed instantaneous power of the samples. Therefore, for a given $n = n_0 \dots$, $|x[n_0]|^2$ is χ^2 distributed with degree of freedom =2. So

$$\Pr \left[|x[n_0]|^2 < \gamma \right] = 1 - e^{-\sigma_x^2 \gamma} \quad (2.12)$$

Furthermore, according to [19], after the IFFT each discrete time sample can be dealt independently. Hence the probability that the power of as a minimum one $x[n]$ among N symbols is above a specific level, γ , can be calculated as

$$\Pr \left[\max_{0 \leq n < N} |x[n]|^2 < \gamma \right] = (1 - e^{-\sigma_x^2 \gamma})^N \quad (2.13)$$

Finally, if $E[|x[n]|^2]$ is normalized to unity, then the CCDF of the PAPR is

$$\Pr \left[\text{PAPR}\{x[n]\} > \gamma \right] = 1 - (1 - e^{-\gamma})^N \quad (2.14)$$

It is natural to wonder what PAPR is of the most interest in an OFDM system. For instance, when an engineer has to specify the dynamic range of the digital-to analog converter (DAC) in an OFDM system, the most important PAPR measurement would be that of the signal input to the DAC which is $x[n/L]$. On the other hand, the PA will have to be designed around the PAPR of the pass band signal $x_{pb}(t) = R\{x(t)e^{j2\pi f_c t}\}$ where f_c is the carrier frequency. If $f_c \gg B$, which is the case in most practical systems, then

$$\max |x_{pb}(t)| = \max |x(t)| \quad (2.15)$$

Also

$$E \left[|x_{pb}(t)|^2 \right] = E \left[\left| \operatorname{Re} \{ x(t) e^{j2\pi f_c t} \} \right|^2 \right] \quad (2.16)$$

$$= E \left[\operatorname{Re} \{ x(t) \}^2 \cos^2(2\pi f_c t) + \operatorname{Im} \{ x(t) \}^2 \sin^2(2\pi f_c t) \right] \quad (2.17)$$

$$= \frac{1}{2} E \left[\operatorname{Re} \{ x(t) \}^2 + \operatorname{Im} \{ x(t) \}^2 \right] \quad (2.18)$$

$$= \frac{1}{2} E \left[|x(t)|^2 \right] \quad (2.19)$$

Therefore,

$$PAPR \{ x_{pb}(t) \} \approx 2PAPR \{ x(t) \} \quad (2.20)$$

Equation (1.15) is a very good approximation to the PAPR distribution of $x[n]$ but differs by one dB from the PAPR distribution of $x(t)$. The distribution of $x(t)$ has determined by different researchers. The first came from [20], where it was claimed that

$$\Pr \left[PAPR \{ x[t] \} > \gamma \right] \approx 1 - (1 - e^{-\gamma})^{2.8N} \quad (2.21)$$

Which is just a intuitive modification to the CCDF that resulted from the Gaussian approximation in (2.14). Later in [21], a more theoretical analysis of the problem was carried out based on level crossing probabilities of $x_{pb}(t)$, where the ratio f_c/B was taken into account as well as the power distribution of $X[k]$. It was concluded that for $f_c \gg B$ and a constant modulus power distribution that

$$\Pr\left[|PAPR\{x[t]\}| > \gamma\right] \leq N \sqrt{\frac{\pi}{3}} \gamma e^{-\gamma} \quad (2.22)$$

In [25], the authors present the approximation

$$\Pr\left[|PAPR\{x[t]\}| > \gamma\right] \approx \left\{ \begin{array}{ll} \left(1 - \frac{\sqrt{\gamma} e^{-\gamma}}{\sqrt{\bar{\gamma}} e^{-\bar{\gamma}}}\right) & \gamma > \bar{\gamma} \\ 0 & \gamma \leq \bar{\gamma} \end{array} \right\} \quad (2.23)$$

Where $\bar{\gamma} = \sqrt{\pi}$ based on the level crossing rates of $x(t)$. The authors further refine equation for large γ to

$$\Pr\left[|PAPR\{x[t]\}| > \gamma\right] \approx e^N \sqrt{\frac{\pi}{3}} \gamma e^{-\gamma} \quad (2.24)$$

In order to test these approximations it is necessary to resort to digital signal theory where it is known that as $L \rightarrow \infty, x[n/L] \rightarrow x(t)$. It follows that the PAPR of $x[n/L]$ approaches the PAPR of $x(t)$ for large L . Thus we should be able to approximate $PAPR\{x(t)\}$ by running simulations on the over sampled signal $x[n/L]$.

Simulation in literature [22] shows $L=4$ gives good approximation.

2.7. Factors Effecting PAPR

Factors effecting PAPR are as follows:

2.7.1. Number of sub carriers

In Multi-Carrier Systems the complex base band signal for one symbol in an OFDM system may be expressed as:

$$\mathbf{x}(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^N a_n e^{j\omega_n t} \quad (2.25)$$

Where N is the number of sub carriers and a_n is the modulating symbol. For moderately large numbers of m-PSK sub carriers the quadrature components of $x(t)$ each tend towards a Gaussian distribution (giving the sum of their power amplitude a Rayleigh distribution). Consequently, whilst the peak value possible is N times the individual sub carrier peak, the probability of any value close to that peak occurring is very low. For example, with only 24 sub carriers, the probability of the PAPR exceeding 4dB is 10^{-2} and of exceeding 8dB is only 10^{-4} [23].

2.7.2. Modulation Order

High data bandwidth efficiency (in terms of b/s/Hz) may be achieved by using higher order modulations based, for example, on QAM. When the sub carrier's modulation is a higher-order QAM type, the PAPR of the summed OFDM signal is increased by the PAPR of the QAM constellation used. However, the probability of these higher peaks occurring is correspondingly less. Moreover, since among advantages of OFDM one is that sub carriers can have their modulation independently varied to adapt to channel conditions, the combined PAPR in any system using this technique may be difficult to predict and control.

PAPR for an unfiltered base band signal is listed in the following **Table 2.2**.

Modulation	PAPR
m-PSK (reference)	0dB
16-QAM	2.55dB
64-QAM	3.68dB
256-QAM	4.23dB
256-QAM (modified)	2.85dB

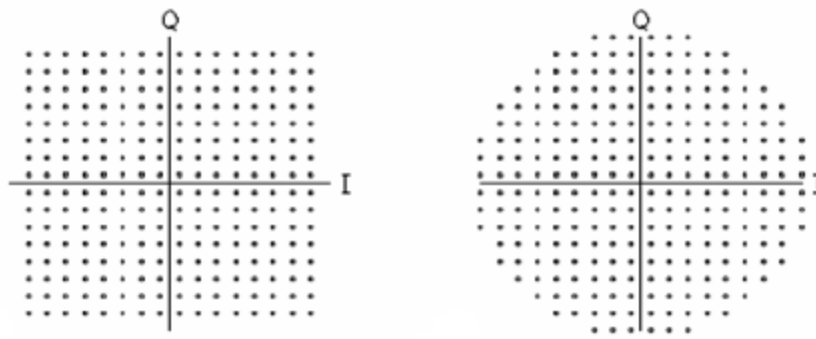


Figure 2.4 256-QAM constellations: (a) regular and (b) modified mapping to reduce PAPR

2.3.7. Constellation shape

The last entry in Table 2.2 is for a constellation obtained by modifying 256-QAM to reduce PAPR. This modified constellation shape is shown in figure 2.4. However, there is an additional processor load associated with encoding and decoding this constellation.

2.7.4. Pulse Shaping

In terrestrial communications it is common to apply pulse shaping to the base band signal, to reduce the bandwidth of the transmitted spectrum, but this causes overshoot and could increase the PAPR of the modulating signal by 4-5dB.

Chapter 03

Peak to Average Power Ratio (PAPR) Reduction Techniques

3.1. Classification of Existing Techniques

Several schemes have been proposed to reduce the PAPR. These techniques can be divided into two main categories

- Signal Scrambling
- Signal Distortion

Scrambling category consists of different variations of codes used for scrambling to achieve PAPR reduction. Amongst coding techniques Barker codes, Msequences, Golay complementary and Shapiro-Rudin sequences have been used for reduction of PAPR. The main drawback is that as number of carriers increases the associated overhead with search for best code increases exponentially. Amongst this category better techniques are selective mapping, partial transmit sequences and block coding. The scrambling category can be subcategorized as follows:

- **Schemes with explicit side information**

Block codes: e.g. linear block code scheme, cyclic code scheme

Probabilistic schemes: e.g. SLM scheme, PTS scheme, Interleaving schemes

- **Schemes without explicit side information**

Examples: Special Block coding scheme, Hadamard transform method, Dummy sequence insertion method etc.

Signal scrambling techniques, with side information reduces the effective throughput since they introduce the redundancy. The signal distortion techniques introduce both in band and out-of-band interference and complexity to the system

The distortion category attempts to reduce PAPR by manipulation of signal before amplification. Clipping of signal prior to amplification is a simplest method but it causes increase in both out-of-bands (OOB) as well as in-band interference thus compromises upon performance of system. Amongst this category better techniques include companding, peak windowing, peak power suppression, peak cancellation, weighted multicarrier transmission etc. Any technique which is used to reduce PAPR should not only have high spectral efficiency but must be compatibility with the existing modulation schemes and at the same time must not be computational complex.

3.2. Reduction techniques

Now we focus more closely on the PAPR reduction techniques for multicarrier transmission with some examples, various techniques which have been used for PAPR reduction are briefly described along with merits and demerits.

3.2.1. Amplitude Clipping and Filtering

The simplest technique for PAPR reduction might be amplitude clipping [24]. Amplitude clipping limits the peak envelope of the input signal to a predetermined value or otherwise passes the input signal through unperturbed [25], that is,

$$B(x) = \begin{cases} x, & |x| \leq A \\ Ae^{j\phi(x)}, & |x| > A \end{cases} \quad (3.1)$$

Where $\phi(x)$ is the phase of x . The distortion caused by amplitude clipping can be viewed as another source of noise. The noise caused by amplitude clipping falls both in-band and out of-band. In-band distortion cannot be reduced by filtering and results in error performance degradation, while out-of-band radiation reduces spectral efficiency. Filtering after clipping can reduce out-of-band radiation but May also cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce overall peak regrowth, a repeated clipping-and-filtering operation can be used [26, 27]. Generally, repeated clipping-and-filtering takes many iterations to reach a desired amplitude level. When repeated clipping-and-filtering is used in conjunction with other PAPR reduction techniques described below, the deleterious effects may be significantly reduced.

There are a few techniques proposed to mitigate the harmful effects of the amplitude clipping. In [28] a method to iteratively reconstruct the signal before clipping is proposed. This method is based on the fact that the effect of clipping noise is mitigated when decisions are made in the frequency domain. When the decisions are converted back to the time domain, the signal is recovered somewhat from the harmful effects of clipping, although this may not be perfect. An improvement can be made by repeating the above procedures. Another way to compensate for the performance degradation from clipping is to reconstruct the clipped samples based on the other samples in the oversampled signals. In [29] oversampled signal reconstruction is used to compensate for signal-to-noise ratio (SNR) degradation due to clipping for low values of clipping threshold. In [30] iterative estimation and cancellation of clipping noise is proposed. This technique exploits the fact that clipping noise is generated by a known process that can be recreated at the receiver and subsequently removed.

3.2.2. Block Coding

The scheme of block coding was proposed [31], to reduce PAPR of multi carrier transmission. The data is block coded in such a way that only those sets of code words are permissible which do not contain high peak envelope powers. The PAPR is reduced and it also enables reduction of nonlinear distortion as well as decreased spectral regrowth. The signal of the system also has higher average transmit power with an upper limit for peak power. The disadvantage includes a rise in the bandwidth with no change in data transfer rate. The other setback is that there is reduction in energy per transmitted bit while the transmit power remains same. The major drawback is that this technique is not suitable for higher number of sub carriers because of exponential rise in complexity as the number of carriers increase.

3.2.3. Sub-Block Coding

It was observed that all $\frac{3}{4}$ rate systematically coded block codes with the last bit as an odd parity checking bit demonstrated lowest peak envelope power. Based on this observation the scheme of sub-block coding (SBC) was proposed [32]. The block code schemes are not suitable for large number of carriers; however, in sub block coding the long information sequence is divided into several sub-blocks. Then sub-blocks are encoded with systematic odd parity checking coding (SOPC).

3.2.4. Block Coding with Error Correction

Based upon the idea that block codes can not only reduce PAPR but if well designed can also be utilized to for error correction, the scheme of block coding with error correction was proposed [33]. A block diagram of the block coding with error correction technique is illustrated in figure 3.1.

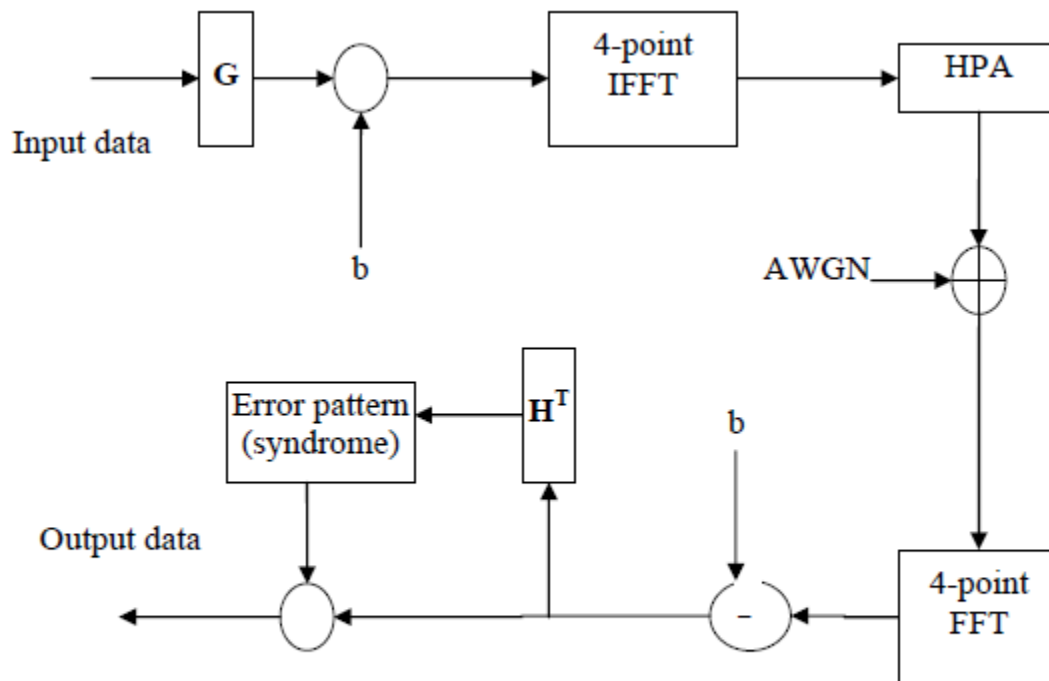


Figure 3.1 Block diagram of block coding with error correction scheme

The authors claim that there is improvement in BER in comparison with uncoded system. But the fact remains that just like block coding this scheme is also not feasible for higher number of sub carriers. Furthermore, there is significant increase in the bandwidth.

3.2.5. Selected Mapping

The technique of selected mapping (SLM) for PAPR reduction was proposed in 1996 [34]. In SLM from a set of candidate signals which are generated to represent the same information, the signal with lowest PAPR is selected and transmitted. The information about this selection also needs to be explicitly transmitted along with the selected signal as side information. The SLM system block diagram is illustrated below.

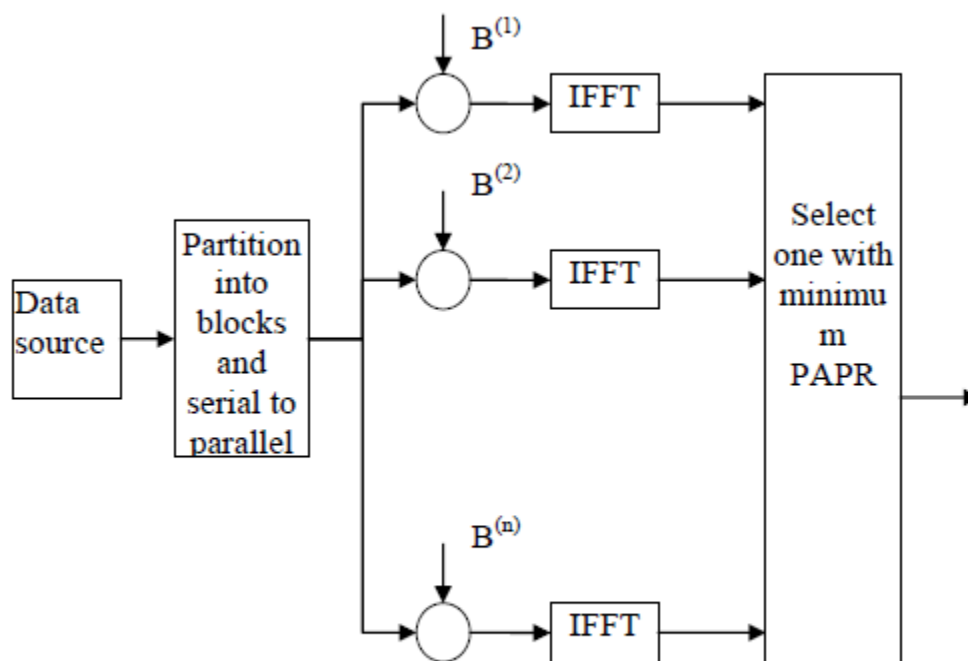


Figure 3.2 Block diagram of Selected Mapping Technique

SLM needs to transmit the information to the receiver, about the selected signal, as side information. If there is a error in the received side information, then the receiver cannot recover the information from the transmitted selected signal. That's why a strong protection against transmission errors is needed regarding side information. Once the receiver has this side information then the decoding process is very simple. SLM is can be employed for larger number of sub-carriers with moderate complexity. The technique uses codes only for PAPR reduction and does not include error correction capabilities of codes. This scheme is aimed at

decreasing the frequency of peak occurrence rather than elimination of peaks. The drawbacks include multiple numbers of IFFT operations leading to increased complexity and the need for transfer of side information to the receiver without any margin for transmission errors. New extension techniques of SLM but with no need for transmission of side information have been proposed [35].

3.2.6. Partial Transmit Sequences

In partial transmit sequences (PTS) [36], initially partitioning of the data block into non-overlapping sub-blocks is done. Then these sub-blocks are rotated with rotation factors which are statistically independent. Subsequently the information about rotation factor, which generates the lowest peak amplitude in time domain data, is transmitted to the receiver. The block diagram of PTS is illustrated in Figure 3.3.

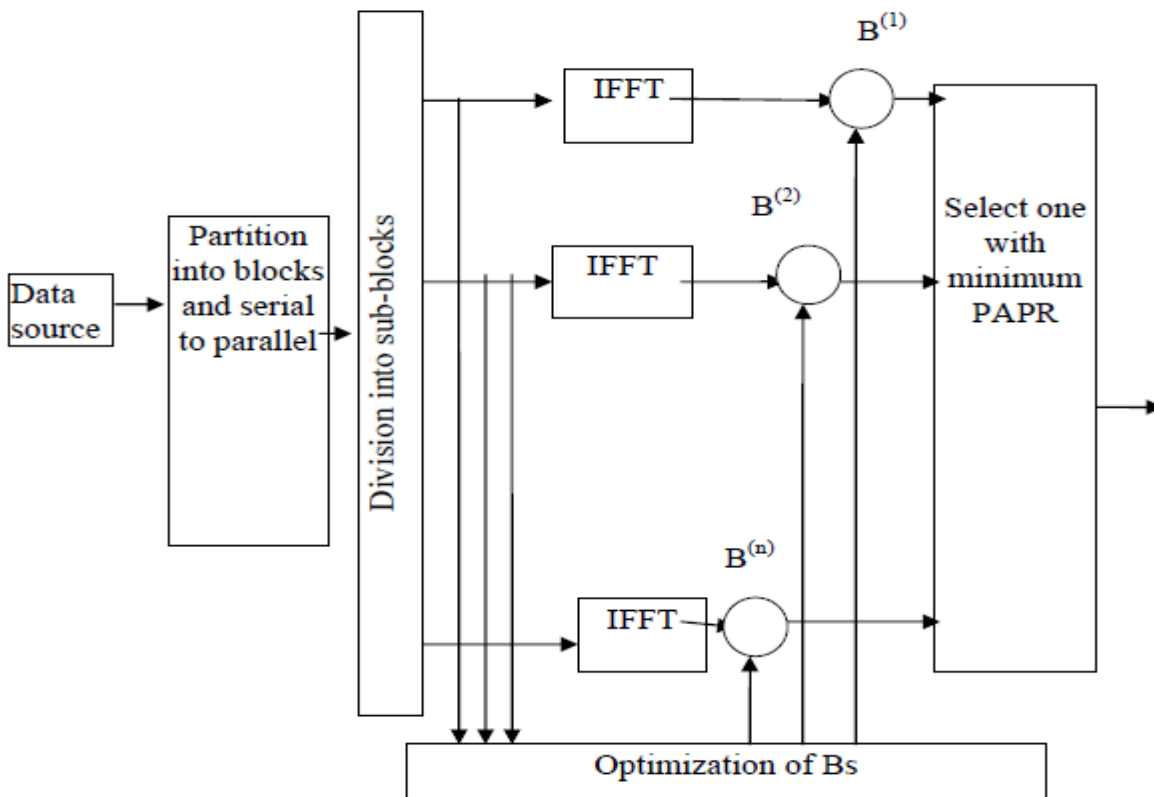


Figure 3.3 Block diagram of PTS technique

The simulation results of PTS for PAPR reduction are better as compared to SLM. The need for transmission of side information can also be overcome if differential modulation is employed for each sub block and block partitioning is known in the receiver.

3.2.7. Interleaving

A technique employing interleaver to reduce PAPR of OFDM signal was proposed [37]. It is observed that data frames having large PAPR are highly correlated. Hence PAPR can be reduced if we can break down the long correlation patterns. This technique utilizes data randomization to reduce PAPR while adaptive interleaving (AIL) is employed to keep the complexity at minimum. The concept of this technique is based on finding an early terminating threshold. Hence the search will stop when the PAPR value arrives at the threshold, and all the interleaved sequences will not be searched. It is understood that higher the threshold value lower will be the number of searched interleaved sequences and vice versa. This scheme achieves PAPR reduction which is comparable to PTS but with less complexity.

3.2.8. Hadamard Transform

The technique of Hadamard Transform [38] is based upon the relationship between correlation property of OFDM input sequence and PAPR probability. The average power of the input sequence represents the peak value of the autocorrelation. Hence the peak value of autocorrelation depends on the input sequence provided that number of sub carriers remains unchanged. The block diagram of Hadamard Transform is illustrated below.

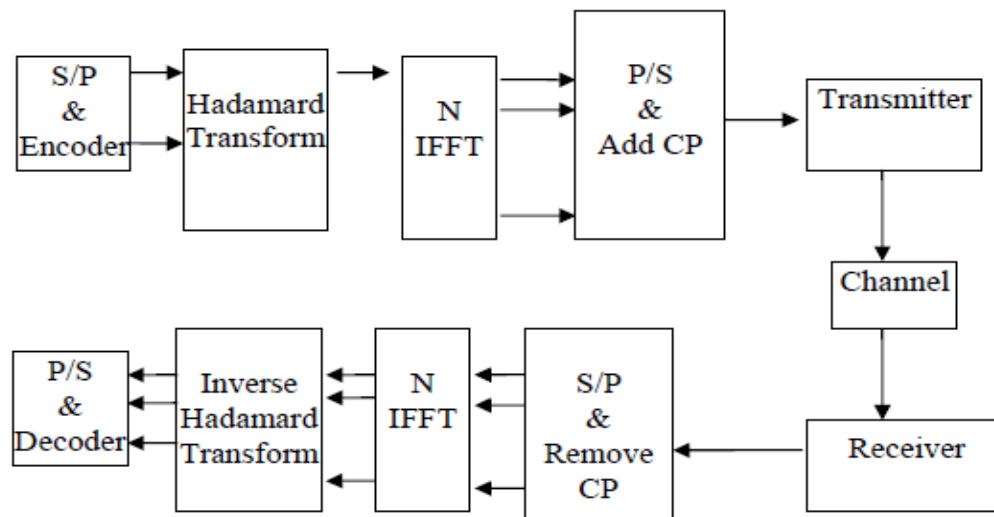


Figure 3.4 Block diagram of an OFDM system using Hadamard Transform

The utilization of Hadamard transforms to reduce the autocorrelation of the input sequence for PAPR reduction is a novel idea. The main advantage is that there is no need to transmit side information to the receiver.

3.2.9. Dummy Sequence Insertion (DSI)

In Dummy sequence insertion (DSI) [39], before IFFT stage in input data a dummy sequence is added. The sequences which are used may be complementary, correlation or any other sequence. Since dummy sequence is not used as side information hence any transmission error does not increase BER. DSI technique is united with PAPR threshold method. After IFFT, if PAPR is below specific threshold then signal is transmitted but if it is more than this specific level then insertion of dummy sequence is done to achieve the required results. The block diagram of DSI system is shown in figure 3.5

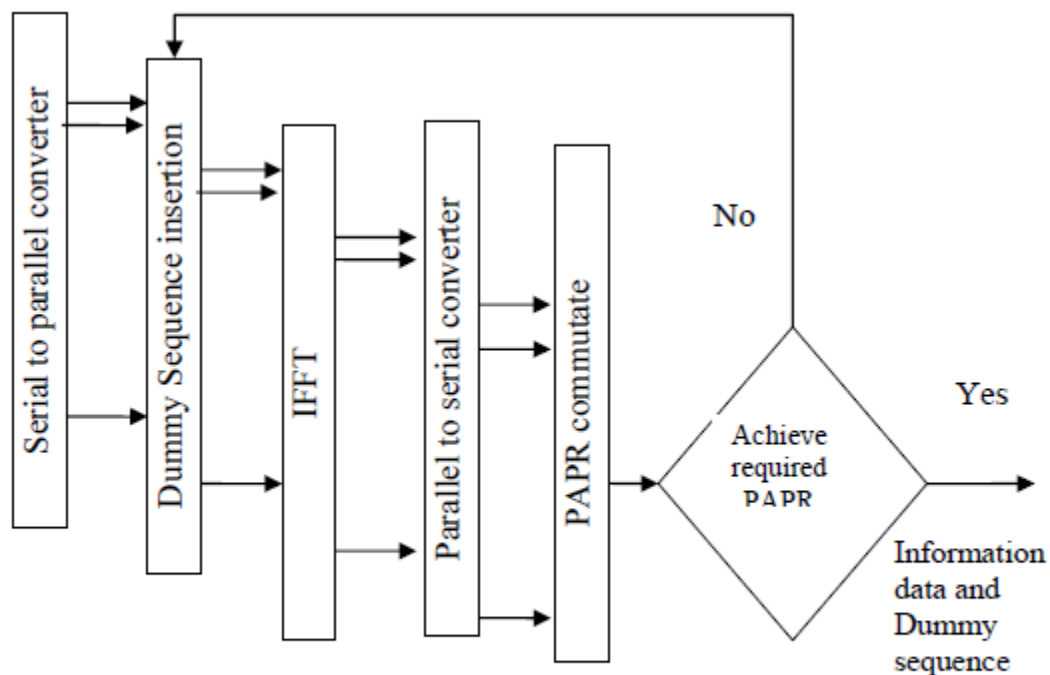


Figure 3.5 Block diagram of DSI system

The main advantage of this technique is that BER is not degraded due to transmission errors in the dummy sequence. So far amongst different sequences, use of complementary sequence produces better results.

3.2.10. A Modified SLM Scheme for PAPR Reduction of OFDM Systems

There is a new peak-to-average power ratio (PAPR) reduction scheme of orthogonal frequency division multiplexing (OFDM) system, called a modified selected mapping (SLM) scheme[26], which considerably reduces the computational complexity with keeping the similar PAPR reduction performance compared with the conventional SLM scheme. The modified SLM scheme for the PAPR reduction of OFDM system, which considerably reduces the computational complexity while it maintains the similar PAPR reduction performance compared with the comparable conventional SLM scheme. The performance of this scheme is numerically confirmed for the OFDM system proposed in the IEEE 802.16 standard. Since the computational complexity reduction ratio increases as the numbers of sub-carriers and binary phase sequences increase, this scheme becomes more efficient for the high data-rate OFDM systems.

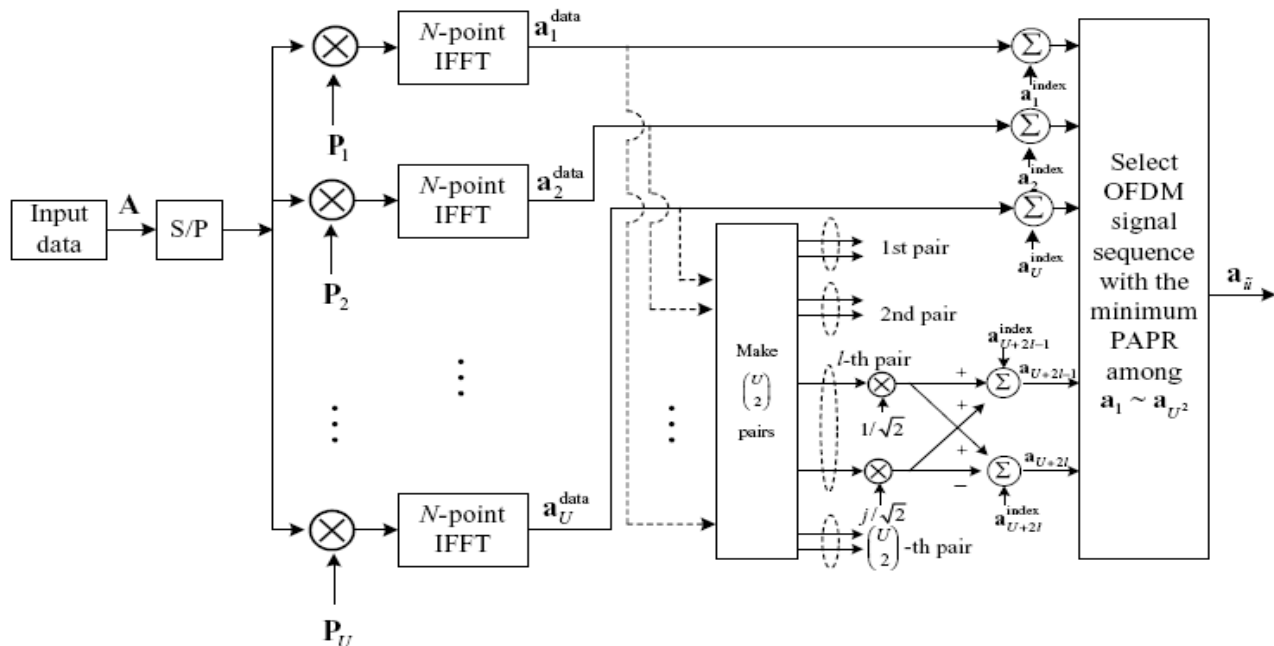


Figure.3.5a. block diagram of the modified SLM scheme of OFDM

In order to achieve large PAPR reduction in the conventional SLM scheme, we have to generate a sufficiently large number of alternative OFDM signal sequences that causes high computational complexity because IFFT should be performed to generate each alternative OFDM signal sequence. Therefore, it is desirable if we can reduce the number of IFFTs without compromising the PAPR reduction performance. In Fig 3.5a, Let \mathbf{a}_i and \mathbf{a}_k be the alternative OFDM signal sequences, which are generated by the conventional SLM scheme as the output after IFFT. Using the linear property of Fourier transform, the linear combination of these two sequences can be given as

Eq. (3.2), where \mathbf{c}_i and \mathbf{c}_k are some complex numbers.

$$\begin{aligned} a_{i,k} &= c_i a_i + c_k a_k \\ &= \text{IFFT}(A \otimes (c_i p_i + c_k p_k)) \end{aligned} \quad (3.2)$$

If each element of the sequence $c_i p_i + c_k p_k$ has unit magnitude, $c_i p_i + c_k p_k$ also be a phase sequence for the SLM scheme and $a_{i,k}$ can be considered as the corresponding OFDM signal sequence. Under the condition each element of the phase sequences p_i and p_k has unit magnitude. The following conditions are satisfied:

(1) Each element of p_i and p_k takes the value in $\{+1, -1\}$;

(2) $c_i = \frac{1}{\sqrt{2}}$ and $c_k = \frac{1}{\sqrt{2}}$

From U binary phase sequences, we can obtain $2 * \binom{U}{2}$ additional phase sequences. Thus, there are total U^2 alternative OFDM signal sequences in this scheme.

	Conventional SLM, $U = 9$	Modified SLM, $U = 3$	CCRR	Conventional SLM, $U = 16$	Modified SLM, $U = 4$	CCRR
# IFFTs	9	3		16	4	
$N = 256$						
# Complex multiplications	11,520	5,376	53.3%	20,480	8,192	60.0%
# Complex additions	18,432	6,400	65.3%	32,768	11,264	65.6%
$N = 512$						
# Complex multiplications	25,344	11,520	54.5%	45,056	17,408	61.4%
# Complex additions	41,472	16,896	59.3%	73,728	24,576	66.7%
$N = 1024$						
# Complex multiplications	55,296	24,576	55.6%	98,304	36,864	62.5%
# Complex additions	92,160	36,864	60.0%	163,840	53,248	67.5%
$N = 2048$						
# Complex multiplications	119,808	52,224	56.4%	212,992	77,824	63.5%
# Complex additions	202,752	77,872	61.6%	360,448	81,920	77.3%

Table 3.1 Computational Complexity of the Conventional SLM and the Modified SLM Schemes
When $N = 256, 512, 1024,$ and 2048

The CCRR of the modified SLM scheme over the conventional SLM scheme with typical values of U and N is given in Table 3.1, which tells us that the proposed scheme becomes computationally more efficient as the N or U increases. Note that the complex multiplication is more complicated than the other operations. When $N = 2048$, the modified SLM scheme with $U = 4$ can reduce the complex multiplications by 63.5% with keeping the similar PAPR reduction performance compared with the conventional SLM scheme with $U = 16$.

3.2.11. Additive Corrective Function

As the name suggests additive correcting function [41], achieves reduction in PAPR through addition of suitable corrective function in OFDM signal. The amplitude peaks are in this approach, the amplitude peaks are manipulated in such a manner that after the correction the amplitude of the signal does not go beyond a specified threshold.

In this method the reduction in PAPR is achieved by manipulation of OFDM signal at transmit side. The correct functions which can be added include additive Sinc functions and multiplicative Gaussian function. This technique can be employed for arbitrary number of carriers without requiring increased redundancy. In this method after correction the out of band interference is reduced but in band interference is increased leading to increase in band distortion. Hence the correcting function which is added should have minimal power to keep interference power within the OFDM band to minimum.

3.2.12. Peak Windowing

Peak Windowing [42], is based upon the fact that frequency of high peaks is infrequent hence these can be removed by minimal increase in self-interference. In this technique signal peaks are multiplied with certain windows. These include Cosine, Hamming, Gaussian shaped and Kaiser Windows. The used window must be shortest possible in time domain and at the same time must be narrowband. Another example of PAPR reduction technique which is based upon self-interference is clipping. The PAPR reduction performance as well as spectral efficiency of peak windowing technique is better as compared to clipping. The major advantage of peak windowing is that PAPR reduction is achieved with minimal complexity for any number of sub carriers. The disadvantages include an increase in out-of-band interference and BER.

3.2.13. Envelope Scaling

In envelop scaling [43], the input envelopes of sub carriers are scaled prior to IFFT. The base for this scheme is the facts that with PSK modulation all the sub carriers input envelopes are equal. Hence input envelop of some sub carriers is scaled in such a way that minimum PAPR is achieved at IFFT output. The input which yields minimum PAPR is fed into the system. The phase information of the input sequence is similar to original however envelopes are not the same. Hence decoding of sequence can be done by receiver without any requirement for side information. The major drawback of this method is that it can only be used when OFDM is employing PSK modulation. On the other hand if we use this method when QAM modulation is implemented by OFDM, then there is severe degradation in BER performance results.

3.2.14. Random Phase Updating

The random phase updating technique for reduction of PAPR has been proposed [44]. In this technique for each carrier the phase is generated randomly and assigned. This process of updating phases keeps running until the PAPR is reduced below a specified threshold level. The PAPR reduction performance is independent of number of carriers and depends mostly upon the chosen threshold level. Although this level can be dynamic however there is an upper limit for the number of iterations which can be used for updating of phases.

Since the receiver must be provided with the information about change of phases that are applied, hence the amount of side information which needs to be provided to the receiver becomes quite significant.

3.2.15. Data Bearing Peak Reduction Carriers

Data bearing peak reduction carriers (PRCs) have been proposed to achieve reduction of PAPR [45]. In this scheme the symbol of lower order modulation is represented by a scheme of higher order modulation. Hence the phase and amplitude of these carriers remains inside the constellation area which represents the data symbols being transmitted.

Amongst drawbacks of PRCs, one is that the overall data transmission efficiency of the system is compromised if we try to achieve maximum PAPR reduction efficiency. At the same time the BER performance is also affected adversely because of employing constellation of higher order for carrying symbols of lower order results in higher probability of error.

3.2.16. Companding Technique

Companding technique was proposed [46], based upon the assumption that the OFDM signal has Gaussian distribution and occurrence of high peaks is infrequent.

In OFDM system on transmitter side after IFFT, the signal undergoes companding and quantization while on receiver side the signal is first digitized and then expanded. The block diagram of OFDM system with companding technique is shown in figure 3.6.

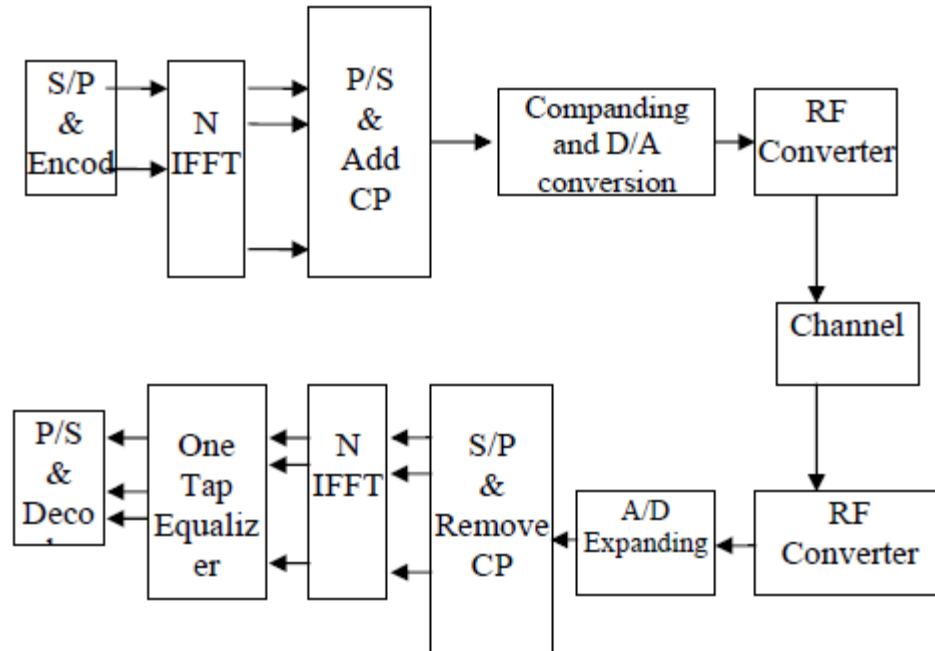


Figure 3.6 Block diagram of OFDM system with companding

Companding was initially employed for speech processing because of infrequent occurrence of peaks. Since the OFDM signal also shows infrequent peaks, hence the authors planned to apply the technique for reduction of peaks in OFDM system. Since frequency of small signals is much more as compared to large ones hence there is improved quantization resolution for small signals as compared to large signals. The overall BER performance is considerably degraded because the quantization error is increased notably for large signals. Hence the overall improvement in PAPR which is achieved by companding is at the cost of BER performance

3.2.17. Tone Reservation

Tone reservation was first introduced in a paper [47], in which the authors implemented a projection onto convex sets (POCS) method. Later, Tellado and Cioffi [48] discussed the idea of tone reservation as a linear programming problem that has an exact solution (the POCS method is suboptimal). The linear programming solution can be reached with complexity $O[N \log N]$.

The idea behind tone reservation is to isolate energy used to cancel large peaks to a predefined set of tones. These tones do not bear any useful information and are orthogonal to the data bearing tones. This orthogonality makes recovering the data trivial.

The advantages of TR technique include:

- No need for side information
- Fewer complex-multiplications as only one time IFFT operation is needed. But multiple iteration operations are needed after IFFT operation.
- No special receiver operation is needed

While promising, tone reservation has several shortcomings. First the data rate is necessarily decreased because some tones are used strictly for PAR reduction. The second problem is the difficulty of selecting which tones to reserve. A random search over all the possible sets, B , would greatly increase the complexity of tone reservation. Often the tones have to be chosen contiguously because fades often affect contiguous sets of sub carriers. These contiguous sets of tones are known to have bad PAR reduction abilities. The third issue is a tradeoff between the quantity of reserved tones and the raise in average power due to tone reservation. More the tones that are reserved, lesser the power needs to be allocated for PAPR reduction. On other hand, more reserved tones mean more unused bandwidth that could be data bearing.

3.2.18. Tone Injection

Motivated by the data rate loss of tone reservation, Tellado introduced a new technique named tone injection [49]. It reduces the PAPR without compromising the data rate. In this method the size of the basic constellation is increased. Hence mapping of original constellation points into numerous corresponding points in the new stretched out constellation becomes possible. The distance between these duplicate points can be calculated as $D = \rho d \sqrt{M}$ where M = constellation size, and $\rho \geq 1$

There is no effect on BER and all we have to do is add a modulo- D subsequent to FFT in the receiver side. Since mapping of each information unit into numerous corresponding constellation points is done, it gives a margin of free will which can be used reduction of PAPR.

3.2.19. Active Constellation Extension

Comparable to tone injection active constellation extension (ACE) [50] is another technique which is used for PAPR reduction. The dynamic extension of data block's outer signal constellations to periphery of original constellation in such a way that PAPR is decreased, is the basic theme of this technique. This idea is schematically represented in figure 3.7 and can be elucidated for multicarrier transmission utilizing QPSK modulation for all sub-carriers. As we can see that the possible number of constellation points for every sub carrier is four. These points have equal distance from imaginary and real axes and are positioned one in each quadrant. These four quadrants delimited by the axis represent the regions of maximum likelihood decision based upon white Gaussian noise assumption. Hence the decision about the received data symbol depends upon the quadrant where it is observed. . Compared to nominal constellation points, those points which are away from decision boundaries result in increased margin, which warrants a decreased BER. Hence constellation points can be modified beyond nominal constellation point but inside the quarter-plan without degrading its performance.

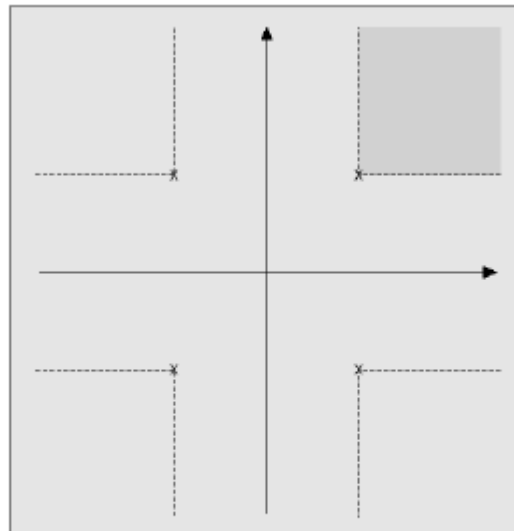


Figure 3.7 The ACE technique for QPSK modulation

This technique can also be used with MPSK and QAM. The main advantages of technique include significant reduction of PAPR without compromising data rate and no need for side information. There is additional slight decrease in BER also. The drawback is that the technique is useful for larger constellation size modulations only.

3.2.20. Zero Forcing Peaks Technique:

Let a_μ be the vector containing sub carriers amplitude within the μ -th OFDM symbol and let N be the numbers of sub-carriers in a symbol. The time domain signal of the μ -th OFDM symbol may be mathematically expressed as

$$x_\mu[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_\mu[k] e^{j\frac{2\pi}{N}kn} \quad 0 \leq n \leq N-1 \quad (3.3)$$

Where $a_\mu[k]$ represents the amplitude and phase of the k^{th} sub carrier within an OFDM symbol and $x_\mu[n]$ represents the n^{th} sample of the time domain OFDM symbol. In ZFP method the original OFDM symbol vector x_μ is multiplied by a window function, in order to satisfy the required PAPR. The window used can be Gaussian or any other. The width of the window is equal to the OFDM symbol duration. This results into the symbol vector x_μ^w . The windowed signal can be written as

$$x_\mu^w[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_\mu^w[k] e^{j\frac{2\pi}{N}kn}, \quad 0 \leq n \leq N-1 \quad (3.4)$$

The phase of every sub carrier within the original complex vector a_μ and the windowed complex vector a_μ^w are compared. Let ϕ_μ be the vector containing the phase differences between the original OFDM signal and the windowed OFDM signal. Then the new sub carrier vector \tilde{a}_μ^w may be expressed as

$$\tilde{a}_\mu^w[k] = \begin{cases} a_\mu[k] & \text{if } \phi_\mu[k] \leq \theta_{\text{tx}} \\ 0 & \text{if } \phi_\mu[k] > \theta_{\text{tx}} \end{cases} \quad (3.5)$$

Where

$$\phi_\mu[k] = \left| \text{phase}\{a_\mu[k]\} - \text{phase}\{a_\mu^w[k]\} \right| \quad (3.6)$$

The resulting OFDM symbol in the time domain may be presented as

$$\tilde{x}_\mu[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_\mu[k] \cdot e^{j\frac{2\pi}{N}kn}, \quad 0 \leq n \leq N-1 \quad (3.7)$$

Only those sub-carriers having more phase difference than θ_{tx} between the original symbol $x_\mu[n]$ and the windowed symbol $x_\mu^w[n]$, are changed and forced to have zero magnitude. In this way whole of the time domain signal is not distorted, as happens in the case of simple windowing but high peaks are efficiently suppressed.

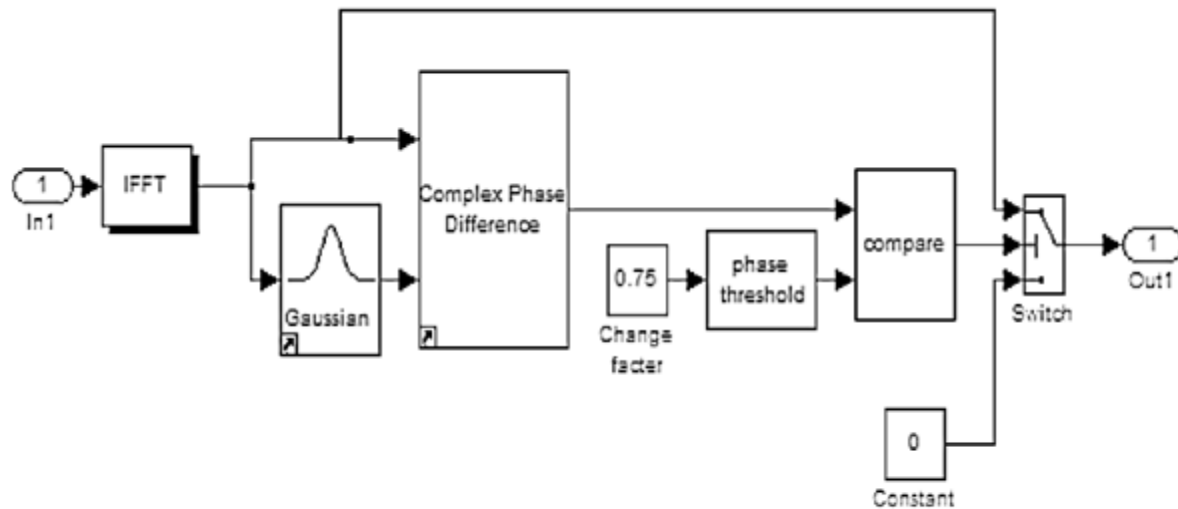


Figure 3.8 Block diagram of ZFP used at the Transmitter end

Block diagram of the ZFP part used in the transmitter side to reduce the PAPR is shown in the Figure 3.8. After IFFT, the signal is windowed. The phase of the windowed signal and the original signal are compared and stored in vector ϕ_n . Using the values of this vector it is decided to transmit $a_\mu[k]$ (the original magnitude of the sub carrier) or zero. The signal is transmitted on the Rayleigh multipath channel. As the sub carriers causing peaks are forced to have zero magnitude so overall PAPR is reduced. The index of sub-carriers which are forced to have zero magnitude is transmitted as side information. In MPSK OFDM system there are almost M^2 patterns that yield the highest PAPR and the probability of observing such PAPR is $M^2/M^N = M^{2-N}$. So numbers of bits which are to be sent as side information for N point IFFT (N sub-carriers) cost only $\log_2 N$ bits.

On the receiver end the signal is demodulated utilizing the side information, for which sub carriers values are to be boosted to have a_{fix} value before demodulation. The a_{fix} is the maximum magnitude which was allowed on the transmitter side. The value of a_{fix} is very important as it decide the value of PAPR allowed in the system and once the system is designed for a certain PAPR then there is no use of reducing PAPR any further.

3.2.21. Learning Vector Quantization –Selected Mapping Technique.

3.2.21.1. The Usefulness of detecting of High PAPR Symbols

One fact about the PAPR ratio problem is that the probability of occurrence of high PAPR symbols is very low. With 24 sub carriers ($N=24$), the probability of the PAPR exceeding 4dB is 10^{-2} and of exceeding 8dB is only 10^{-4} [51]. It can be shown that for IEEE 802.11.a (Hiperlan 2) $N=58$, QPSK ($M=4$), $Pr [PAPR=N]= 4^{-56}$ which implies that it happens once every $5.2 \cdot 10^{20}$ years. The idea of this technique is that we can efficiently deal with the issue of high PAPR by first detecting these less probable, high PAPR symbols and then processing only them, leaving rest of the data to be transmitted unchanged. In this way we reduce the number of symbols to be operated which is highly useful to reduce computational complexity. We use unconventional computing technique (i.e. non-parametric classification using learning vector quantization) to decide what set of operations the selected symbols have to undergo to achieve reduced PAPR. More precisely we use pattern classification to achieve efficient PAPR reduction.

3.2.21.2. Memory Based Pattern Recognizer

First we concentrate on learning pattern recognition in the light of current developments in machine learning. The goal in pattern recognition is to synthesize reliable machines that group complex input data (or patterns) into categories (or classes) with the help of a (supervised) learning device that uses a set of labeled patterns. The core of a pattern recognizer is usually composed of a feature extractor and a classifier. The feature extractor reduces the input by measuring certain invariant “features” or “properties”. (This helps to reduce the complexity of the original problem and the design of the classifier). The classifier then uses these features to make the decision of assigning the input pattern to a class. Efficient feature extraction is crucial for reliable classification and if possible these two subsystems should be matched optimally in the design of a complete pattern recognition system.

In this chapter, we concentrate on classification only. As in our application we have digital signal at the input. There is no noise or interference and we have to segregate fewer high PAPR valued symbols from a large number of low PAPR valued symbols and classify them in different classes depending on what set of operations they have to undergo to have reduced PAPR before transmission. Furthermore, supervised classification using LVQ neural network methods is considered.

There are many non-parametric classification methods but among them, Learning Vector Quantization (LVQ) classifiers are among the most famous in machine learning. Particular attention to this method in several fields like statistics, machine learning and pattern recognition has been paid. This memory-based method reveal, in spite of its simplicity, as very powerful non-parametric classification system in real world problems [52]. When an input pattern is presented to these classifiers, they compute the closest prototypes to it, using a distance metric defined by the user. Then the classifier assigns a class label to the input vector which is same as to the nearest prototypes. The parameters of these classifiers to be estimated in the learning phase are often the set of prototypes and (sometimes) the distance metric. The most direct approach to compute the prototypes is to store the whole training database as the set of prototypes (this would be the case of Nearest Neighbor classifier). However, storage and computational requirements advocate the use of more condensed sets as done in Learning Vector Quantization (LVQ) classifier.

3.2.21.3. LVQ-SLM Technique

As described in chapter three, SLM is one of the most promising PAPR reduction technique, however, the method also suffered from two major limitations. One is the side information required to be transmitted. This has been resolved by use of Blind SLM (BSLM) [53] and SLM without explicit side information [54]. The other shortcoming of the SLM scheme is its computational complexity that seriously affects its utilization in a practical OFDM system. The conventional SLM scheme reduces the PAPR values at the expense of using “D” independent phase sequences. Compared with the standard system, an SLM scheme integrated OFDM system will need “D –1” additional IFFTs. Utilizing the SLM scheme in an OFDM system with a large number of sub carriers will usually bring significantly high computational complexity. At the same time, the high cost to implement it will also counteract its PAPR reduction performance. Thus, to find an efficient way to reduce the computational complexity while keeping a decent PAPR reduction performance is a very important research topic for SLM implementation.

Some methods are shown in the literature. In [55], Wanget et al. proposed low-complexity conversions to replace the IFFT blocks in the conventional SLM method. Their method uses a

few IFFT blocks to generate the set of candidate signals. In [56], Lim et al proposed a low computational complexity SLM scheme which transforms an input symbol sequences to the signal after a certain intermediate stage of IFFT. Then the OFDM signal with the lowest PAPR is selected for transmission. Most recently Seok-Joong Heo in [57] propose a modification of SLM to reduce the computational complexity of the SLM.

Differing from these approaches, we focus on reducing the number of mappings to only one, by the use of a pre-trained classifier resulting, significant reduction in computational complexity.

3.2.21.4. Parameters of proposed LVQ.

The input nodes of the LVQ neural network depend on number of bits in OFDM symbol which further depends on total number of IFFT points. The output layer consists of “C” the number of different classes in which the input data was labeled (e.g. four classes representing the four different mappings). This layer represents the target classes. The first layer is a competitive layer representing the prototype weight vectors. The prototype weight vectors classify the input vectors into subclasses. Both the competitive and linear layers have one neuron per (sub or target) class. The structure of the neural network is N-CV-C assuming “N” is the number of bits in an OFDM symbol, CV is number of code vectors or prototypes (subclasses), and “C” is the total number of classes in which the input data space is labeled.

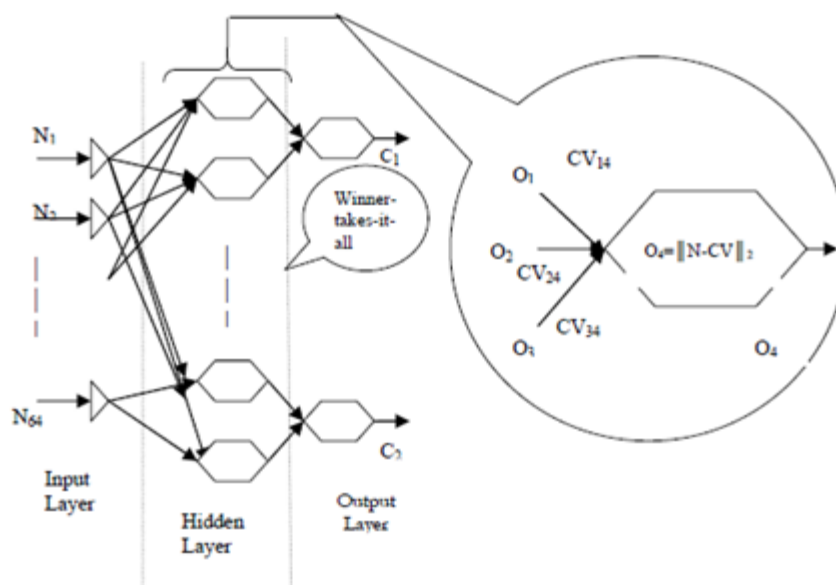


Figure 3.9 LVQ architecture: adjustable weight between input and hidden layer and a winner takes it all (WTA) mechanism.

We create a two-layer network. The first layer uses the “**compet**” transfer function, calculates weighted inputs with “**negdist**” (which compute the negative Euclidean distance), and net input with “**netsum**” (Net input functions calculate a layer's net input by combining its weighted inputs and biases.). The second layer having “**purelin**” neurons calculates weighted input with “**dotprod**” and net inputs with “**netsum**”. Neither layer has biases. First layer weights are initialized with midpoint. The second layer weights are set so that each output neuron “**i**” have unit weights coming to it from **PC(i)** percent of the hidden neurons. Adaption and training are done which both update the first layer weights with the specified learning functions.

3.2.22. Other Techniques

There are some other techniques which cannot be categorized in any one class of reduction techniques. These are briefly described below.

In **clustered OFDM technique** [58], clustering of sub carriers into a number of smaller blocks is done before transmitting them on different antennas. PAPR reduction is achieved because the number of sub-carriers for each transmitter is reduced. The main disadvantage is that number of power amplifiers is increased.

Two-dimensional pilot symbol assisted modulation (2D-PSAM) [59] is a distortion free technique for reduction of PAPR and can also be used for channel estimation. Although the scheme is complex, however it can be reduced if the sequence used is properly designed.

Carrier-Interferometry OFDM (CI/OFDM) [60] reduces PAPR by utilizing CI phase codes for transmitting each bit on every N carriers. The phase codes applied to the N carriers result in one bit's power reaching a maximum, when the powers of the remaining N-1 bits are at a minimum.

Another technique, **artificial signals**, is based on the fact that data is not carried on all **M** frequencies when M-point FFT/IFFT is used by transceiver. Hence few empty carriers are present in OFDM symbol. Sine waves are added in empty carriers to reduce PAPR of the OFDM signal. The addition of only two artificial signals can achieve a 6 dB reduction in PAPR for 16 carriers [61]. The disadvantages include additional power for transmission of artificial signals and increased complexity.

3.3. Comparison of PAPR Reduction Techniques

A number of techniques which were developed in last two decades to combat the PAPR problem in OFDM systems have been discussed. As we have seen that although the target is same but a lot of diverse approaches have been applied. Besides this, different views are presented of the impact of high PAPR on the system performance in the literature. Hence there is no consensus over the best solution for the problem at hand. A more detailed report [62] reviews the criteria which can be used to compare and select different PAPR reduction techniques. Table 3.1 summarizes advantages and drawbacks of various techniques presented to reduce PAPR in OFDM systems.

3.4. Essential Features in PAPR Reduction Technique

A number of factors need to be well thought-out in evaluation of any PAPR reduction technique.

3.4.1. PAPR Reduction performance

A vital factor in selection of any technique is its performance as regards to PAPR reduction. This means how much capable the technique is as regards to reduction of PAPR. This capability of any technique is algorithm dependent. For example Interleaver technique PAPR reduction capability is less than that of SLM. However this PAPR reduction performance must be judged while giving a cautious thought to other detrimental effects which may result. Take an instance of clipping technique which has very high performance as regards to PAPR reduction but the amount of resultant in-band distortion and out-of-band radiation is intolerable.

3.4.2. Transmit Signal power increase

It is necessary in a number of techniques that the power of transmit signal should be increased. Take an instance of TR, which needs additional power because PRCs employed in this technique also need power. The original constellation point is replaced by equivalent constellation points in TI. These equivalent constellation points need additional power as compared to original point, hence the power of transmit signal is increased in TI. The normalization of transmit power to original level, results in degradation of BER performance.

3.4.3. Increased BER at the Receiver

An increase in BER at receiver end is another significant factor. The increase in transmit signal power and increased BER at receiver are interrelated. For instance in few techniques e.g. ACE, the BER is increased when transmit power is fixed. BER may also be increased due to other reasons like errors in side information. For example the error in side information in PTS, SLM and interleaving can result in loss of entire data block hence a resultant increase in BER.

3.4.4. Data Rate loss

Few techniques while tackling the PAPR result decrease in data rate. As shown in the previous example, the block coding technique requires one out of four information symbols to be dedicated to controlling PAPR. In PTS, SLM and interleaving, the loss in data rate is due to transmission of side information utilizing some of the carriers. Since transmission of side information without errors is critical for retrieval of data, sometimes channel coding is used to protect against side information errors which further augments the problem of data rate loss.

3.4.5. Computational Complexity

When considering the most appropriate algorithm for a PAPR reduction technique of OFDM system the issues of computational complexity, hence power consumption and latency involved due to computations, are of major concern and have to be evaluated against potential increase in data bandwidth. In some schemes the signal processing needed for minimizing PAPR may be sufficiently complex that even leading edge technology gate arrays would not meet the speed requirements for broadband data transmission within acceptable battery power and cost constraints. PAPR reduction may also incur an additional processor load in the receive path, for example, where a complex decoding or error-correction scheme is necessitated. So to check the applicability of any proposed technique employed for reduction of PAPR, it is to be seen that how much penalty in terms of additional computations it will cost. For example the performance for PAPR reduction is increased by increasing the number of inter-leavers but with every Interleaver addition we have additional computational cost. Generally, more complex techniques have better PAPR reduction capability.

3.4.6. Other considerations

Many of the PAPR reduction techniques do not consider the effect of the components in the transmitter such as the transmit filter, digital-to-analog (D/A) converter, and transmit power amplifier. In practice, PAPR reduction techniques can be used only after careful performance and cost analyses for realistic environments.

Table 3.2 Comparison of PAPR reduction techniques

	distortion less	Power Increase	Data rate Loss	Computational Complexity	
				Transmitter processing	Receiver processing
Clipping	No	No	No	Amplitude clipping and filtering	none
Coding	Yes	No	Yes	Encoding or table search	Decoding or table search
PTS	Yes	No	Yes	M IFFTs, WM-1complex vector sums	Side information extraction, inverse PTS
SLM	Yes	No	Yes	U IFFTs	Side information extraction, inverse SLM
Interleaving	Yes	No	Yes	K IFFTs, K-1 interleaving	Side information extraction, inverse interleaving
TR	Yes	Yes	Yes	IFFT and find values of PRCs	Ignore non data bearing sub carriers
TI	Yes	Yes	No	IFFTs, search for maximum point in time, tone to be modified	Modulo D operation
ACE	Yes	Yes	No	IFFTs, projection onto shaded area	none

Chapter 04

Proposed Combined Hadamard and Discrete Cosine Transform in SLM

4.1. The Selected Mapping Technique

In the SLM technique, the transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and selects the most favorable for transmission [15, 16]. A block diagram of the SLM technique is shown in Fig. 4.1 Each data block is multiplied by U different phase sequences, each of length N ,

$$B^{(u)} = [b_{u,0}, b_{u,1}, \dots, b_{u,N-1}]^T, u = 1, 2, \dots, U, \quad (4.1)$$

Resulting in U modified data blocks. To include the unmodified data block in the set of modified data blocks, we set $B^{(1)}$ as the all-one vector of length N . Let us denote the modified data block for the u -th phase sequence

$$X^{(u)} = [X_0 b_{u,0}, X_1 b_{u,1}, \dots, X_{N-1} b_{u,N-1}]^T, u = 1, 2, \dots, U. \quad (4.2)$$

After applying SLM to X , the multicarrier signal becomes.

$$x^{(u)}(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n b_{u,n} e^{j2\pi n \Delta f t}, 0 \leq t \leq NT, u = 1, 2, \dots, U \quad (4.3)$$

Among the modified data blocks $X^{(u)}$, $u = 1, 2, \dots, U$, the one with the lowest PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information. At the receiver, the reverse operation is performed to recover the original data block. For implementation, the SLM technique needs U IDFT operations, and the number of required side information bits is $\lceil \log_2 U \rceil$ for each data block.

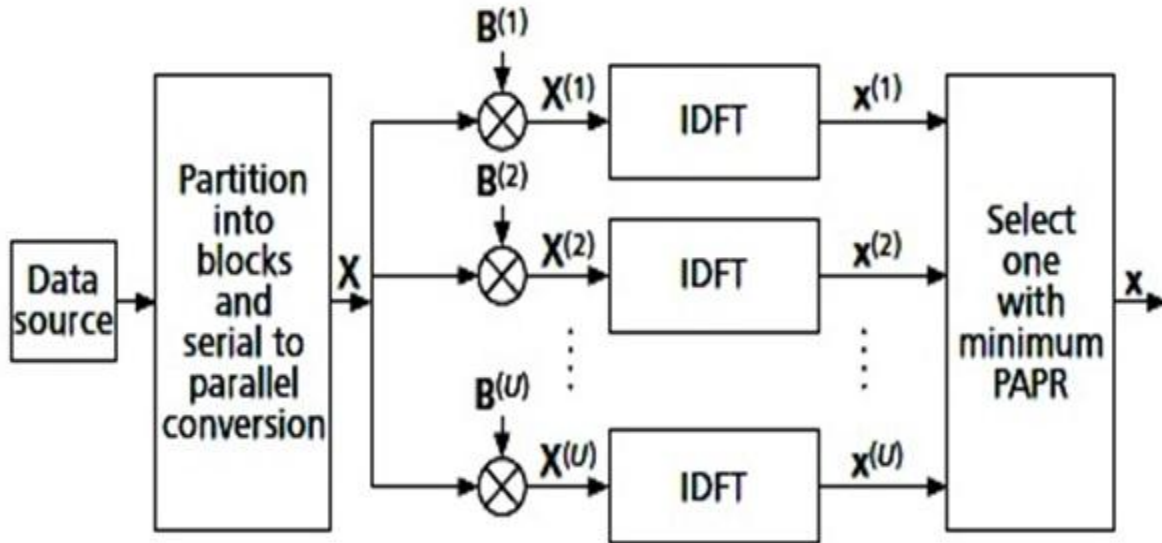


Figure 4.1 Block diagram SLM technique

This approach is applicable with all types of modulation and any number of subcarriers. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the design of the phase sequences. In [63] an SLM technique without explicit side information is proposed.

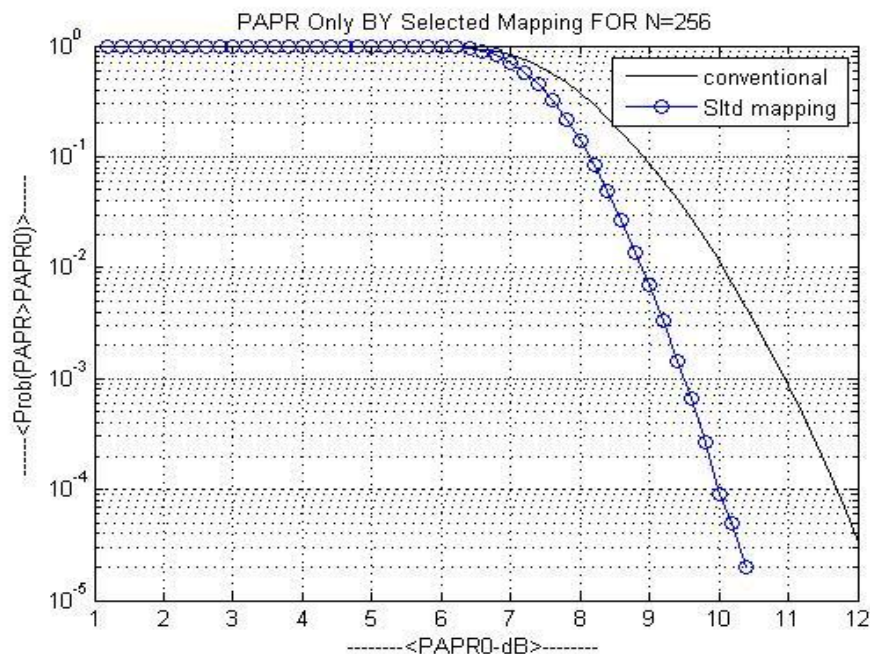


Figure 4.2 Performance of selected mapping technique

Figure 4.2 shows the performance of selective mapping technique. The technique shows that the PAPR is reduced by 2.133 dB

4.2. Hadamard Transform

The paper, by Park proposes a scheme for PAPR reduction in OFDM transmission using Hadamard transform. The proposed Hadamard transform scheme may reduce the occurrence of the high peaks comparing the original OFDM system. The idea to use the Hadamard transform is to reduce the autocorrelation of the input sequence to reduce the peak to average power problem and it requires no side information to be transmitted to the receiver.

The technique of Hadamard Transform is based upon the relationship between correlation property of OFDM input sequence and PAPR probability. The average power of the input sequence represents the peak value of the autocorrelation. Hence the peak value of autocorrelation depends on the input sequence provided that number of sub carriers remains unchanged.

We assume H is the Hadamard transform matrix of N orders, and Hadamard matrix is standard orthogonal matrix. Every element of Hadamard matrix only is 1 or -1. The Hadamard matrix of 2 orders is stated by

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (4.4)$$

Hadamard matrix of $2N$ order may be constructed by.

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \quad (4.5)$$

Where $-H_N$ is the complement of H_N . Hadamard matrix satisfy the relation

$$H_{2N} H_{2N}^T = H_{2N}^T H_{2N} = I_{2N} \quad (4.6)$$

Where H_{2N}^T is the transport matrix, I_{2N} is the unit matrix of $2N$ order.

After the sequence $X = [X_1 X_2 \dots X_N]^T$ is transformed by Hadamard matrix of N order, the new sequence is $Y = HX$

4.3. Discrete Cosine Transform (DCT)

Like other transforms, such as the Hadamard transform, the DCT de-correlates the data sequence. To reduce the PAPR in an OFDM signal, a DCT is applied to reduce the autocorrelation of the input sequence before the IFFT operation is applied. In this section, we briefly review the DCT. The formal definition of a one-dimensional DCT of length N is given by the following formula:

$$X_c(k) = \alpha(k) \sum_{n=0}^{N-1} X(n) \cos\left[\frac{\pi(2n+1)k}{2N}\right] \quad K = 0, 1, 2, \dots, N-1 \quad (4.7)$$

Where

$$\alpha(k) = \frac{1}{\sqrt{N}}$$

And

$$X_c = C_n x$$

Where X_c and x are both vectors of dimension $N \times 1$, and C_n is a DCT matrix of dimension $N \times N$. The rows (or column) of the DCT matrix, N , are orthogonal matrix vectors. We can use this property of the DCT matrix and reduce the peak power of OFDM signals.

4.4. Combined Hadamard & Discrete Cosine Transform With Selected Mapping Technique (SLM)

In our technique we used combined Hadamard Discrete Cosine transform and selective mapping methods as shown in Figure below.

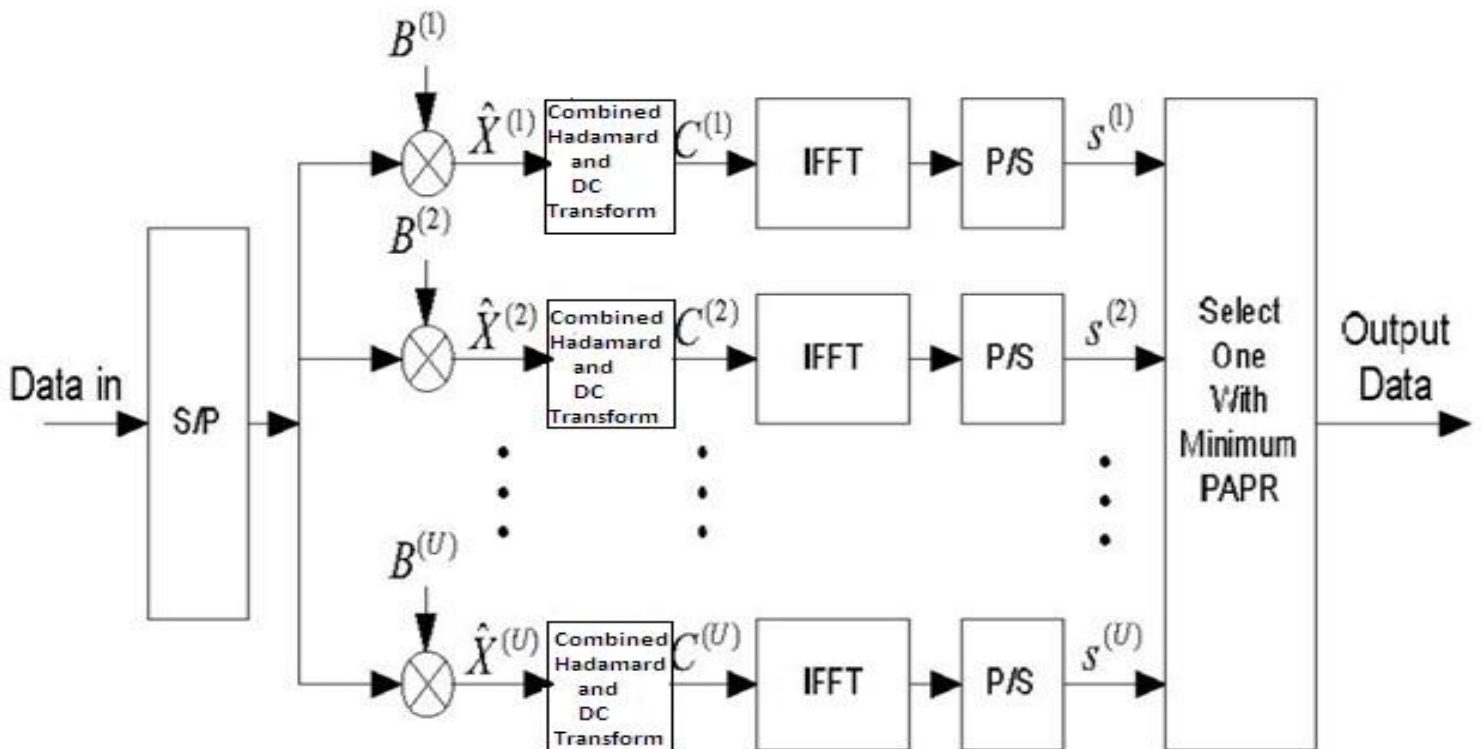


Figure 4.3 Block diagram of Proposed scheme

The input data stream is processed with a Hadamard transformation matrix & a DCT then with an IFFT signal processing unit and then by selective mapping method the data with lowest PAPR is been selected and transmitted through the channel. The function of Hadamard and Discrete Cosine transform is to reduce the occurrence of high peaks compared to original OFDM.

The PAPR reduction of SLM method and combined Hadamard DCT can get about 1 dB better PAPR reduction performance than conventional SLM Also, it can be expected that the more robust data transmission is available in nonlinear HPA and multi-path fading channel by PAPR reduction function and frequency diversity effect.

$$X = [X_0, X_1, \dots, X_{N-1}]^T \quad (4.8)$$

Thus, phase rotated vector can be written as

$$X^{(U)} = [X^{(U)}_0, X^{(U)}_1, \dots, X^{(U)}_{N-1}]^T, \quad (4.9)$$

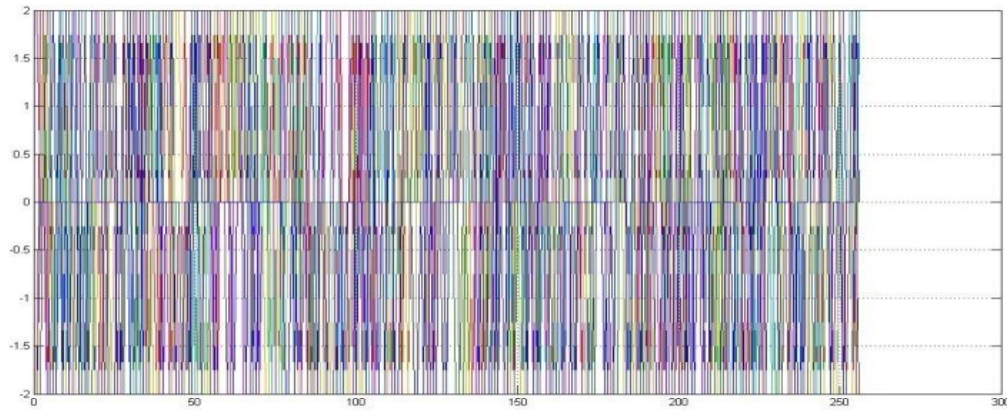


Figure 4.4 Time domain representation of OFDM signal before Hadamard transform

So, each branch signal through Hadamard transform can be defined as

$$C1_K^U = \sum_{n=0}^{N-1} h_{k,n} X^{(U)}_n, \quad K = 0, 1, 2, \dots, N - 1 \quad (4.10)$$

Then after DCT

$$C_k^U = \alpha(k) C1_k^U \sum_{n=0}^{N-1} X_n^U \cos\left[\frac{\pi(2n+1)k}{2N}\right] \quad K = 0, 1, 2, \dots, N - 1 \quad (4.11)$$

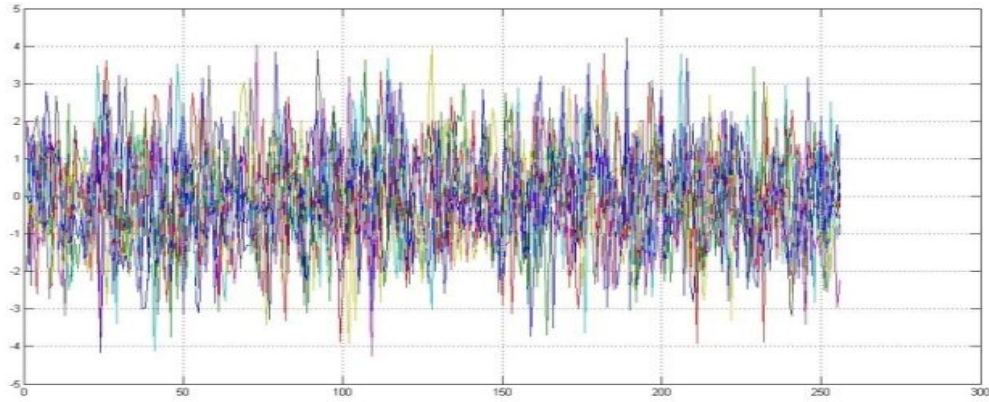


Figure 4.5 Time domain representation of OFDM after Hadamard transform

Thus, the signals after passing through IFFT can be written as

$$S^{(U)}(t) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} C_K^{(U)} e^{j2\pi\Delta ft} \quad 0 \leq t \leq NT \quad (4.12)$$

By this method, we can get more PAPR reduction and we can additionally expect the frequency diversity benefit because of hadamard transform

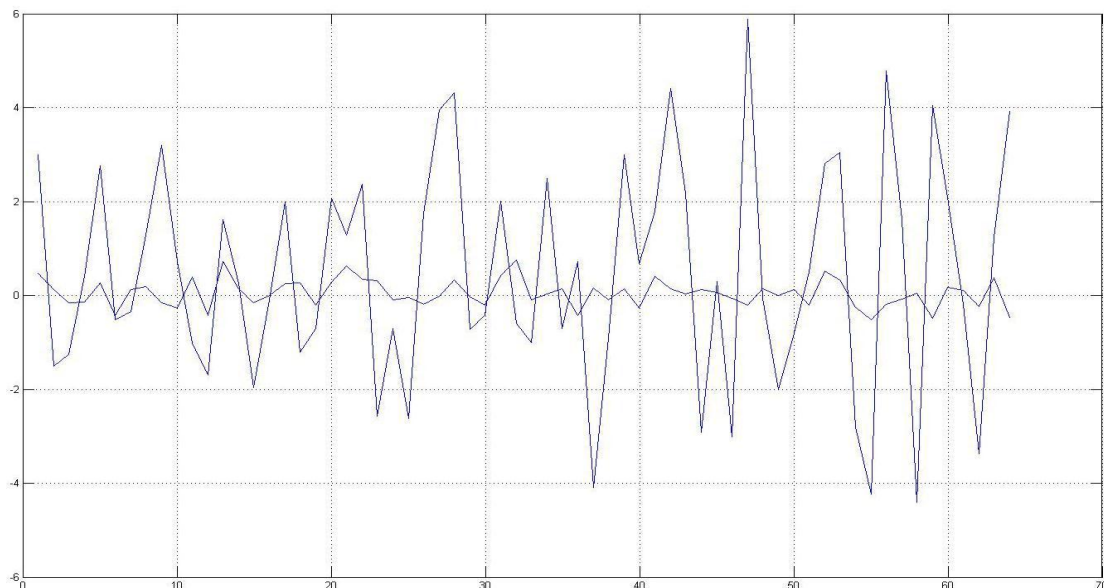


Figure 4.6 Comparisons of time-domain signals before and after technique

The figure above shows two time-domain signals, one without application of our scheme and the other after the application of our proposed scheme. Clearly it can be seen that without PAPR improvement, there is high peak compared to the average power of the signal. But after applying our proposed scheme, the high peaks are greatly reduced and they become comparable to the average power of the signal. The PAPR is reduced since the peak power becomes comparable to the average power of the signal.

4.4.1. CCDF Performance

We can evaluate the performance of PAPR using the cumulative distribution of PAPR of OFDM signal. The cumulative distribution function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of and PAPR technique. The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(-z) \quad (4.13)$$

However, the complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. A plot of CCDF vs PAPR of the data block is desired in our case to compare outputs of various reduction techniques. This is given by

$$P(\text{PAPR} > z) = 1 - P(\text{PAPR} \leq z) = 1 - (1 - \exp(-z))^N \quad (4.14)$$

Simulation Results are shown on next pages

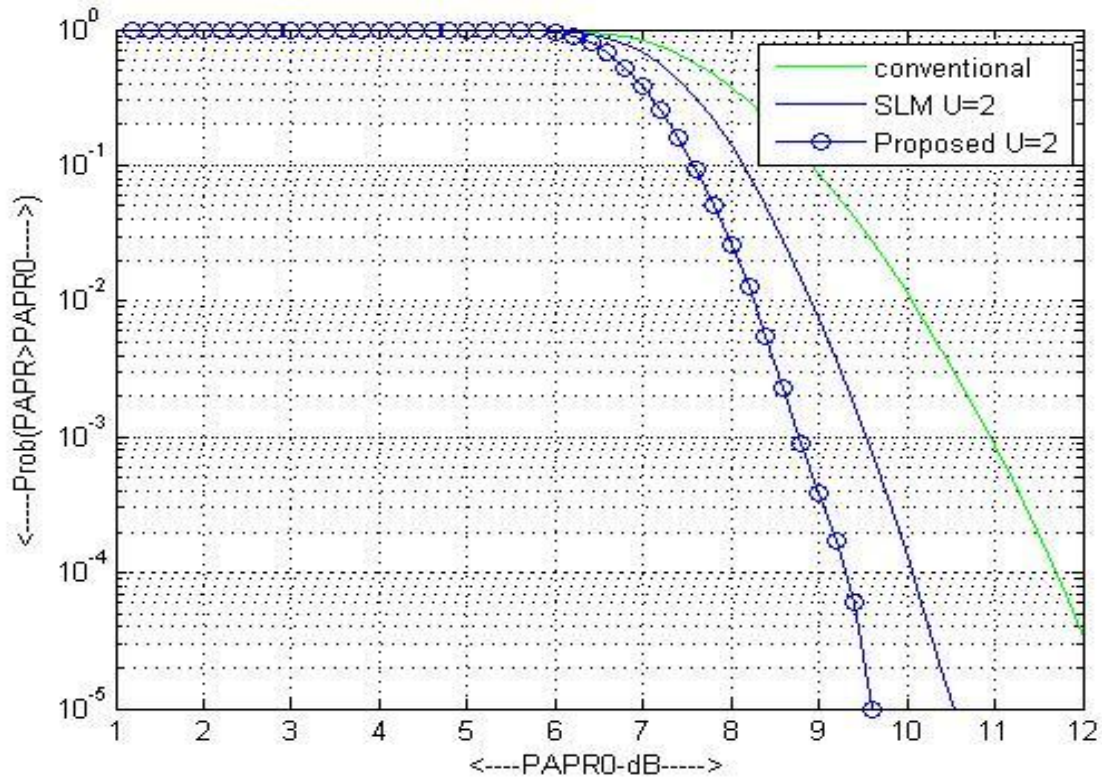


Figure 4.7 PAPR reduction performance of SLM and Proposed technique For N=256 and U=2

The complementary cumulative distribution functions (CCDFs) of PAPR are numerically obtained for the conventional SLM scheme and the Proposed scheme. The simulation results are shown in Fig. for 1,00,000 input symbol sequences, where show the probabilities that the PAPR of OFDM signal sequences exceeds the given PAPRO for N = 256. The performance of the proposed scheme with U = 2 is better than that of the conventional SLM scheme with U = 2 by 0.63 dB

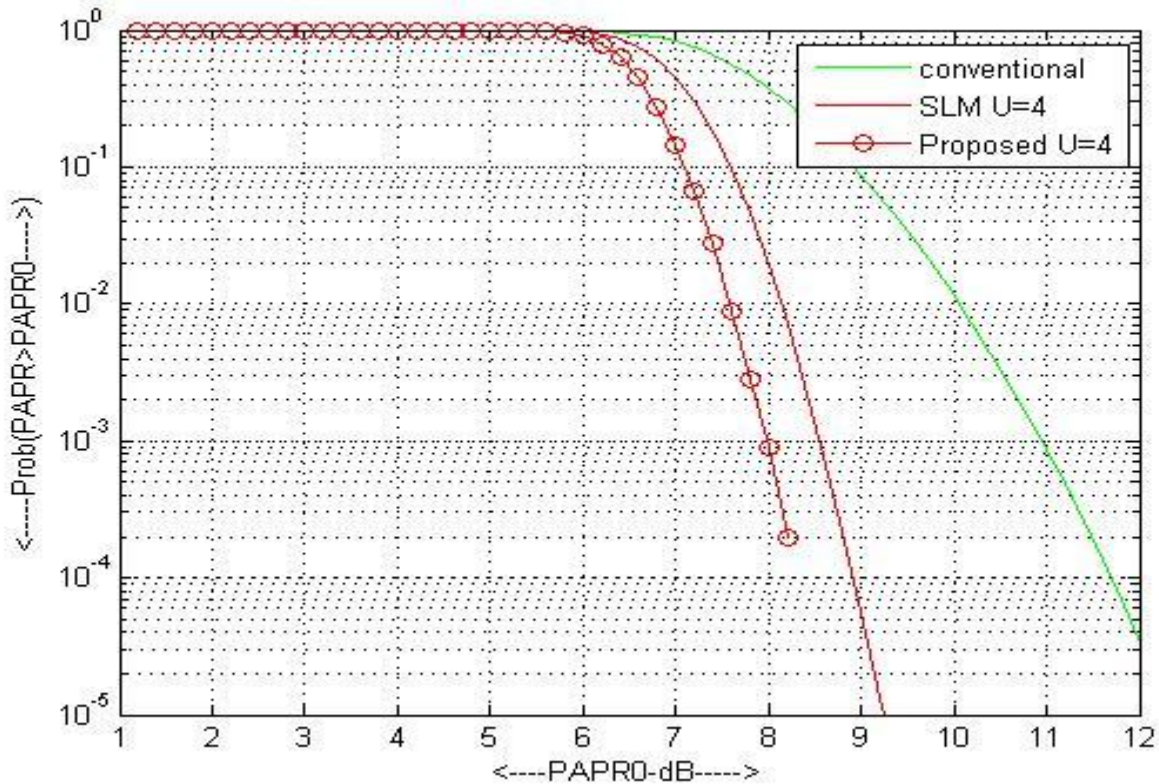


Figure 4.8 PAPR reduction performance of SLM and Proposed technique For N=256 and U=4

The complementary cumulative distribution functions (CCDFs) of PAPR are numerically obtained for the conventional SLM scheme and the proposed scheme. The simulation results are shown in Fig for 1,00,000 input symbol sequences. The performance is 0.92dB better in the proposed scheme than the conventional SLM scheme for N=256 and U=4.

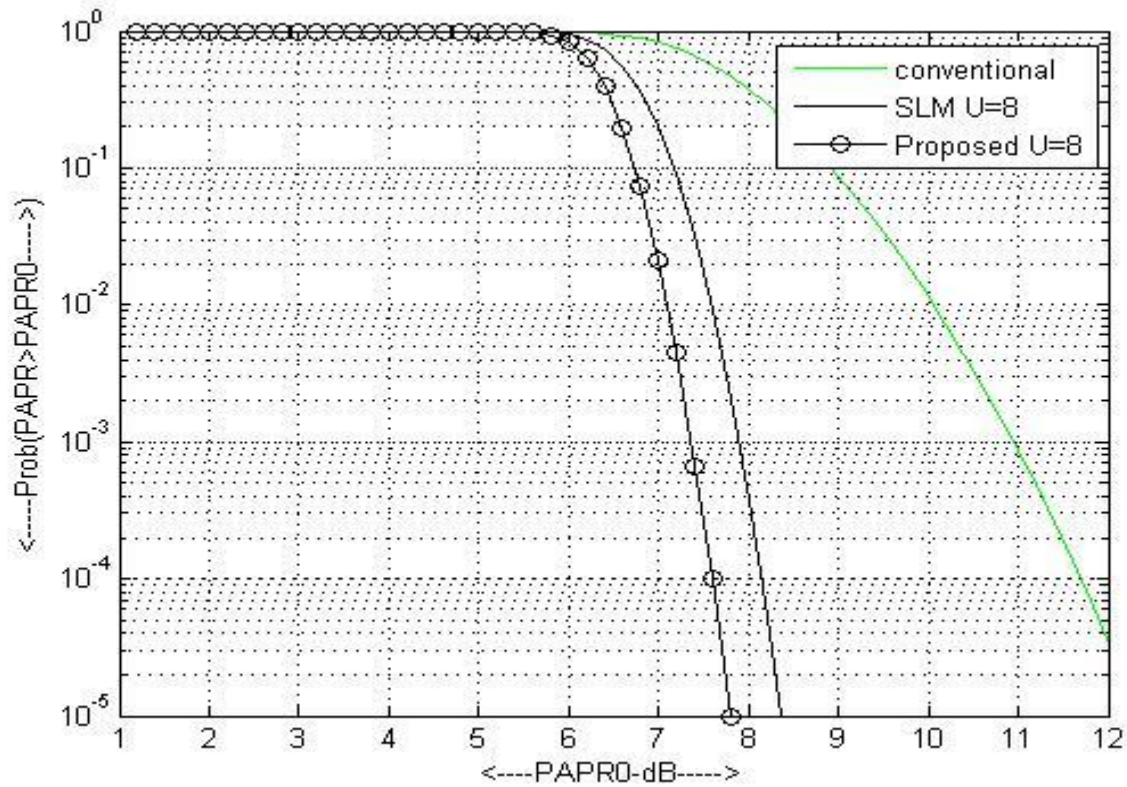


Figure 4.9 PAPR reduction performance of SLM and Proposed technique For N=256 and U=8

The complementary cumulative distribution functions (CCDFs) of PAPR are numerically obtained for the conventional SLM scheme and the proposed scheme. The simulation results are shown in Fig for 1,00,000 input symbol sequences. The performance is 0.39dB better in the proposed scheme than the conventional SLM scheme for N=256 and U=8.

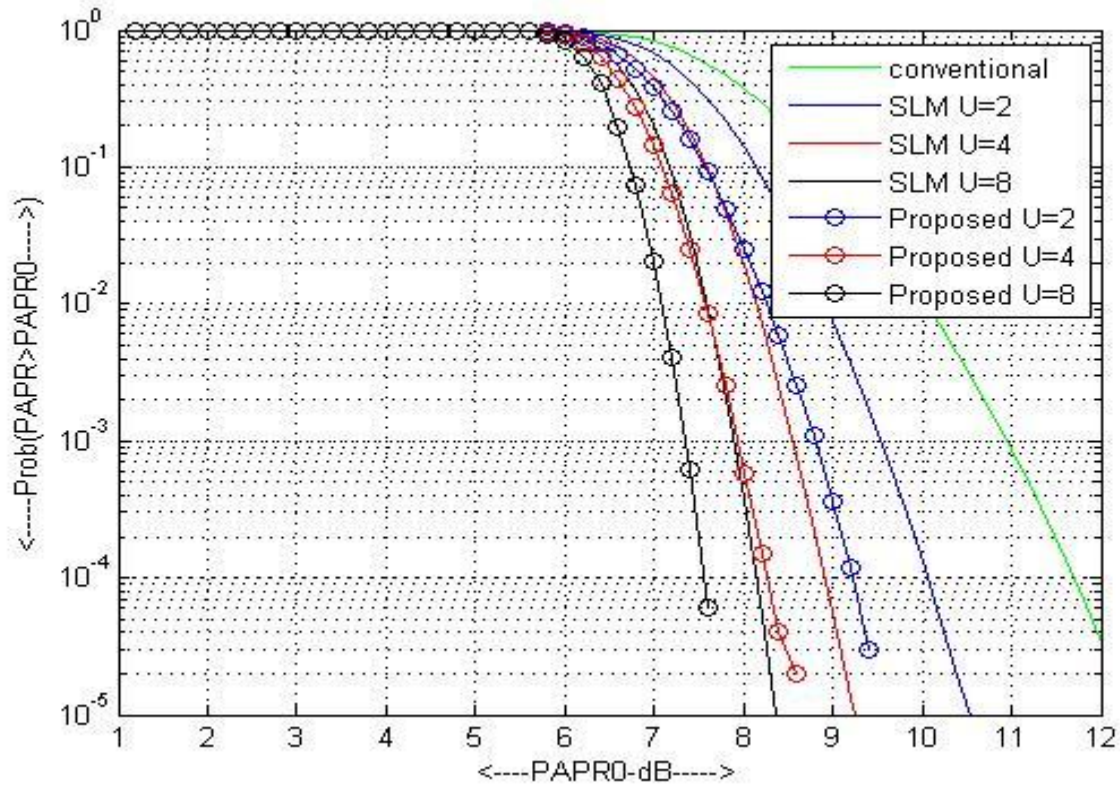


Figure 4.10 PAPR reduction performance of SLM and Proposed technique For N=256

The figure shows that the proposed scheme with $U = 2, 4$ and 8 has better performance compared with the conventional SLM scheme with $U = 2, 4$ and 8 respectively.

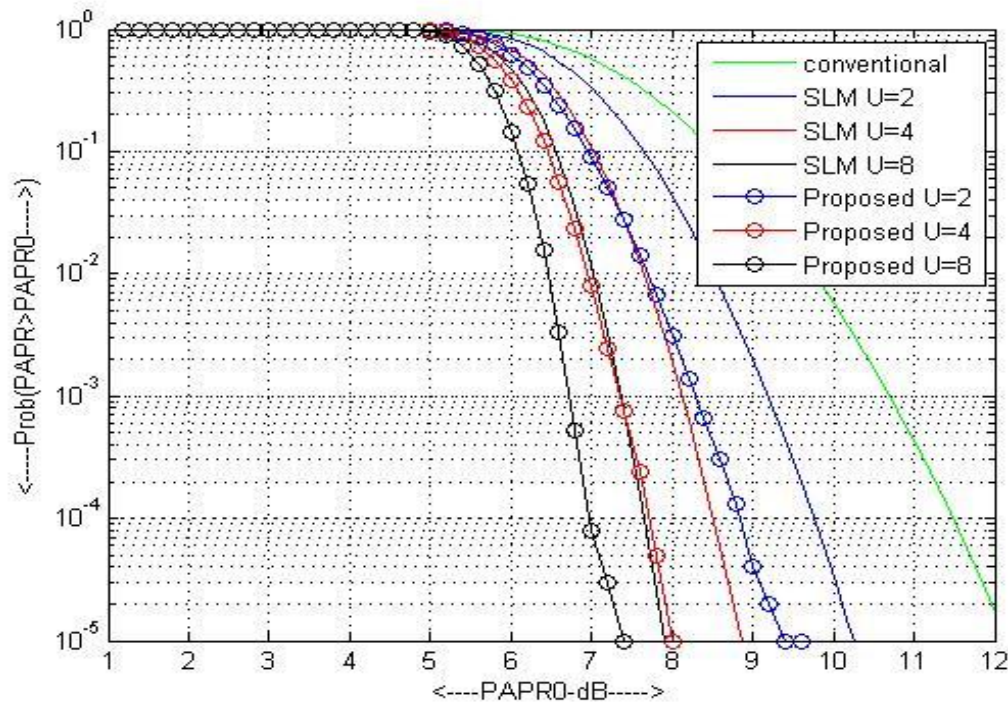


Figure 4.11 PAPR reduction performance of SLM and Proposed technique For N=128

The above figure shows that for decreasing N the PAPR is reduced more.

4.4.2. Analytical Results

Technique Used	U	PAPR Before Technique	PAPR After Technique	
			N=256	N=128
Selective mapping Alone	2	12.643	10.51	10.23
	4		9.19	8.9
	8		8.27	7.93
Proposed Technique	2	12.643	9.88	9.33
	4		8.27	8.0
	8		7.88	7.45

Table.4.1 Result for PAPR of both Conventional and Proposed techniques

Table.1 shows the result for PAPR of selective mapping technique and proposed technique for different number of U and N. Here the original PAPR of OFDM signal is 12.643 and the simulated result is different for different number of U and N. For N=256 the selective mapping gives improvement of PAPR by 2.133 for U=2, 3.453 for U=4 and 4.373 for U=8 for probability of 10^{-5} . And the Proposed (combined Hadamard and discrete cosine transform with selective mapping) technique gives improvement of PAPR by 2.763 for U=2, 4.373 for U=4 and 4.763 for U=8 for probability of 10^{-5} .

Similarly for N=128 the selective mapping gives improvement of PAPR by 2.413 for U=2, 3.743 for U=4 and 4.713 for U=8 for probability of 10^{-5} . And the Proposed technique gives improvement of PAPR by 3.313 for U=2, 4.643 for U=4 and 5.193 for U=8 for probability of 10^{-5} .

U	PAPR reduced by SLM		PAPR reduced by proposed Technique		Greater PAPR reduced by
	N=256	N=128	N=256	N=128	
2	2.133	2.413	2.763	3.313	Proposed Technique
4	3.453	3.743	4.373	4.643	Proposed Technique
8	4.373	4.713	4.763	5.193	Proposed Technique

Table.4.2 Comparison between Conventional SLM and Proposed technique

Table.4.2 shows the comparison between conventional SLM and Proposed method. It is clear from the table that greater PAPR is reduced by the proposed technique

4.4.3. Comparison of BER and SNR

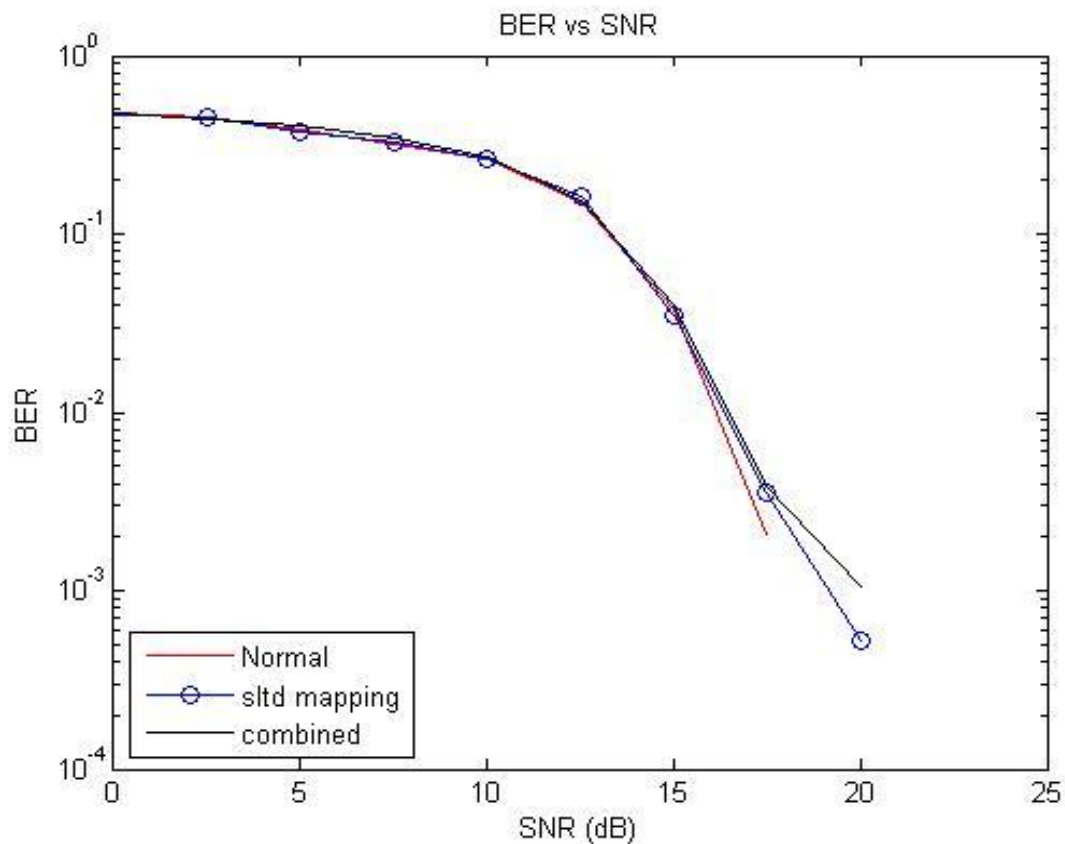


Figure 4.12 Performance of BER VS SNR

Figure 4.12 is the BER performances of OFDM system with proposed PAPR reduction scheme. We can see that the performance of system is not degraded compared to the system with SLM technique.

4.4. Analysis of Algorithm Complexity

The computational complexity of the proposed scheme is increased because the DCT is used. However, like FFT, there are many fast methods to computer DCT.

And another method to decrease complexity is the modification in selected mapping technique.

Chapter 05

Conclusion

5.1. Conclusion

OFDM is a very appealing technique for achieving high-rate wireless data transmission. However, the potentially large PAPR of an OFDM signal has limited its application for practical implementations. In order to solve this OFDM peak power problem, many techniques have been proposed. SLM seems to be a powerful and efficient method known to reduce OFDM peak power without nonlinear distortion, but the computational complexity has been increased. In this thesis, a combined scheme for reducing the PAPR is proposed. By this method, we can get more PAPR reduction and we can additionally expect the frequency diversity benefit because of DCT and Hadamard transform.

In this thesis, a joint discrete cosine transform (DCT) and Hadamard transform method with SLM technique is proposed to reduce peak-to-average of OFDM signal. Simulation results show that the proposed scheme may obtain significant PAPR reduction while maintaining good performance in the BER compared to ordinary Hadamard method. So, we expect that this method can reduce peak to average power ratio (PAPR) of an OFDM signal without much degradation of system performance.

The computational complexity of the proposed scheme is increased because the DCT is used. However, like FFT, there are many fast methods to compute DCT. And another method to decrease complexity is the modification in selected mapping technique which is described in section.

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