ISLAMIC UNIVERSITY OF TECHNOLOGY

ORGANIZATION OF ISLAMIC COOPERATION

Analysis of Different types of cracks on Beams and Plates

B.Sc Engineering (Mechanical) Thesis

Authors

Soad Shajid (160011079)

Sazzadul Islam Sajol (160011076)

Supervised By

Dr. Md. Zahid Hossain
Professor
Department of Mechanical and Production Engineering
Islamic University of Technology (IUT)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING
ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

March, 2021

Candidate's Declaration

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Signature of the Candidates'

Soad Shajid

Student No: 160011079

Sazzadul Islam Sajol

Student No: 160011076

Department of Mechanical and Production Engineering (MPE)

Islamic University of Technology (IUT), OIC

Board Bazar, Gazipur, Dhaka, Bangladesh.

Signature of the Supervisor

Dr. Md. Zahid Hossain

Professor

Department of Mechanical & Production Engineering (MPE)

Islamic University of Technology

ACKNOWLEDGEMENT

The authors express gratefulness to the Almighty Allah (SWT) for his blessings, which enabled to complete this thesis successfully.

The authors express gratitude to their supervisor Professor, Dr. Md. Zahid Hossain, Department of Mechanical and Production Engineering (MPE), Islamic University of Technology (IUT) for his continuous guidance, helpful suggestions and supervision at all stages of this thesis work.

And the authors are indebted to their family members for providing the financial and mental support in perusing the Bachelor’s degree in Mechanical Engineering.

Table of Contents

1. Abstract:

Beam and plates in existence of crack is vulnerable to failure reckoning on the different mode of vibration. The natural frequency of the beams and plates and the superposition of periodic force acting on beams and plates form resonance which is the main cause of failure. It is crucial to find out natural frequency to remain aware about resonance that occurs due to periodic load. In this research work, we have used "Solidworks" software to design beams and plates without and with different types of cracks like slit crack, triangular crack, angular crack, semi-circular crack etc. Afterwards, we have used "ANSYS" simulation software to find the natural frequency and mode shapes of cracked and Non-cracked beams and plates for first six modes through modal analysis and made a through investigation of the vibrational behavior of different modes and mode shapes and made different comparison which can be used as one of the criteria for identification of cracks on beams and plates. The analysis has been extended to evaluate the effect of mesh refinement and crack opening size. For cracked beam, analysis is performed for various crack location and crack depth. In our work, we have observed that due to existence of crack, natural frequency changes. The amount of change varies depending on crack location, depth and crack opening size. Furthermore, we used harmonic analysis to find the deflection curves which can be used as one of the other criteria for crack identification.

2. Introduction:

In engineering applications, beams are one of the most widely used structural components. Beams may be circular or non-circular, and their supports are used to classify them into various forms. Since cracks are a major cause of component failure/damage, it is critical to examine any cracks that occur in the structure early on protecting it from potentially catastrophic failures. Cracks can form as a result of a variety of factors, including poor manufacturing, stress corrosion, hydrogen damage, and failures of different kinds of fatigue. The majority of system failures are currently caused by material fatigue. The load which is repeated in any kinds of mechanical element, fatigue failure is a possibility. Cracks appear on the component's surface in this mode of fatigue failure. If a crack develops in the component during service or usage, it may lead to failure of the component if it grows to a critical size. The shape and geometry of cracks has a major impact on the cracked rotor's properties which are dynamic basically.[1]

Cracks in structural elements such as beams and columns are caused by a number of factors. It's possible that they're fatigue cracks that form in operation due to the fatigue strength's limitations. Mechanical flaws, such as those found in jet engine turbine blades, may also be to blame. The cracks in thoses engines are created by small stones which are actually sucked from the runway surface. Another type of crack is one that occurs inside the material. Due to manufacturing processes, they are made. Vibrations inflict cyclic stresses on structures and system parts, which results the fatigue of the material and its's failure. Individual elements and the beam strength effects actually under loading condition have been extensively studied using experimental methods. Although this approach generates real-life responses, it basically consumes time and also those materials can be very expensive. These components have also been studied using finite element analysis. However, due to advances in understanding and capabilities of computer software and different kinds of hardware, the use of finite element analysis has increased in recent years. It is now the tool of choice for analyzing certain structural components.[2]

The occurrence of cracks in a structural member, such as a beam, results in local stiffness variations, the extent of which is primarily determined by the position and depth of the cracks. These variations, in turn, have a major impact on the overall structure's vibrational activity. It is critical to know if structural members are free of cracks and, if any are present, to determine their extent in order to ensure the secure operation of structures. Direct techniques such as ultrasound and X-rays are often used for identification.However, since these approaches necessitate costly and minutely thorough inspections, they have proven ineffective and unsuitable in some cases [3]. To overcome these drawbacks, researchers have concentrated in recent decades on more effective crack detection procedures based on vibration [4]. A critical feature of these approaches is crack modeling.

The majority of published studies [5-9] believe that a structural member's crack is still open during vibration. When dynamic loadings are dominant, however, this assumption may not be true. During vibration, the crack breathes (opens and closes) on a regular basis, creating changes in structural stiffness. The structure exhibits non-linear dynamic behavior as a result of these variations [10]. The existence of higher harmonic components is the most distinguishing feature of this action. A beam with a breathing crack, in particular, exhibits natural frequencies that are similar to those of a non-cracked beam and a faulty beam with an open crack.As a result, vibration-based approaches can use breathing crack models to provide reliable conclusions about the state of damage in these situations. Several researchers [11-13] have created breathing crack models that only consider the fully open and fully closed states of the crack. Experiments have shown, however, that the transition between these two crack states does not take place instantly [14]. Reference [15] used time-varying attachment matrices to describe the interaction forces between two segments of a beam separated by a crack. To simulate the opening and closing of a crack, these matrices were extended in Fourier series.However, the implementation of this study necessitates a significant amount of machine time. A basic periodic function was used to model the time-varying stiffness of a beam in references [16, 17]. This model, however, is restricted to the fundamental mode, necessitating the solution of the beam's equation of motion.

It is likely that buildings will be affected. As a result, early identification of these vulnerabilities tends to be important for both protection and economics. Mechanical omic reasons, such as mechanical vibration, environmental attack, and long-term operation, etc., mechanical omic reasons, as their identification will greatly prolong the structure's life, while also increasing its reliability. Several studies have been presented in recent years to identify damage from changes in different kinds of properties which are both static and dynamic basically. Andreaus and Casini [18] proposed a static deflection-based multiple damaged detection system.The method's basic principle is that the crack induces a local singularity in the displacement response, which can be identified using wavelet analysis. The static damage identification methods, on the other hand, have the disadvantage of providing less detail than dynamic damage identification methods. Harm can be detected using vibration-based detection methods by changes in the linear response (modal parameters) or the presence of nonlinear effects [19–21].

For a variety of mode forms, modal analysis yields various frequencies of the vibration. Modal analysis is critical for a cracked structure since cracks cause discontinuities in the structure. Discontinuities in a structure result in unique physical features when a structure fails. Local flexibility is introduced by a crack in a structural member, which affects the vibration response.

Basically, the structural members which is most commony uses and widely renowned is the cantilever beam. In addition, this structural function can be used in the design of stadiums, bridges, homes, high-rise towers, and a variety of other structures. As a result, a single crack in a cantilever beam may lead to the collapse of a massive structure. It is difficult to conduct modal analysis of a cantilever beam using an empirical method when there are discontinuities. The Finite Element Analysis method is the most powerful method to solve those types of problems to date, and in this research, “Abaqus” is used to perform all of the computations.Because the structural discontinuity and it's significant role, naval architects, offshore and ocean mechanics, hydro dynamologists, and mathematicians have performed extensive research. H. S. Rane, R.B. Barjibhe, and A.V. Patil [22] proposed a method for detecting the position and size of a crack in a cantilever beam based on natural frequency measurements. Numerical calculations were carried out by solving the equation of Euler for both crack & uncracked beams for obtaining the first three natural frequencies of different modes of vibration for the beam at different positions in the crack. The ANSYS 12 software package was used to perform finite element analysis on a total of ten models, with the opening size of the crack basically not being specified. M Quila, S. C. Mondal, and S. Sarkar [23] investigated the mode form frequency analysing those numerically at a relevant rate by using ANSYS. It had been presented the model where the analysis of free vibration in any kind of beam with an open edge crack. Normal frequency variations due to cracks at different locations and with differing crack depths were investigated. A parametric analysis was also carried out. However, there was not enough knowledge about mesh element type, scale, and refinement to duplicate their work.M. J. Prathamesh and M. A. Chakrabarti [24] investigated “Free Vibration Analysis of Cracked Beams” under different types of boundary conditions. The results of previous studies' experiments were compared to finite element analysis results. ABAQUS software was used to carry out the analysis. However, data on the effects of opening size of the crack and the refinement of mesh actually is missing from the analysis. ”A New Approach for Vibration Analysis of a Cracked Beam” was investigated by M. Behzad, A. Meghdari, and A. Ebrahimi [25].The Hamilton theory was used to establish different types of motion equation and perspective boundary conditions to bend vibration of a beam with an open edge crack in that article. In that study, a uniform Euler-Bernoulli beam was used. Using the model in conjunction which is basically newly developedwith the Galerkin projection process, the natural frequencies of the beam were measured. The findings revealed that increasing crack depth reduces the natural frequencies of a cracked beam.

Several researchers [26-28] have looked at the problem of a beam which has a condition of breathing in the relative crack by using models that reflect the crack in its fully open or either fully closed state. However, experimental evidence has shown that the transition case and its fluency between these two crack states is smoother. [No. 29] Abraham and Brandon [30] used time-varying link matrices to bind two segments of a beam separated by a crack. To model the alternation of crack opening and closing, these matrices were extended into several Fourier sequences. However, the computational effort required for this method is not insignificant.To model the time-varying stiffness of a beam, Cheng et al.[31] and Douka and Hadjileontiadis[32] used a simple periodic function. This model, however, is limited to the fundamental mode, requiring the solution of the beam's equation of motion. Kisa and Brandon [33] created a finite element model of a cracked beam with varying degrees of closure. They measured the natural frequencies of the cracked beam using modal superposition to model the transition field.Their model contains crack nonlinearities to help predict the natural frequencies of a cracked beam, but they didn't look at frequency changes caused by changes in oscillation amplitude. Instead, it was thought that the vibration would change linearly with amplitude.

Owing to a lack of basic understanding of certain aspects of the breathing system, creating a practical model of a breathing crack is difficult. This includes not only identifying variables that influence breathing crack activity, but also issues with assessing the fractured material's structural dynamic response. It's also not clear how partial closure interacts with the problem's main variables. Obviously, in the real world, a model that takes into account the breathing mechanism as well as the interaction between external loading and dynamic crack behavior is required.The field singular activity, the contact area, and the distribution of contact tractions on the closed region of the crack are all unknowns when crack contact occurs. Without crack closure, the above class of unknowns does not occur in the situation. This type of complicated crack surface deformation is a nonlinear problem that is too difficult to solve using traditional analytical methods. When partial crack closure occurs, a suitable numerical implementation is needed.

Finite element procedure : A 2D beam with a nonpropagating edge crack is considered in the following debate. The crack surfaces are considered to be smooth, and the crack thickness to be insignificant. The material properties of the beam are thought to be linear elastic, with slight displacements and strains. A collection of conventional finite elements is used to discretize the area around the crack. The breathing crack behavior is modeled as a complete frictional contact problem between crack surfaces, which is a nonlinear problem by definition. Coulomb's law of friction is believed to apply to all potential slipping, and penetration between touching areas is not permitted.Dynamic loading is applied to the structure. An incremental iterative method is used to solve this nonlinear problem. Fourier or wavelet transforms are used to evaluate the derived response.

A thesis on progress in the analysis of laminated composite beam vibration was prepared by Raciti and Kapania (1989). The investigation is focused on plate theory and shear deformation theory in first order. It has proved unsuccessful in forecasting thick laminate responses to the hypothesis that displacements are linear functions of the thickness direction coordinate[34]. The laminated finite element beam model was developed by Yuan and Miller (1990). The model has enough liberty to cause the cross sections of each lamina to deform into a form that can be coordinated in thickness to cubic terms.Shear deformation is permitted for any lamina up to quadratic terms, but not for interfacial slip or delamination [35]. Maiti & Sinha (1994) used theory of higher order shear deformation to analyze composite beams. Nine iso-parametric nodes are used in the analysis. Natural frequencies of composite beams are contrasted with various stacking sequences, different ratios (l/h) and different boundary conditions. They noticed the normal frequency decline as the ply angle rises and the ratio (l/h) decreases[36].Teboub and Hajela (1995) adopted the symbolic calculating technique for studying the free vibration of generally layered composite beams in the first-order sharp deformation principle. All of which have been taken into consideration in the model were the fish effects, coupling extended, twisting, and torsional deformations and rotary inertia[37]. The complex rigidity matrix method has been employed by Banerjee (1999) to investigate the free vibration of Timoshenko beams with axial lamination.

This is achieved by constructing an exact dynamic rigidity matrix of a composite beam that takes axial force, shear deformation and inertia into account. The effects of axial strength, shear deformation and rotating inertia are shown on natural frequencies. The theory implemented include composite wings and helicopter blades[38]. Bassiouni (1999) proposed a finite element model for analysis of the natural frequencies and mode forms of laminated composite beams. In a standard cross section, the model FE allowed all laminas to rotate in a different degree to have equivalent lateral displacement.[39] Taken into consideration transverse shear deformations. The effects on natural frequencies and modeforms of a beam with transverse non-propagating open cracks, fiber volume fraction and orientation, were investigated by Kisa (2003). The results of the study led to the conclusion that the suggested method was appropriate for the analysis of vibration of cracked cantilever composite beams and that a reduction in natural frequencies and mode change would identify the present and presence of crack in the structure.[40].

The crack in a broken structural member does not always stay open during vibration. The combination of static detection due to some loading portion on the cracked beam (residual loads, body weight of a structure, etc.) and vibration effect can cause the crack to open at all times, open and close periodically, or fully close depending on various loads at a given time. If the static detection due to any loading part on the beam (dead loads, own weight, etc.) is greater than the vibration amplitudes, the crack will remain open all of the time or will open and close on a regular basis, and the problem will be linear.If the static detection is low, the crack will open and close over time, depending on the amplitude of the vibration. The method is non-linear in this case. It's difficult to explain the experimental findings because there isn't a formal theory for the breathing crack. Long ago, it was understood that the breathing crack has an impact on the vibration response of cracked structural members.According to Kirmsher [41], if a crack in a concrete beam is "lled with soil or crystallized material, or is narrow enough to cause interference, the effect on the natural frequency is the same as that of a crack of lesser depth." This discovery sparked a more thorough inquiry into the effects of crack opening and closing.

Different forms of loading have various effects on concrete structural elements. The process of identifying and calculating these responses is extremely time-consuming and expensive.However, there are many methods available to solve this problem nowadays, the most commonly used of which is the finite element method.

The finite element method is a numerical analysis method for evaluating the concrete response and pre-stressed concrete members by dividing the structural element into different kinds of smaller parts and simulating different kinds of static loading conditions. Because of tremendous advancements in engineering and computer knowledge, this methodology is becoming more widely used. Since each variable has a different stress strain behavior, this approach responds well to non linear analysis.The program ANSYS effectively tackles this behavior by providing different types of elements for modeling materials and applying loads to determine the response.

The aim of this research was to compare an experimentally tested RC beam to one that was modelled using ANSYS using a discrete method as suggested by Dahmani et al (2010). The Ayman and Banerjee (2007) model beam was used as the reference beam for our research, and shear cracks were compared to it using ANSYS.

Breathing cracks can describe the vibratory broken system more objectively than open cracks. Chondros et al.[41] assumed a beam with a transverse breathing crack to be a partially linear, twofold structure: totally open and closed. A continuous cracked beam vibration theory anticipated the transverse vibration changes in a simply assisted beam with a breath break. A transverse crack split a lifting beam into two segments, using time-varying correlation matrices to connect all segments; then, the breathing crack was analyzed using linear and nonlinear modelling methods. The dynamic reaction of a transverse airbreak in a one-degree system and the vibrative action of a crunched beam were investigated by Benjamin et al.[44]. Wu [45] developed an iterative computational model to examine the forced vibration features of a transverse breathing-cracking cantilever beam, and could further use the model to measure the cracked beam's fatigue existence in the crack spot. In conclusion, numerous models of breathing cracks have been suggested for the study of natural vibrations in transverse broken beams.

Faults are renowned for being the leading cause of structural collapse. The costs of loss of structural components are substantial, and human lives and collateral damage can be catastrophic. In order to mitigate or even avoid structural collapse, cracks are better detected when still small. For this reason, ultrasound screening, X-ray, acoustic emission, and other non-destructive measurement techniques are also used. As the inspector may have access to the object under examination for the identification of holes, in some situations the bulk of such methods are uncomfortable.[46] Because vibration parameters such as natural frequencies are easy to quantify and collect, vibration-based inspection can avoid this discomfort (VBI). In addition, VBI solutions do not require sweeping up of urban environments, unlike other techniques. In areas with inaccessible surfaces, VBI can be used to detect defects in cracks far removed from the sensors. Also low-cost and fast are VBI techniques. [47]

There have been a host of crack models and vibrational techniques. Rizos etal. [48] used an analytical method, by modeling the crack as a local versatility, to relate measured vibration modes with their location and scale. In order to detect a break and a depth in a beam by modeling the crack in rotational spring, Nandwana and Maiti[49], Chaudhari and Maiti[50] have employed a semi-analytical method. Since FEMs are now well-known as a standard protocol for resolving problems of cracks, more investigators are using these approaches to detect structure crashes[51].Chinchalkar[52] used finite variable depth elements of two nodes and two degrees of freedom per node to model a beam of varying depth, and then the direction and size of cracks were observed. Ostachowicz and Krawczuk [53] and Lele and Maiti [54] have used triangular finite elements as well as eight nodal isoparametric elements respectively for more accurate measurements in crack detection. Because of the intrinsic defects of standard finite elements, an incredibly fine mesh in the breaking area and a substantial number of calculation work in references are impotent for explaining the singled behavior of breakings[55].

Spaces of the wavelet were used to resolve traditional FEM challenges in an approximate way, followed by the Wavelet Finite Element (WFEM) methods[56–64]. WFEM provides many advantages for modal investigation of crack issues relative to conventional FEMs. One of its most attractive characteristics is the ability of WFEM to represent reasonably general functions with a few wavelet coefficients correctly and to characterize the flatness of such functions from the numerical action of these coefficients[57].In addition, as Jaffard and Laurencot [58] have shown, WFEM can avoid number instability caused by traditional FEM in crack analysis as the numeric conditions of WFEM remain separate. In addition, WFEM produces sparse stiffness matrices, which can greatly minimize computing time, while orthogonal wavelet functions are used as interpolation functions with compact supports.

This paper also includes a simple technology to identify crack location and scale, which uses WFEM entirely to analyze the singularity problems such as a cracked beam. The crack detection protocol based on the WFEM is referenced. The first three natural frequencies of the strap with various crack locations and sizes are specifically identified by WFEM. FRFs are presented with 3-dimensional images and surface-assignment technology to approximate the FRF[65] feature in accordance with the location and size of the crack and scale. When natural frequencies of crack beams are determined as a details, the three frequency contour lines of a beam are obtained for a given crack position and duration. The direction and scale of the crack was shown by finding the intersection points for the three contour lines.

2.1. Significance of our research work:

In most of the literature reviews, they have found the results through experimentation. While this is a method that produces real life response, it is extremely time consuming and the use of materials can be quite costly. The use of finite element analysis to study these components has also been used. There are only few of them. In those papers, they have worked with only one type of crack but we have shown results for different types of cracks like-slit, angular, triangular and semi-circular and made a comparison between them which helped us to find better results. They have worked with either beams or plates but we have analyzed both of them.

2.2. Objective of this research work:

- To find the Natural Frequencies of Cracked and un-cracked Plates and Beams using Modal Analysis
- Analyzing the vibrational behaviour of Different types of cracks like- slit crack, triangular crack, semi-circular crack comparing their natural frequencies keeping the volume constant
- Analyzing the vibrational behavior by changing the size of the crack i.e. increasing the length of slit crack
- Analyzing vibrational behavior considering different boundary conditions like- one end fixed, both end fixed and both end hinged
- To find the Deflection behavior of cracked and un-cracked Beams using Harmonic Analysis
- Comparing the deflection curve for beams without and with cracks

3. Material and other Properties:

Elastic, slender beams and plates with length L, width W, and height H is considered for frequency analysis. The model has been designed in Solidworks 2018 and Ansys workbench 2019 R2 is used for vibration analysis. Material of the beams and plates are considered as aluminum alloy and its properties taken are Young's elastic modulus as 71 GPa, Poisson's ratio as 0.33 and density as 2770 kg/m^3.

Aluminum alloy:

An aluminum alloy is a chemical composition where other elements are added to pure aluminum in order to enhance its properties, primarily to increase its strength. These other elements include iron, silicon, copper, magnesium, manganese and zinc at levels that combined may make up as much as 15 percent of the alloy by weight. Alloying requires the thorough mixing of aluminum with these other elements while the aluminum is in molten liquid form.

Young's elastic modulus:

Young's modulus is a measure of the ability of a material to withstand changes in length when under lengthwise tension or compression. Sometimes referred to as the modulus of elasticity, Young's modulus is equal to the longitudinal stress divided by the strain. **Here, Young's elastic modulus is taken as 71 GPa.**

Poisson's ratio:

Poisson's ratio is defined as the ratio of the change in the width per unit width of a material, to the change in its length per unit length, as a result of strain. **Here, Poisson's ratio is taken as 0.33.**

Density:

Density is a measure of mass per volume. The average density of an object equals its total mass divided by its total volume. An object made from a comparatively dense material (such as iron) will have less volume than an object of equal mass made from some less dense substance (such as water). **Here, Density is taken as 2770 Kg/m^3.**

****These material and material properties are kept constant in all of the cases of beams and plates which are shown in modelling****

4. Modelling:

4.1. Thin plate with slit crack:

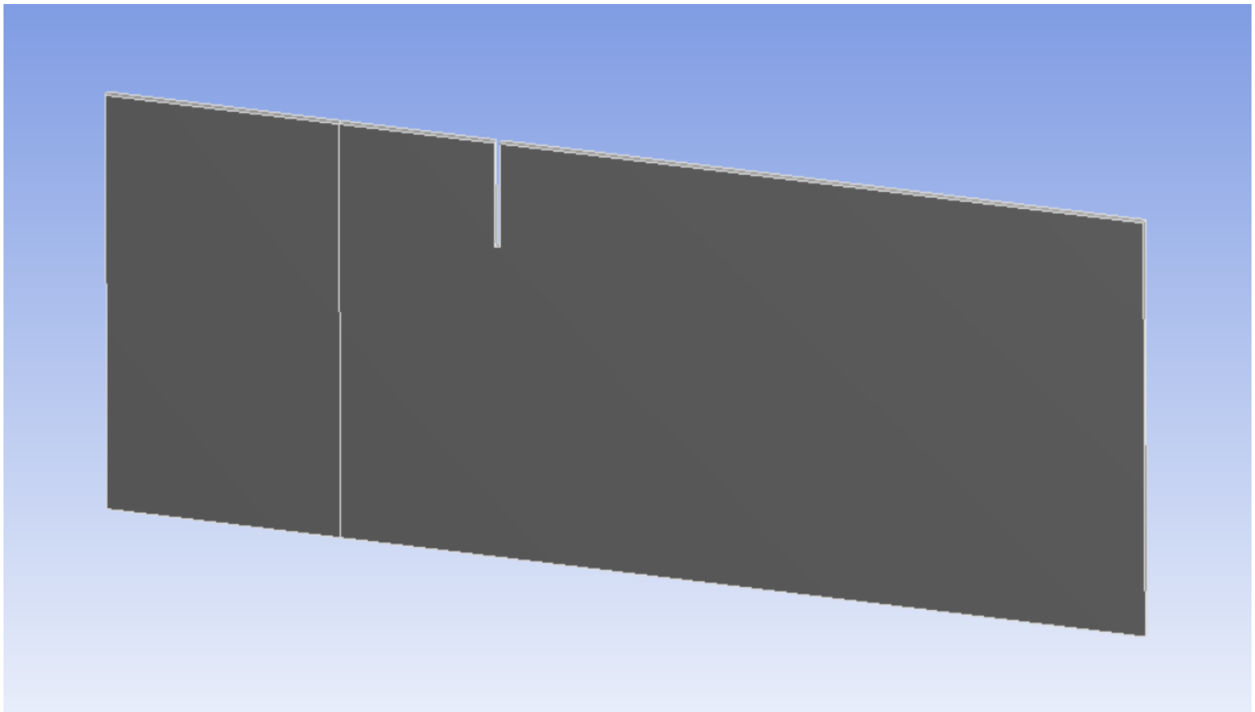

Fig 01: Thin plate with slit crack [66]

In fig 01, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 10mm, breadth 2mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 01: Natural Frequencies found for different location of slit crack for thin plate

Natural frequencies without any crack	Natural frequencies with crack at 30 mm	Natural frequencies with crack at 60 mm	Natural frequencies with crack at 90 mm	Natural frequencies with crack at 120 mm	Natural frequencies with crack at 150mm	
50.322	48.066	49.31	50.038	50.359	50.487	
206.68	196.82	197.66	201.22	205.14	207.95	
313.01	309.82	304.7	301.52	308.51	313.94	Crack dimentions:
676.83	653.95	668.23	651.54	641.41	678.49	L= 10mm
877.31	862.84	852.17	850.92	843.82	879.07	b= 2mm
1280.8	1255.9	1243.2	1263.8	1235.2	1286.2	t= 1.43mm
1312.3	1294.2	1287.1	1276.2	1289.7	1302.5	
1699	1667.6	1651.6	1602.8	1599.1	1705.8	Beam Length = 200mm
1789.3	1745.5	1769	1790.5	1737.4	1769.3	
2184.1	1862.1	2046	2061.3	2121.1	2063.8	
2308.4	2133.7	2064.3	2231.6	2299.4	2315.2	
2495.2	2483.4	2413.3	2457.1	2302.3	2419.8	

4.2. Thin plate with Slit crack in the middle:

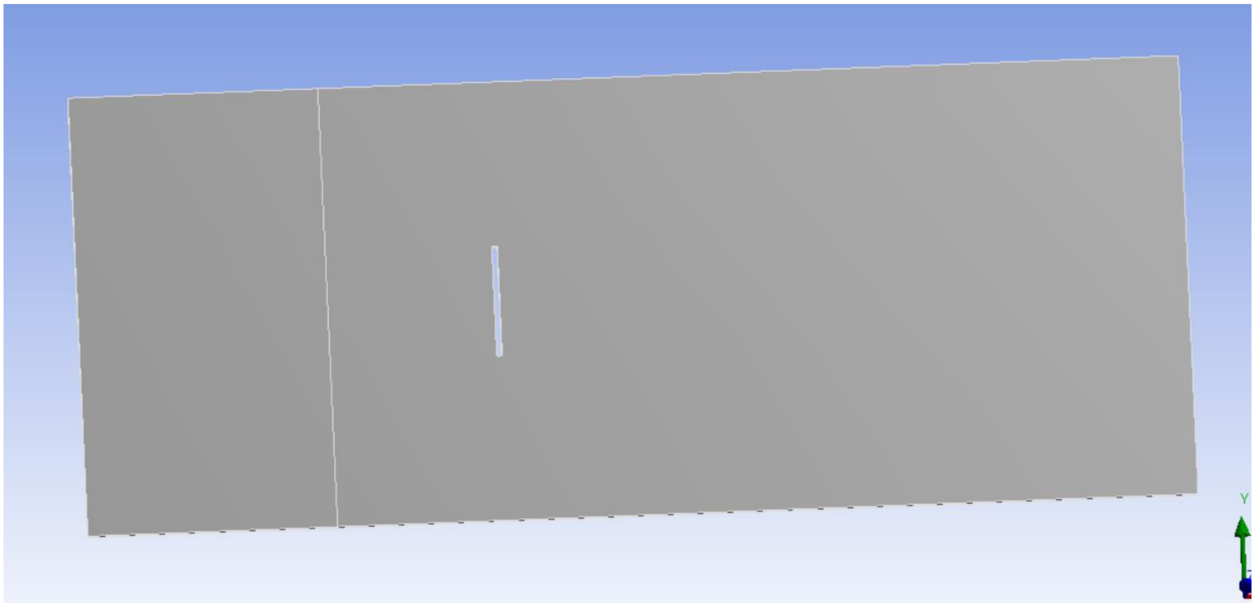

Fig 02: Thin plate with Slit crack in the middle [66]

In Fig 02, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack in the middle of the beam with length 10mm, breadth 2mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 02: Natural Frequencies found for different location of slit crack in the middle for thin plate

Natural frequencies without any crack	Natural frequencies with crack at 30 mm	Natural frequencies with crack at 60 mm	Natural frequencies with crack at 90 mm	Natural frequencies with crack at 120 mm	Natural frequencies with crack at 150mm	
50.322	48.066	49.31	50.038	50.359	50.487	
206.68	196.82	197.66	201.22	205.14	207.95	
313.01	309.82	304.7	301.52	308.51	313.94	Crack dimentions:
676.83	653.95	668.23	651.54	641.41	678.49	L= 10mm
877.31	862.84	852.17	850.92	843.82	879.07	b= 2mm
1280.8	1255.9	1243.2	1263.8	1235.2	1286.2	t= 1.43mm
1312.3	1294.2	1287.1	1276.2	1289.7	1302.5	
1699	1667.6	1651.6	1602.8	1599.1	1705.8	Beam Length = 200mm
1789.3	1745.5	1769	1790.5	1737.4	1769.3	
2184.1	1862.1	2046	2061.3	2121.1	2063.8	
2308.4	2133.7	2064.3	2231.6	2299.4	2315.2	
2495.2	2483.4	2413.3	2457.1	2302.3	2419.8	

4.3. Thin plate with angular crack:

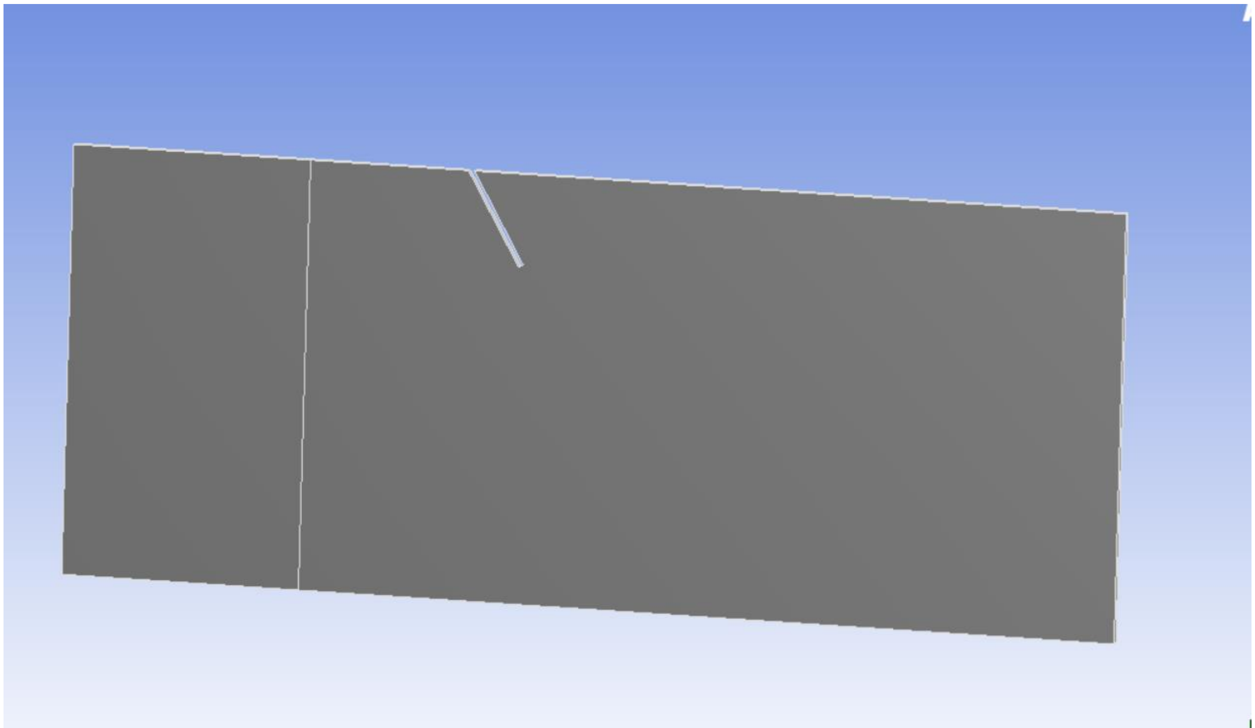

Fig 03: Thin plate with angular crack at 30mm with angle 60^0 [66]

In Fig 03, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, an angular crack with length 10mm, breadth 2mm and thickness 1.43mm is placed at 30mm with angle 60^0 with horizontal. Similarly, fundamental frequencies and mode shapes are obtained.

Table. 03: Natural Frequencies found for angular crack at 30mm with angle 60^0 for thin plate

	Natural frequestion with crack at 30 mm with angle 60
	48.796
	199.13
Crack dimentions:	310.61
L= 10mm	659.22
b= 2mm	856.86
t= 1.43mm	1259.8
	1290.9
Beam Length = 200mm	1668.2
	1751.1
	2005.4
	2142.1
	2479.8

4.4. Thin plate with triangular crack:

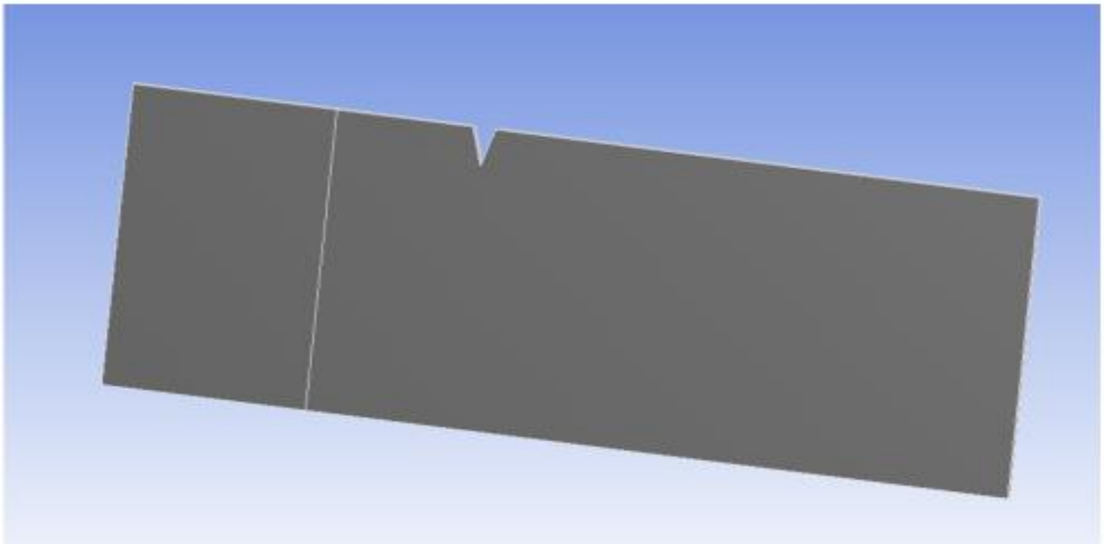

Fig 04: Thin plate with triangular crack

In fig 04, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a triangular crack with length 10mm, breadth 2mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 04: Natural Frequencies found for different location of triangular crack for thin plate

Natural frequencies without any crack	Natural frequencies with triangular crack at 30mm	Natural frequencies with triangular crack at 60mm	Natural frequencies with triangular crack at 90mm	Natural frequencies with triangular crack at 120mm	Natural frequencies with triangular crack at 150mm	
50.322	49.483	49.95	50.2	50.33	50.434	
206.68	203.59	204.35	205.72	207.04	207.92	Crack dimensions:
313.01	311.47	310.21	309.49	311.51	313.53	Base = 5mm
676.83	670.54	675.15	670.39	669.29	681.51	height = 10mm
877.31	870.74	869.71	869.57	868.48	877.66	thickness= 1.43mm
1280.8	1277.7	1280.2	1280.6	1283	1287.4	
1312.3	1301.7	1298.2	1301.4	1300.3	1323.6	v = .5*base*height*thickness
1699	1683	1697.6	1680.4	1677.7	1698.8	
1789.3	1776.8	1782.8	1786.1	1770.3	1805.9	Beam length=200mm
2184.1	2159	2154.2	2155.4	2160.3	2195.6	
2308.4	2167.4	2244.6	2291.5	2308.8	2313.5	
2495.2	2493.4	2461.1	2496.4	2457	2510.6	

4.5. Thin plate with semi-circular crack:

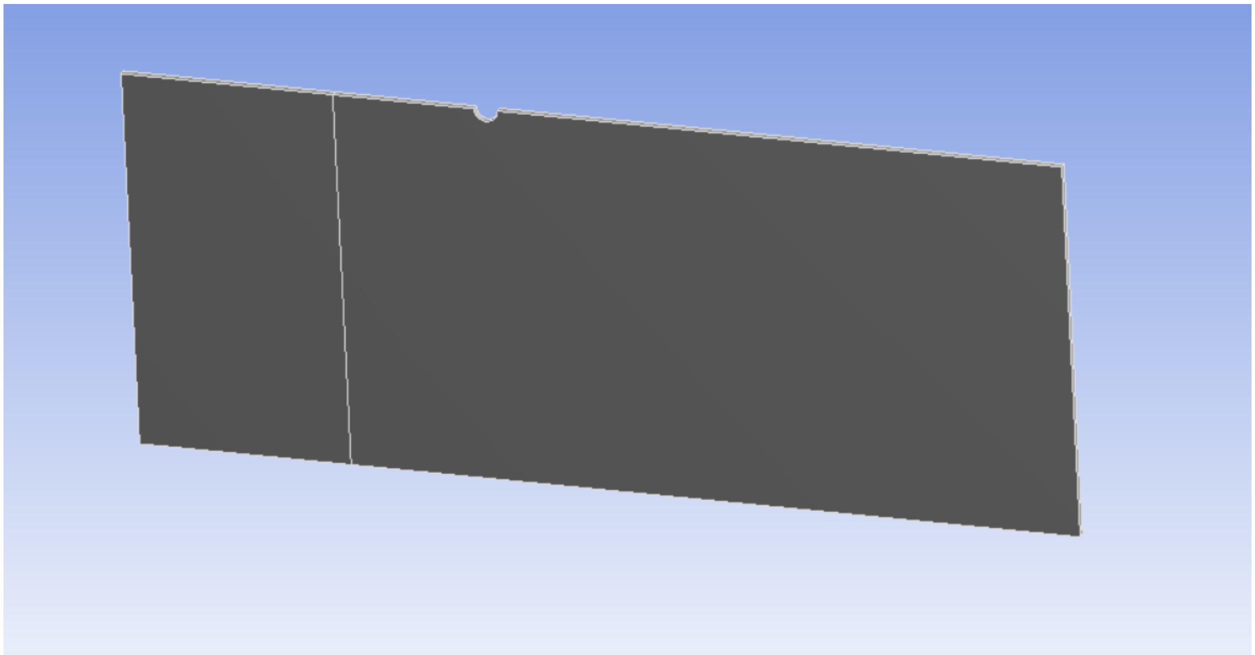

Fig 05: Thin plate with semi-circular crack

In fig 05, One end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a semi-circular crack with radius 2.5mm and height 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 05: Natural Frequencies found for different location of semi-circular crack for thin plate

Natural frequencies without any crack	Natural frequencies with semi-circular crack at 30mm	Natural frequencies with semi-circular crack at 60mm	Natural frequencies with semi-circular crack at 90mm	Natural frequencies with semi-circular crack at 120mm	Natural frequencies with semi-circular crack at 150mm	
50.322	50.156	50.208	50.246	50.279	50.314	
206.68	206.41	206.53	206.78	207.05	207.22	Crack dimensions:
313.01	312.51	312.42	312.27	312.4	312.89	Radius = 2.5 mm
676.83	676.69	677.66	676.43	676.15	678.9	h = 1.43mm
877.31	876.07	875.63	875.66	875.56	876.61	
1280.8	1281.1	1281.9	1282.1	1284.2	1285	v =(pi* r^2*h)/2
1312.3	1312.8	1311.2	1312.1	1311.1	1316.7	
1699	1697.3	1701.8	1698.6	1697.4	1699	Beam length=200mm
1789.3	1790.5	1788.9	1788.1	1786.4	1796.4	
2184.1	2184.2	2182	2181.4	2183.1	2188.5	
2308.4	2292.8	2300.5	2305.1	2307.2	2308.4	
2495.2	2501	2494.4	2501.6	2494	2504.5	

4.6. Changeable length of slit crack:

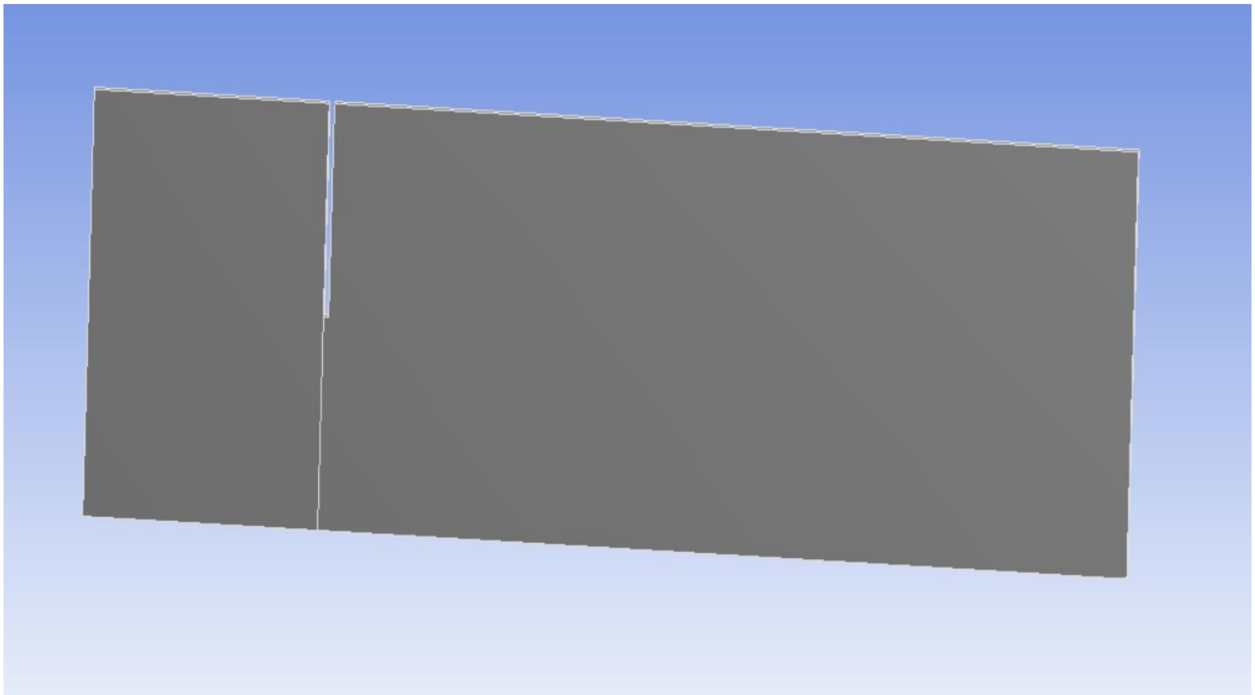

Fig 06: Thin plate with slit crack where the crack length varies [66]

In fig 06, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 10mm, breadth 2mm and thickness 1.43mm is placed at fixed end whose length 10mm and we continually increased the size of the crack by 10mm.

Table. 06: Natural Frequencies found for different size of slit crack for thin plate

Natural frequencies without any crack	Natural frequencies with crack of 20mm at clamped	Natural frequencies with crack of 40mm at clamped	Natural frequencies with crack of 50mm at clamped	Natural frequencies with crack of 60mm at clamped	Natural frequencies with crack of 70mm at clamped	
50.247	48.494	43.402	39.62	34.948	28.72	
206.58	196.22	170.38	151.52	129.55	103.78	
312.59	302.06	281	268.52	255.22	239.41	
676.32	638.68	527.13	464.16	417.08	258.1	Crack dimentions:
876.14	846.89	778.87	747.44	556.78	382.83	L= 10mm
1279.3	1205.9	981.75	876	720.75	688.5	b= 2mm
1311.1	1286.2	1209.7	934	909.71	893.03	t= 1.43mm
1697.5	1649.3	1282.8	1279.9	1271.4	1212	
1786.9	1694	1421.3	1380.2	1332.3	1285.9	Beam Length = 200mm
2181.7	1865.8	1662.2	1624.6	1565	1504.1	
2304.5	2020.8	1849.3	1818.9	1791.5	1762.5	
2493.3	2395.7	2265.3	2225.9	2061.8	1930.4	

4.7. Thin plate with slit crack of volume 28.6 mm^3:

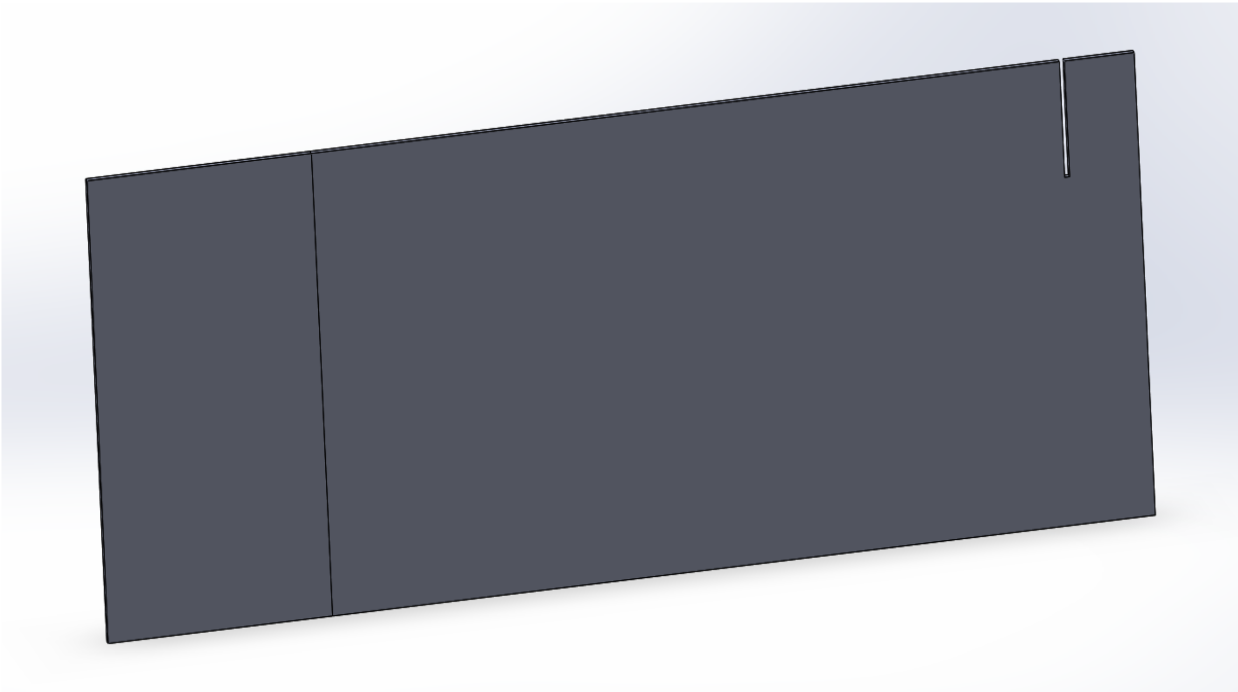

Fig 07: Thin plate with slit crack of volume 28.6 mm^3 where the crack length varies

In fig 07, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 10mm, breadth 2mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 07: Natural Frequencies found for different location of slit crack of volume 28.6 mm^3 for thin plate

Natural frequencies without any crack	Natural frequencies with slit crack of vol 28.6 mm3 at 30mm	Natural frequencies with slit crack of vol 28.6 mm3 at 60mm	Natural frequencies with slit crack of vol 28.6 mm3 at 90mm	Natural frequencies with slit crack of vol 28.6 mm3 at 120mm	Natural frequencies with slit crack of vol 28.6 mm3 at 150mm	
44.293	42.122	43.2	43.908	44.25	44.38	
192.24	182.19	182.7	185.73	189.34	192.22	
275.72	272.42	268.68	263.89	269.27	275.19	
625.09	599.68	614.19	602.42	586.89	611.06	Crack dimentions:
773.87	759.4	747.55	751.86	736.21	764.56	L= 10mm
1197.8	1152.4	1135.3	1135.5	1144.8	1129.3	b = 2mm
1265.8	1259.9	1266.2	1255.4	1257.4	1266.8	t = 1.43 mm
1531.6	1487.9	1460.3	1464.8	1433.3	1484	v = L*b*t
1692.4	1651.6	1691.9	1668	1635.6	1628.2	
1973.1	1661.1	1820.8	1866.7	1886.8	1820.6	Beam length = 210mm
2067.6	1912.1	1830	1979.3	2054.4	2072.6	
2356.1	2313	2276.1	2200.4	2181.9	2233.4	

4.8. Thin plate with semi-circular crack of volume 28.6 mm^3:

Fig 08: Thin plate with semi-circular crack of volume 28.6 mm^3 where the crack length varies

In fig 08, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a semi crack of radius 3.57 mm and height 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 08: Natural Frequencies found for different location of semi-circular crack of volume 28.6 mm^3 for thin plate

Natural frequencies without any crack	Natural frequencies with semi-circular crack of vol 28.6 mm3 at 30mm	Natural frequencies with semi-circular crack of vol 28.6 mm3 at 60mm	Natural frequencies with semi-circular crack of vol 28.6 mm3 at 90mm	Natural frequencies with semi-circular crack of vol 28.6 mm3 at 120mm	Natural frequencies with semi-circular crack of vol 28.6 mm3 at 150mm	
44.293	44.139	44.224	44.284	44.336	44.393	
192.24	191.83	192.01	192.42	192.87	193.2	
275.72	275.61	275.53	275.23	275.35	275.96	Radius = 3.57mm
625.09	625.28	627.08	625.31	623.88	627.61	h = 1.43mm
773.87	773.8	773.05	773.06	772.63	773.76	
1197.8	1200	1197.3	1197.8	1197.4	1200	v =(pi* r^2*h)/2
1265.8	1267.6	1268.8	1268.9	1273	1275.4	
1531.6	1529.6	1532	1532.7	1528.1	1531.2	
1692.4	1697.9	1701.1	1694.9	1691.3	1702.7	Beam Length = 210mm
1973.1	1976.5	1971.3	1971.9	1974.3	1973.8	
2067.6	2046.2	2058.9	2066.7	2070.9	2073.2	
2356.1	2365.3	2356.3	2364.3	2357.1	2363.1	

4.9. Thin plate with triangular crack of volume 28.6 mm^3 :

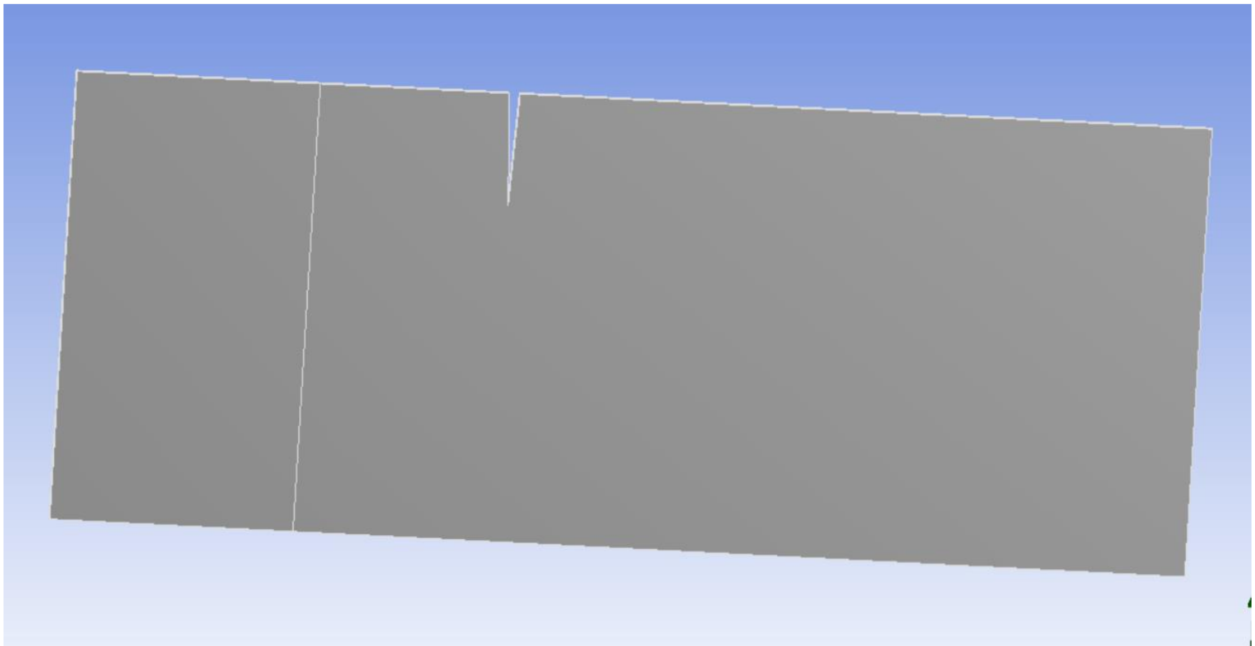

Fig 09: Thin plate with triangular crack of volume 28.6 mm^3 where the crack length varies

In fig 09, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a triangular crack of base 2mm, height 20 mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 09: Natural Frequencies found for different location of triangular crack of volume 28.6 mm^3 for thin plate

Natural frequencies without any crack	Natural frequencies with triangular crack of vol 28.6 mm3 at 30mm	Natural frequencies with triangular crack of vol 28.6 mm3 at 60mm	Natural frequencies with triangular crack of vol 28.6 mm3 at 90mm	Natural frequencies with triangular crack of vol 28.6 mm3 at 120mm	Natural frequencies with triangular crack of vol 28.6 mm3 at 150mm	
44.293	42.279	43.289	43.949	44.26	44.383	
192.24	182.97	183.52	186.4	189.78	192.43	
275.72	272.78	269.07	264.84	269.9	275.32	
625.09	602.26	615.45	604.24	590.55	613.73	Base = 2mm
773.87	760.42	749.7	753.93	739.33	766.15	height = 20mm
1197.8	1157.8	1140.9	1142.5	1148.5	1141.1	thickness= 1.43mm
1265.8	1260.3	1266.8	1256.5	1260.8	1269	
1531.6	1489.9	1470.3	1471.9	1444.7	1493.3	v = .5*base*height*thickness
1692.4	1656.7	1693.7	1670.7	1639	1641.1	
1973.1	1672.6	1836.3	1875.3	1895.8	1835	
2067.6	1916.7	1840.2	1984.6	2055.5	2072.3	Beam Length = 210mm
2356.1	2320.8	2278.8	2228	2201.7	2246.5	

4.10. Thin plate with triangular crack of diff. volume:

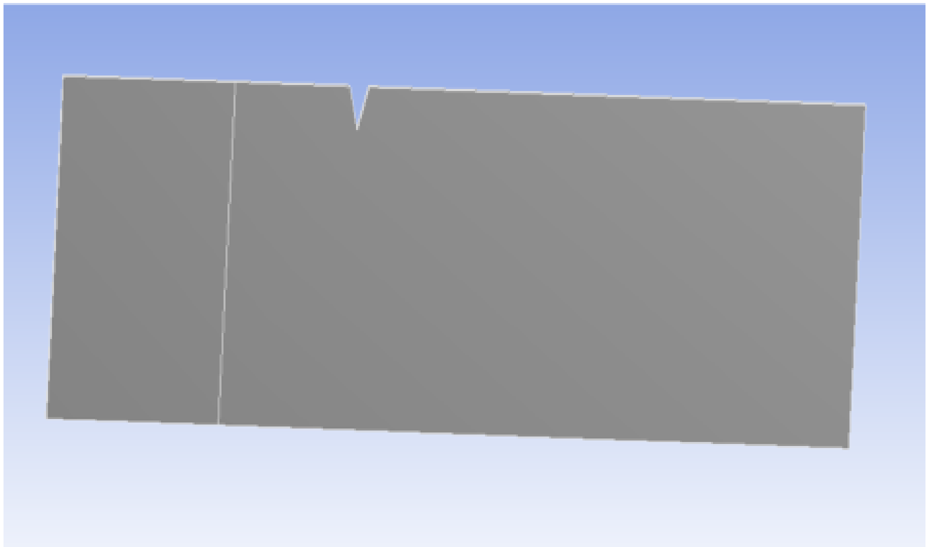

Fig 10: Thin plate with triangular crack of diff. volume where the crack length varies

In fig 10, one end of the plate is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a triangular crack of base 5mm, height 10 mm and thickness 1.43mm is placed at 30mm. The location of crack is varied between 30 mm to 150 mm with an increment of 30 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 10: Natural Frequencies found for different location of slit crack of diff. volume mm^3 for thin plate

triangular crack with diff. vol at 30mm	triangular crack with diff. vol at 60mm	triangular crack with diff. vol at 90mm	triangular crack with diff. vol at 120mm	triangular crack with diff. vol at 150mm	
43.72	44.034	44.222	44.336	44.418	
190.03	190.32	191.28	192.38	193.23	
275.17	274.17	272.98	274.34	275.95	
621.22	624.79	621.04	617.82	625.93	
771.3	768.46	769.76	765.79	773	Crack dimensions:
1192.6	1187.4	1187.6	1187.4	1194.1	b = 5mm
1265.6	1268.5	1266.6	1271.6	1274.9	h= 10mm
1520.7	1524.8	1522	1513.8	1528	t = 1.43mm
1689.2	1700.6	1689.8	1679.5	1697.4	
1947.4	1950.9	1953	1955.2	1959.8	Beam length = 210mm
1961	2010	2050.8	2069.1	2074.3	
2359.8	2336.4	2347.3	2334.9	2347.2	

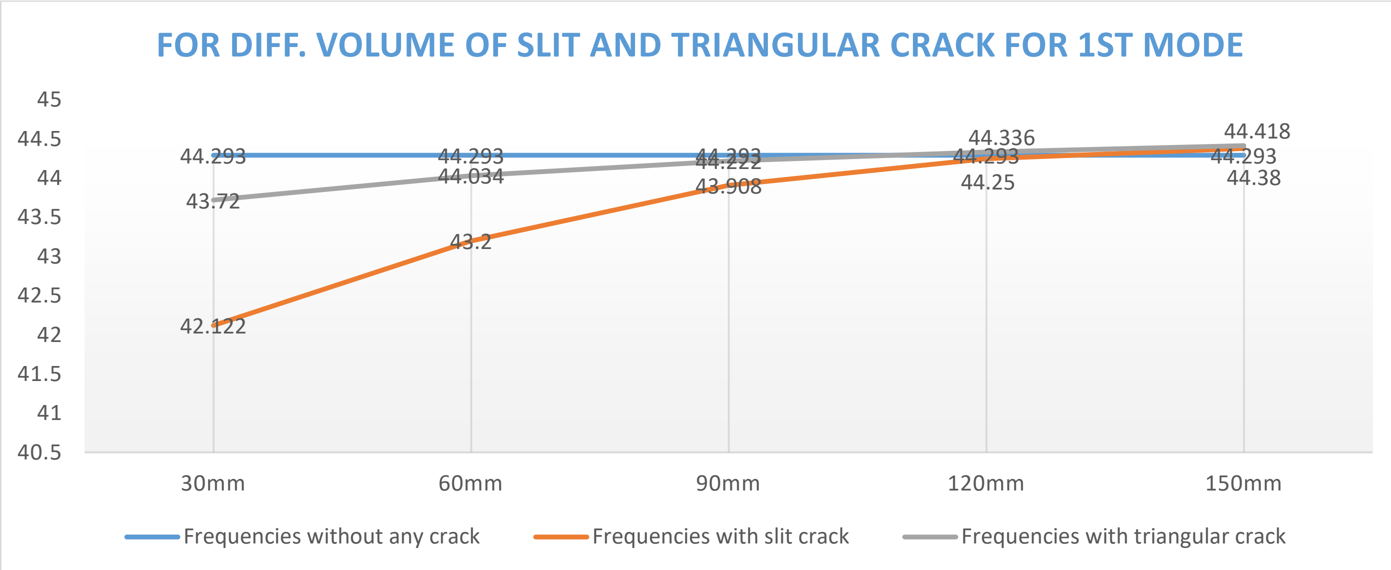

Figure:11

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 1st mode. The volume of the slit and triangular crack are are different. We can see when the crack is near the fixed end, frequencies are lower and it increases as it moves away from the fixed and reaches the frequency of un-cracked plate at 140mm and afterwards, crosses a little more.

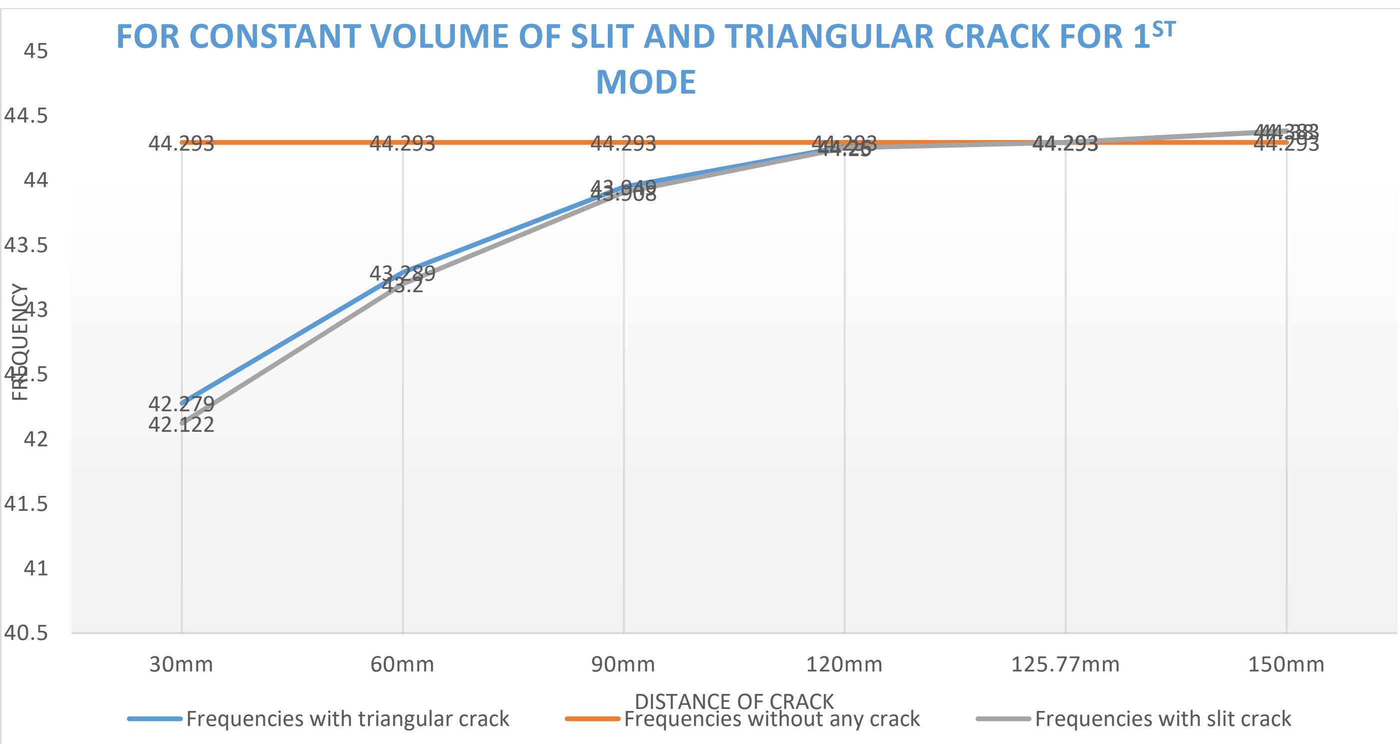

Figure: 12

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 1st mode. The volume of the slit and triangular crack are kept constant. We can see when the crack is near the fixed end, frequencies are lower and it increases as it moves away from the fixed and reaches the frequency of un-cracked plate at 125.77 mm and afterwards, crosses a little more.

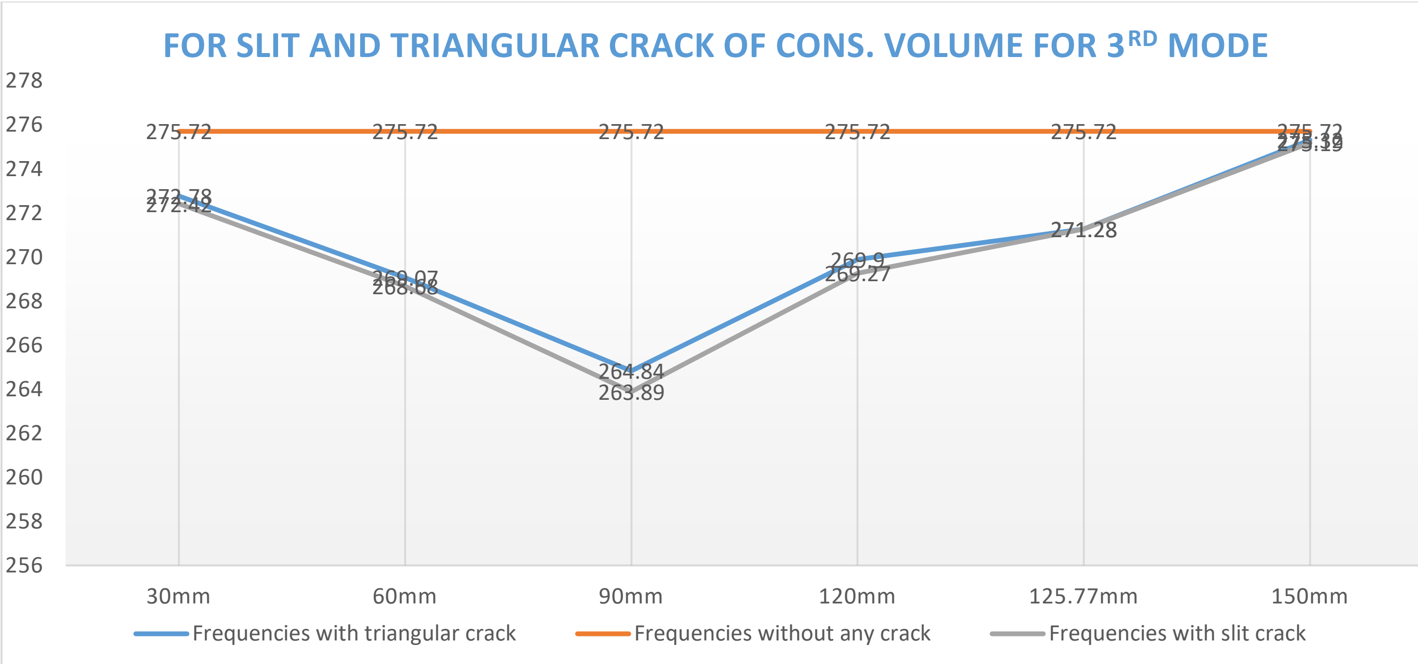

Figure: 13

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 3rd mode. The volume of the slit and triangular crack are kept constant. We can see frequencies of slit and triangular crack are almost same and never crosses the frequency of un-cracked plate. The frequency of cracked plates reaches almost equal to the frequency of nu-cracked pate when the crack is at 150mm.

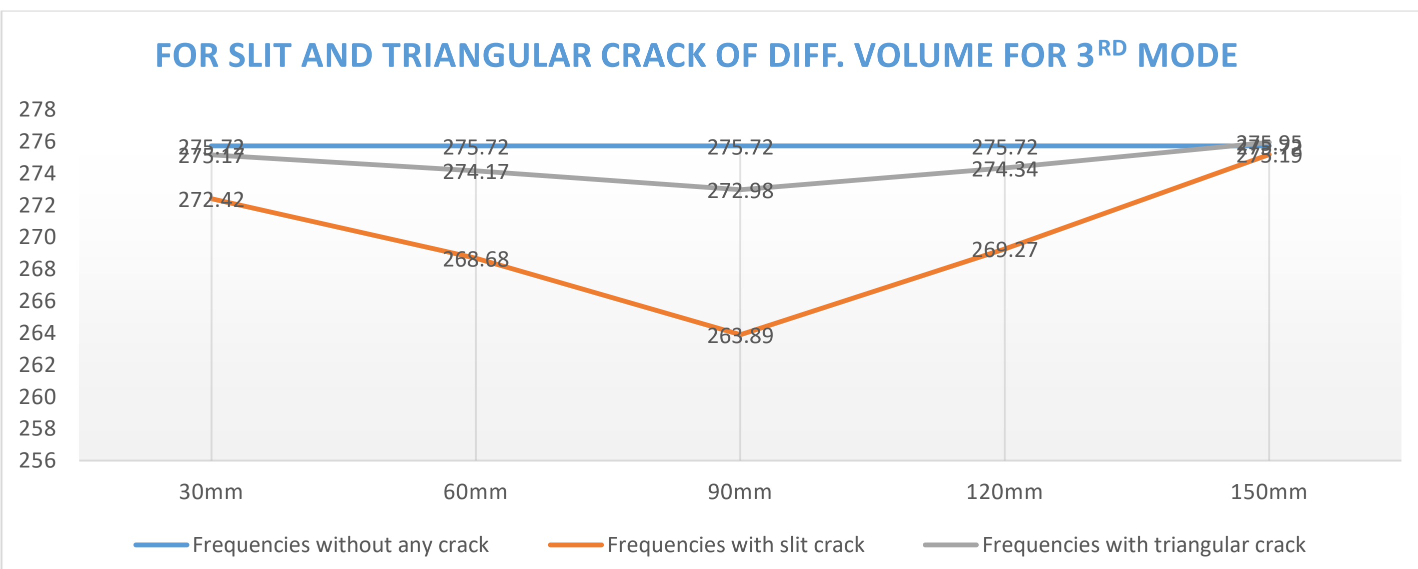

Figure: 14

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 3rd mode. The volume of the slit and triangular crack are different. We can see frequencies of slit and triangular crack are almost same and never crosses the frequency of un-cracked plate. The frequency of cracked plates reaches almost equal to the frequency of nu-cracked pate when the crack is at 150mm.

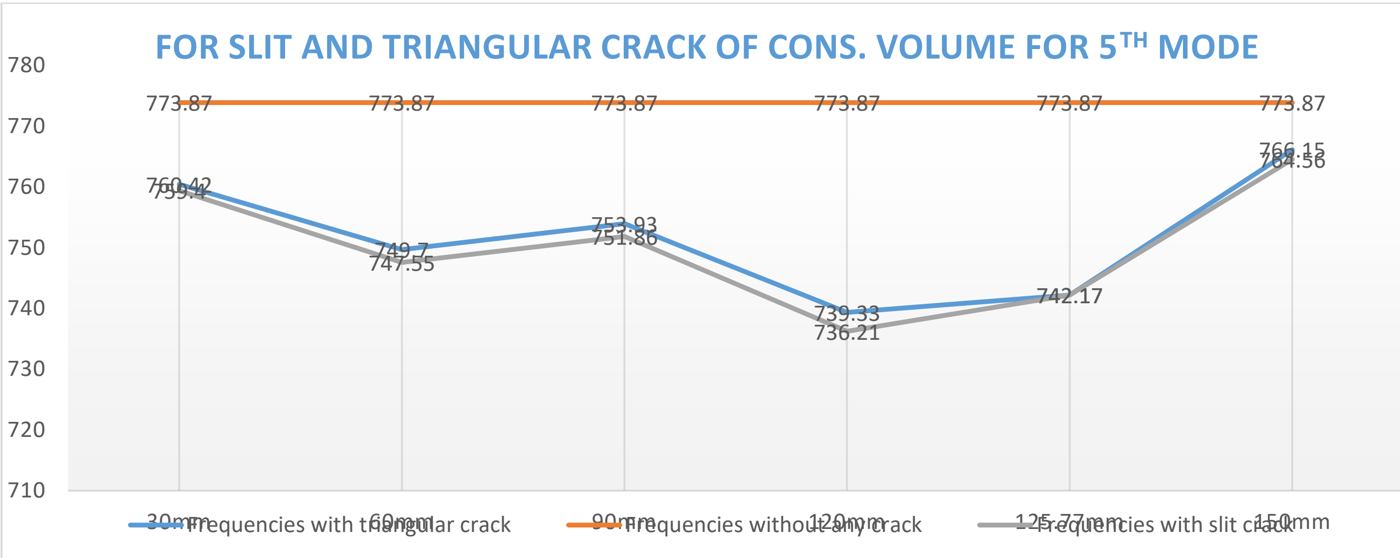

Figure: 15

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 5th mode. The volume of the slit and triangular crack are kept constant. We can see the frequencies of slit and triangular crack are almost similar but are quite away from the frequency of un-cracked plate. It never crosses the frequency of un-cracked plate.

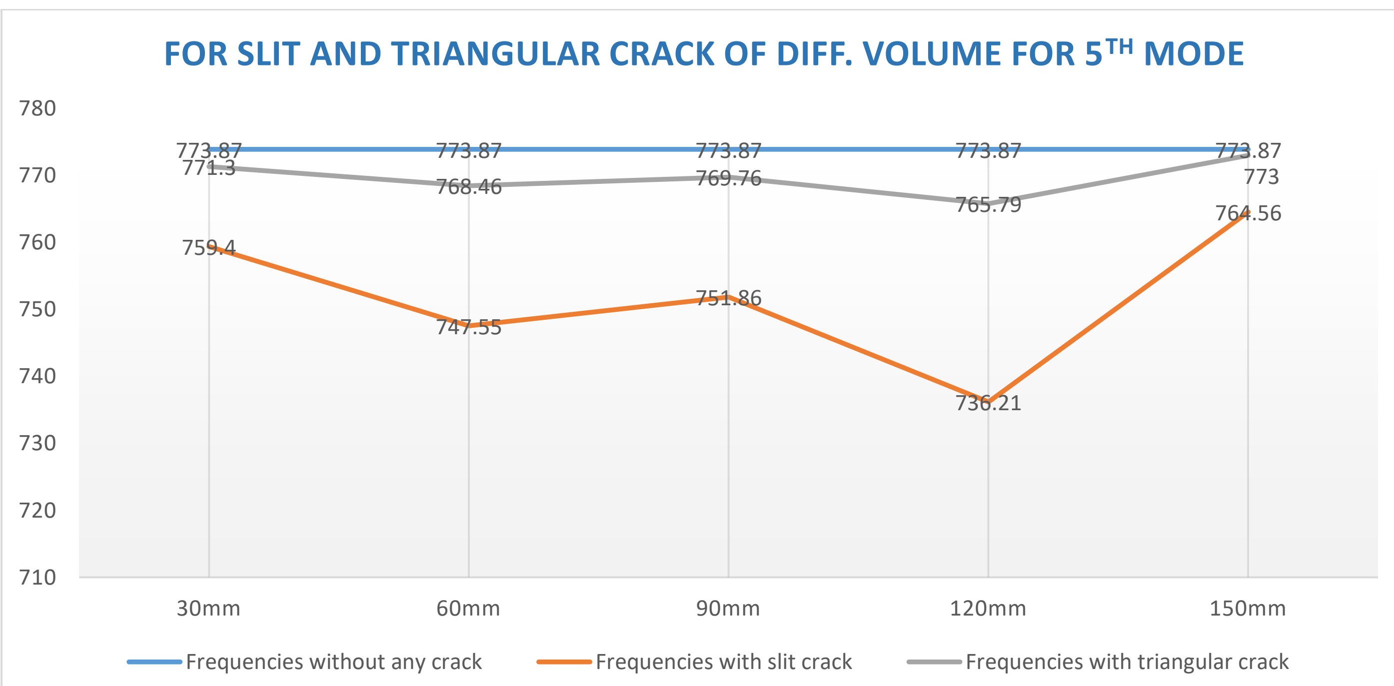

Figure: 16

In the following graph, natural frequencies of plate without any crack is compared with plates that have slit crack and triangular crack for 5th mode. The volume of the slit and triangular crack are different. We can frequency of slit and triangular crack are quite different when their volumes are different and never reaches the frequency of un-cracked plate.

4.11. Both end fixed beam of 500mm with triangular crack:

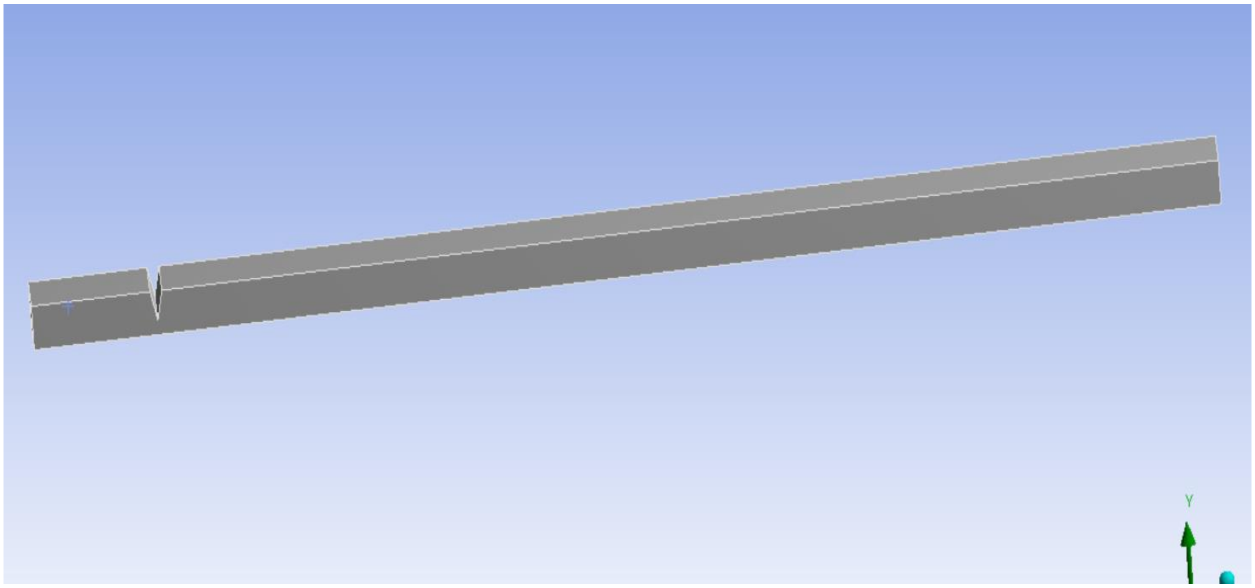

Fig 17: Both end fixed beam with triangular crack where the crack length varies

In fig 17, a beam of length 500mm, breadth 20mm and height 30mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a triangular crack of base 5mm, height 10 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 445 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 11: Natural Frequencies found for different location of triangular crack for Both end fixed beam

two end fixed without any crack	two end fixed with crack at 50 mm from right	two end fixed with crack at 100 mm	two end fixed with crack at 150 mm	two end fixed with crack at 200 mm	two end fixed with crack at 250 mm	two end fixed with crack at 295 mm	two end fixed with crack at 345 mm	two end fixed with crack at 395 mm	two end fixed with crack at 445 mm	Crack dimentions:
316.31	297.8	315.93	309.61	293.34	286.64	293.38	309.64	315.93	297.91	b= 5mm
521	508.77	520.56	517.89	509.58	505.84	509.55	517.89	520.56	508.86	h= 10mm
865.13	858.26	827.73	778.83	821.39	864.76	821.49	779.04	827.54	858.24	t= 25mm
1406.3	1399	1386.6	1359.6	1381.9	1404	1381.8	1359.7	1386.7	1399.1	
1678.7	1676.4	1532.5	1633.8	1626.1	1501.2	1626.3	1633.8	1531.8	1676.4	
2612.5	2524	2555.3	2586	2500.8	2583.8	2501.2	2586	2555.5	2524.6	Beam Length = 500mm

4.12. Both end fixed beam of 500mm with slit crack:

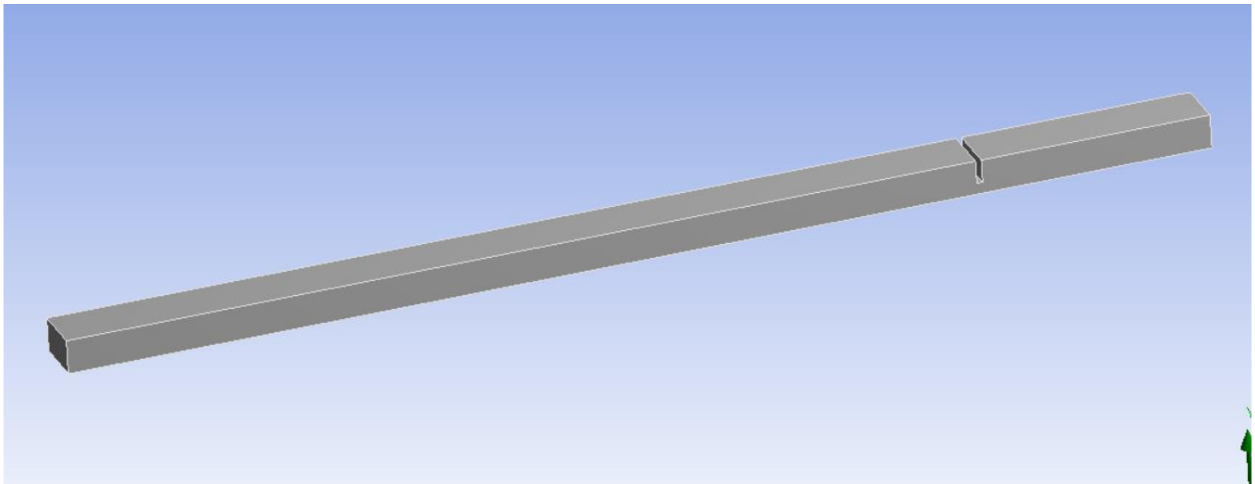

Fig 18: Both end fixed beam with slit crack where the crack length varies

In fig18, Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 2.5mm, breadth 25mm and thickness 10mm is placed at 50mm. The location of crack is varied between 50 mm to 447.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 12: Natural Frequencies found for different location of slit crack for Both end fixed beam

two end fixed without any crack	two end fixed with straight crack at 50 mm from right	two end fixed with straight crack at 100 mm from right	two end fixed with straight crack at 150 mm from right	two end fixed with straight crack at 200 mm from right	two end fixed with crack at 250 mm from right	two end fixed with crack at 297.5 mm from right	two end fixed with crack at 347.5 mm from right	two end fixed with crack at 397.5 mm from right	two end fixed with crack at 447.5 mm from right	Crack Dimentions:
316.31	293.86	315.58	308.45	288.95	280.76	288.93	308.44	315.59	293.91	L= 2.5mm
521	505.89	520.14	517.23	507.39	502.79	507.43	517.25	520.15	505.94	b= 25mm
865.13	855.29	821.77	763.12	811.42	864.65	811.46	763.15	822.02	855.3	h= 10mm
1406.3	1395.2	1383.1	1350.6	1375.9	1403	1376	1350.5	1383.1	1395.2	
1678.7	1675.7	1507.8	1621.7	1621.5	1473	1621.5	1621.9	1508.6	1675.8	Beam Length= 500mm
2612.5	2476.3	2521.9	2564.8	2461.9	2566.4	2462	2564.6	2522.1	2476	

Comparison of 1st mode for triangular cracked beam, slit cracked beam and beam with no crack

two end fixed with straight crack from right	Frequencies with straight crack	Frequencies without any crack	Frequencies with triangular crack
50mm	293.86	316.31	297.8
100mm	315.58	316.31	315.93
150mm	308.45	316.31	309.61
200mm	288.95	316.31	293.34
250mm	280.76	316.31	286.64
297.5mm	288.93	316.31	293.38
347.5mm	308.44	316.31	309.64
397.5mm	315.59	316.31	315.93
447.5mm	293.91	316.31	297.91

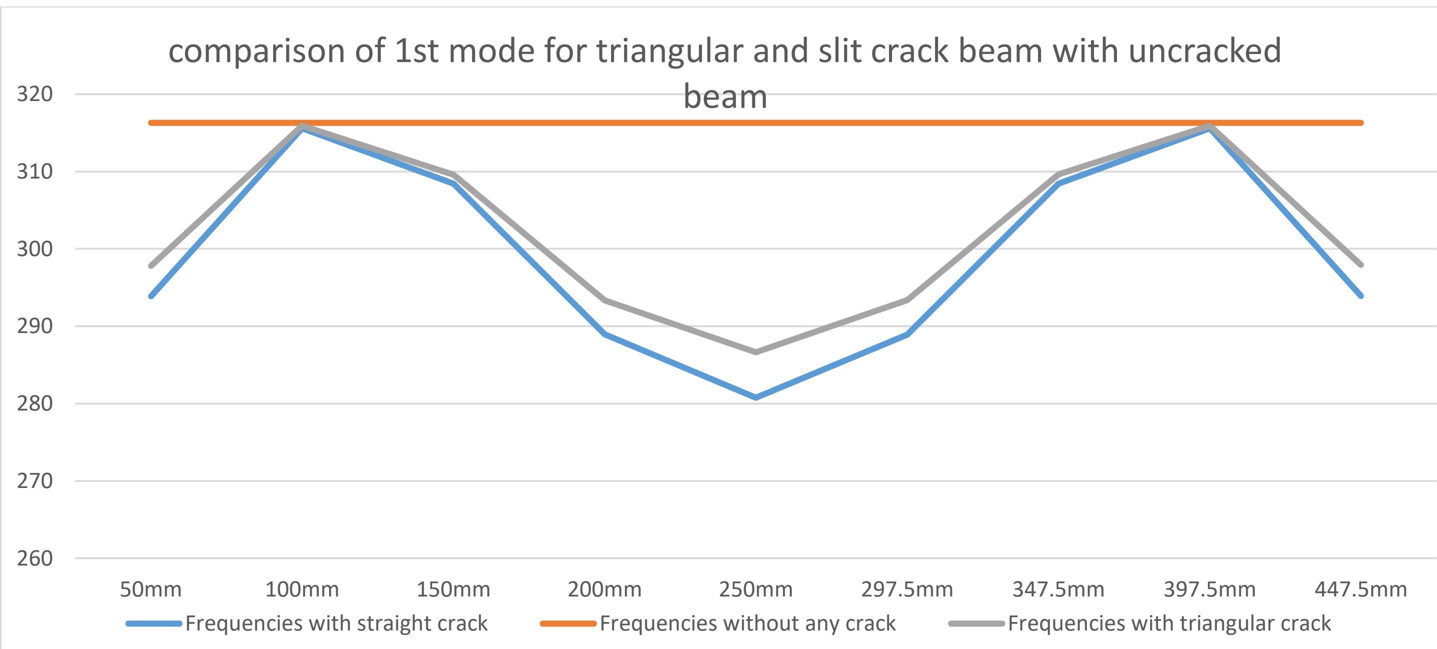

Figure: 19

In the following graph, we can see natural frequencies of slit and triangular crack are almost similar. Since both end of the beam is fixed, symmetric behavior can be seen. Frequency first increases and reaches the frequency of un-cracked beam at 100mm and decreases and becomes minimum at the middle and afterwards, increases and shows a symmetric behavior.

4.13. Both end fixed beam of 400mm with slit crack:

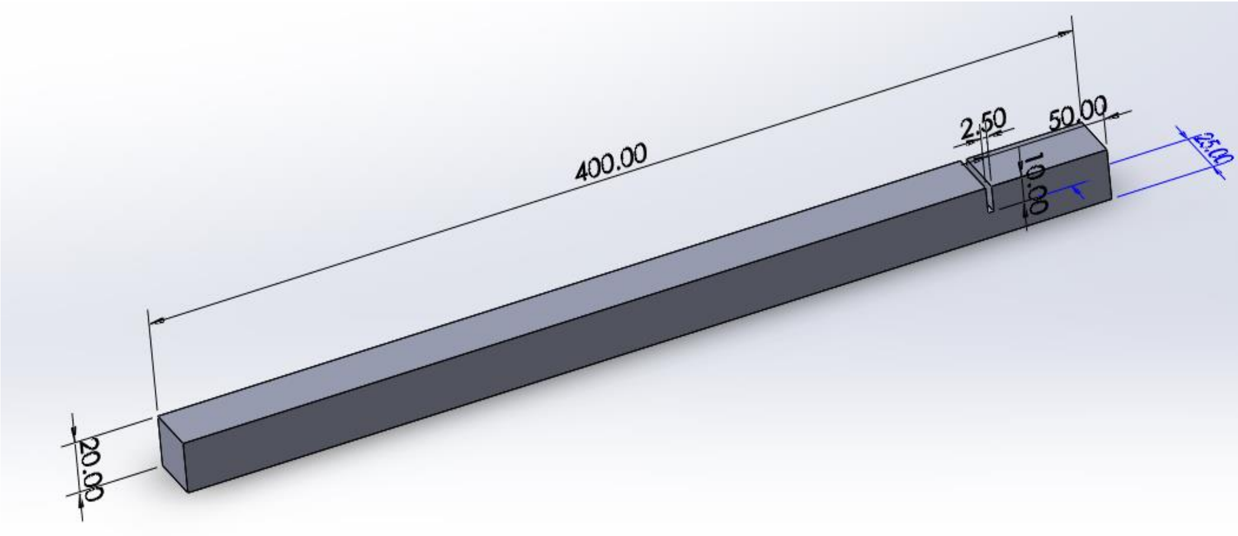

Fig 20: Both end fixed beam with slit crack where the crack location varies

In fig 20, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 10mm, height 2.5 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 13: Natural Frequencies found for different location of slit crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 50mm from right	Two end fixed beam of 400mm with crack at 100mm from right	Two end fixed beam of 400mm with crack at 150mm from right	Two end fixed beam of 400mm with crack at 200mm from right	Two end fixed beam of 400mm with crack at 247.5mm from right	Two end fixed beam of 400mm with crack at 297.5mm from right	Two end fixed beam of 400mm with crack at 347.5mm from right	
644.2	620.9	642.99	610.54	591.39	610.35	642.93	621	Slit Crack Dimentions:
797.14	785.36	796.33	784.77	777.13	784.78	796.36	785.38	L=10mm
1738.1	1736.7	1610	1630.4	1736.8	1630.2	1610.7	1736.8	b= 2.5mm
2127.3	2119.1	2077.7	2082.6	2121.7	2082.5	2078	2119.1	t= 25mm
3316.1	3256.4	3109.9	3289.9	3007.8	3289.6	3111.2	3256.8	
3493.7	3361	3420.5	3483.2	3509.5	3483	3421.9	3360.9	Beam length = 400mm

4.14. Both end fixed beam of 400mm with angular crack:

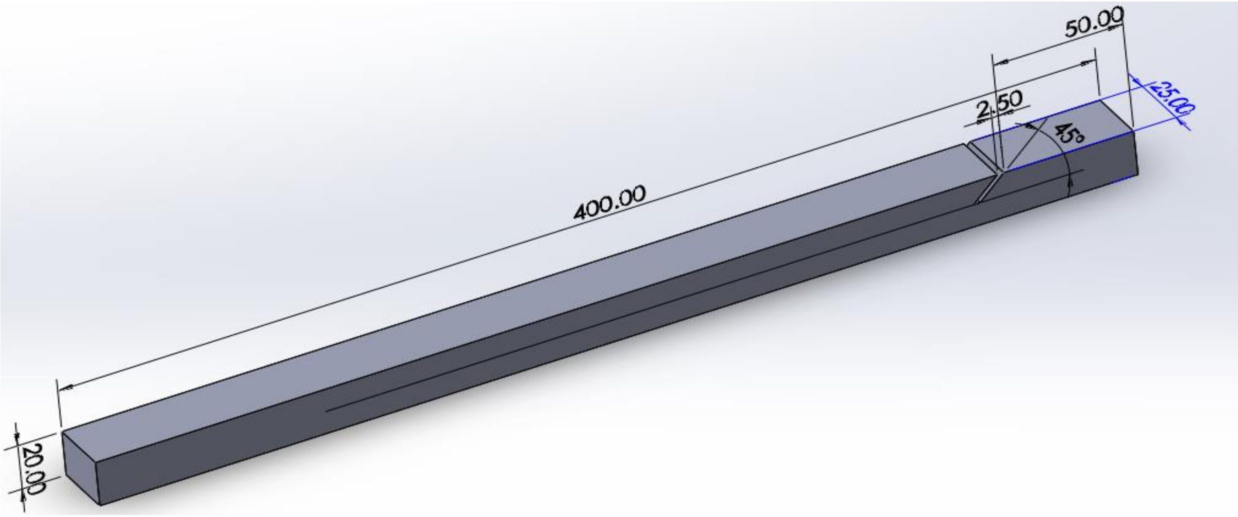

Fig 21: Both end fixed beam with angular crack where the crack location varies

In fig 21, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, an angular crack of length 10mm, height 2.5 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 14: Natural Frequencies found for different location of slit crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 50mm from right and angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 100mm from right and angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 150mm from rightand angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 200mm from right and angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 247.5mm from right and angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 297.5mm from right and angle 45 with vertical	Two end fixed beam of 400mm with slit crack at 347.5mm from right and angle 45 with vertical
644.2	629.3	638.41	601.32	588.51	601.37	638.39	629.43
797.14	788.48	794.99	781.76	776.19	786.11	796.55	779.89
1738.1	1730.5	1579.7	1650.6	1728	1597.3	1626.7	1722.8
2127.3	2116.7	2067	2088.7	2118.5	2071.1	2082.7	2112.5
3316.1	3160	3151.2	3224.5	3007	3308.1	3051.1	3303
3493.7	3329.1	3398.3	3476.4	3504.4	3465.2	3387.5	3313.8

4.15. Both end fixed beam of 400mm with triangular crack:

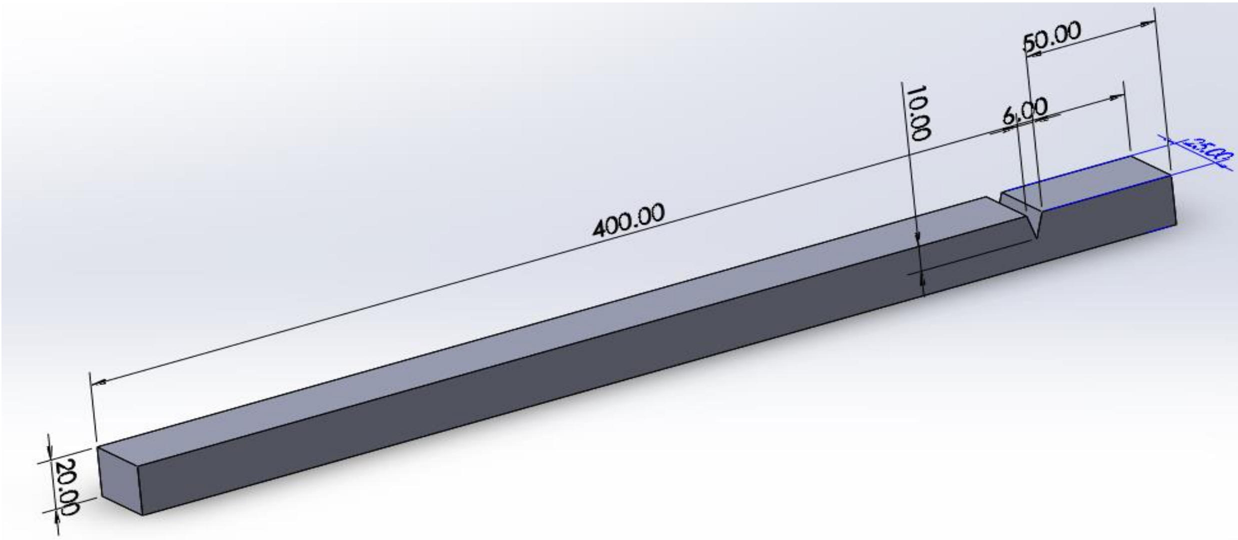

Fig 22: Both end fixed beam with triangular crack where the crack location varies

In fig 22, a beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, an triangular crack of base 6mm, height 10 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 15: Natural Frequencies found for different location of triangular crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with triangular crack at 50mm from right	Two end fixed beam of 400mm with triangular crack at 100mm from right	Two end fixed beam of 400mm with triangular crack at 150mm from right	Two end fixed beam of 400mm with triangular crack at 200mm from right	Two end fixed beam of 400mm with triangular crack at 244mm from right	Two end fixed beam of 400mm with triangular crack at 294mm from right	Two end fixed beam of 400mm with triangular crack at 344mm from right		
644.2	626.05	643.09	615.02	600.15	615.26	643.07	625.86	Triangular crack dimention:	
797.14	788.01	796.81	786.68	780.86	786.73	796.76	787.85	b = 6mm	
1738.1	1738.7	1627.4	1649.9	1737.1	1650.5	1627.3	1738.5	h = 10mm	
2127.3	2122.8	2085.3	2091	2123.1	2091.2	2084.7	2122.7	t = 25mm	
3316.1	3258.6	3147.1	3288.1	3052.9	3288.3	3146.9	3257.4		
3493.7	3400.1	3446.8	3495.3	3514.3	3495.5	3447.1	3399		

4.16. Both end fixed beam with semi-circular crack:

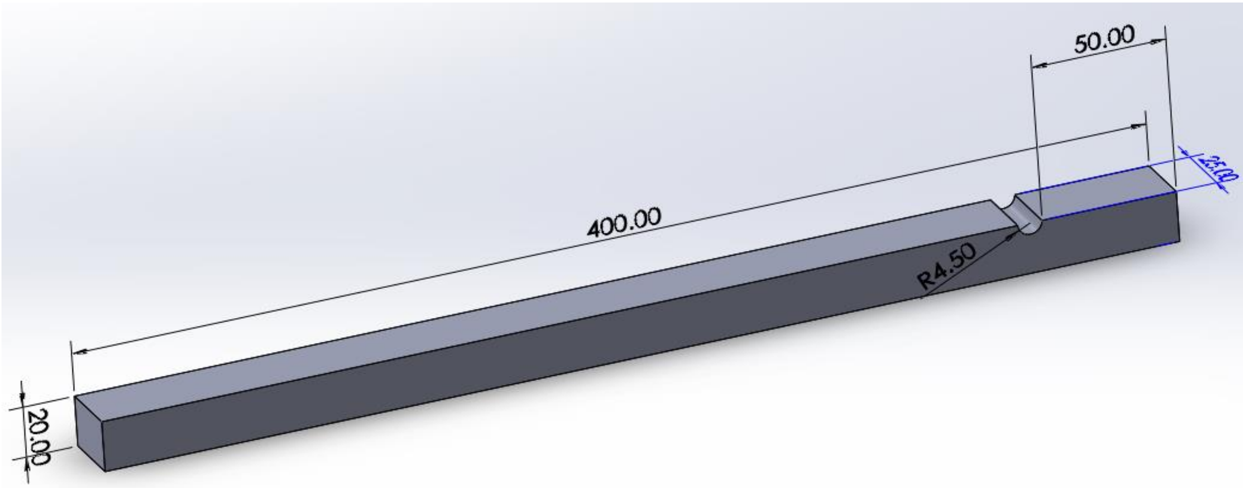

Fig 23: Both end fixed beam with semi-circular crack where the crack location varies

In fig 23, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a semi-circular crack of radius 4.5mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 16: Natural Frequencies found for different location of semi-circular crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with semi-circular crack at 50mm from right	Two end fixed beam of 400mm with semi-circular crack at 100mm from right	Two end fixed beam of 400mm with semi-circular crack at 150mm from right	Two end fixed beam of 400mm with semi-circular crack at 200mm from right	Two end fixed beam of 400mm with semi-circular crack at 241mm from right	Two end fixed beam of 400mm with semi-circular crack at 291mm from right	Two end fixed beam of 400mm with semi-circular crack at 341mm from right	
644.2	638.86	644.52	637.23	633.17	637.23	644.52	638.86	semi-circular crack dimention:
797.14	794.54	797.92	796.17	794.95	796.17	797.92	794.54	r = 4.5mm
1738.1	1739.6	1708.2	1714.6	1737.7	1714.6	1708.2	1739.6	t = 25mm
2127.3	2127.5	2120.4	2120.4	2125.9	2120.4	2120.4	2127.5	
3316.1	3302.2	3267.9	3306.5	3238.4	3306.5	3267.9	3302.2	
3493.7	3452.1	3478.1	3503.9	3512.8	3503.9	3478.1	3452.1	

4.17. Both end fixed beam with slit crack:

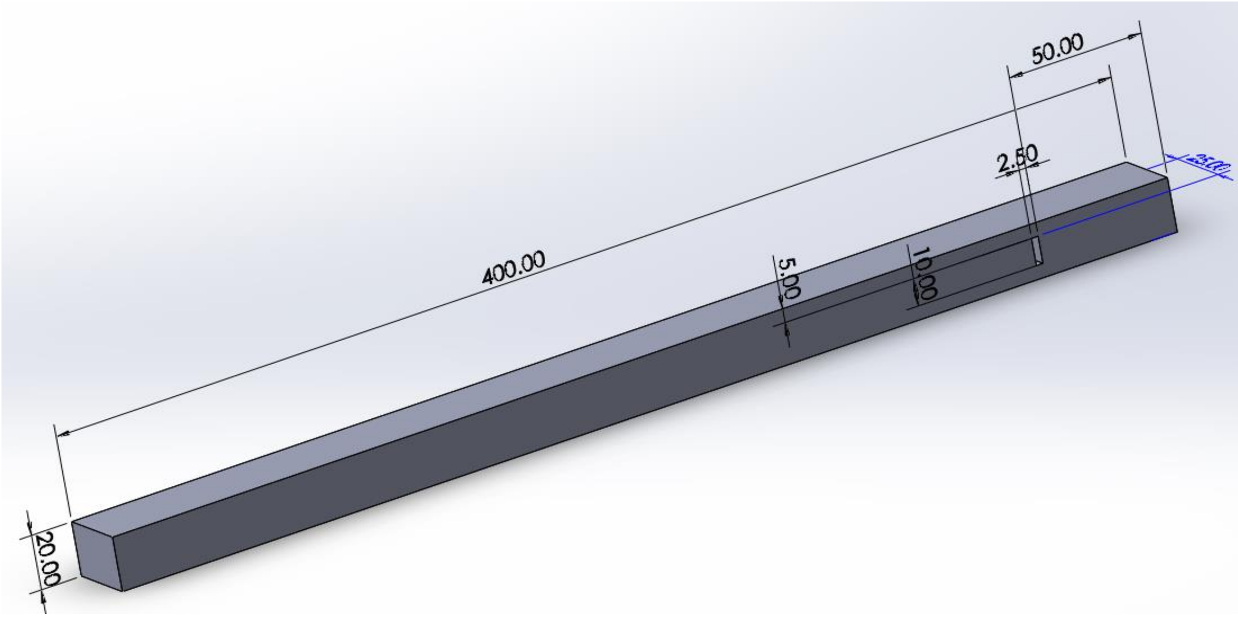

Fig 24: Both end fixed beam with slit crack where the crack location varies

In fig 24, a beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a slit crack of length 10mm, height 2.5 mm and thickness 25 mm is placed at 50mm in the middle of the beam. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 17: Natural Frequencies found for different location of slit crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with slit crack in middle at 50mm from right	Two end fixed beam of 400mm with slit crack in middle at 100mm from right	Two end fixed beam of 400mm with slit crack in middle at 150mm from right	Two end fixed beam of 400mm with slit crack in middle at 200mm from right	Two end fixed beam of 400mm with slit crack in middle at 247.5mm from right	Two end fixed beam of 400mm with slit crack in middle at 297.5mm from right	Two end fixed beam of 400mm with slit crack in middle at 347.5mm from right
644.2	643.19	644.57	645.32	645.57	645.32	644.57	643.17
797.14	790.21	797.3	790.52	785.85	790.52	797.3	790.18
1738.1	1736	1740.5	1738.9	1736.1	1738.9	1740.5	1736
2127.3	2125.7	2099.2	2101.4	2125.5	2101.4	2099.1	2125.7
3316.1	3313.7	3319	3309.2	3318.5	3309.2	3318.9	3313.8
3493.7	3462.7	3478.8	3494.9	3501.5	3494.9	3478.7	3462.7

4.18. Both end fixed beam with angular crack in the middle:

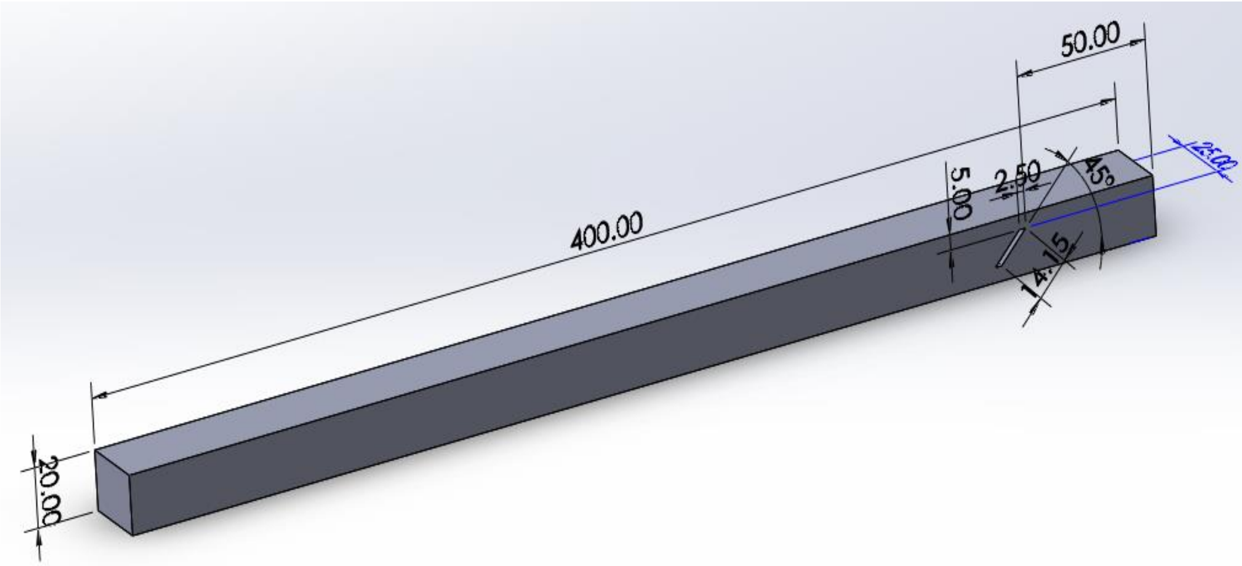

Fig 25: Both end fixed beam with angular crack in the middle where the crack location varies

In fig 25, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, an angular crack of length 10mm, height 2.5 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm in the middle of the beam. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 18: Natural Frequencies found for different location of angular crack in the middle for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 50mm from right and angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 100mm from right and angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 150mm from rightand angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 200mm from right and angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 247.5mm from right and angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 297.5mm from right and angle 45 with vertical in middle	Two end fixed beam of 400mm with slit crack at 347.5mm from right and angle 45 with vertical in middle
644.2	642.91	644.39	645.26	645.53	645.27	644.39	642.88
797.14	791.58	796.9	789.22	785.42	790.97	797.41	787.65
1738.1	1734.3	1740.2	1737.8	1734.7	1738.8	1739.4	1732.6
2127.3	2125.9	2094.1	2103.4	2124.7	2096.9	2100.2	2123.5
3316.1	3311.1	3315.4	3305.7	3317.6	3303.5	3319	3304.5
3493.7	3439.8	3466.9	3493.1	3501.3	3488.8	3460.4	3436.4

4.19. Both end fixed beam with triangular crack in the middle:

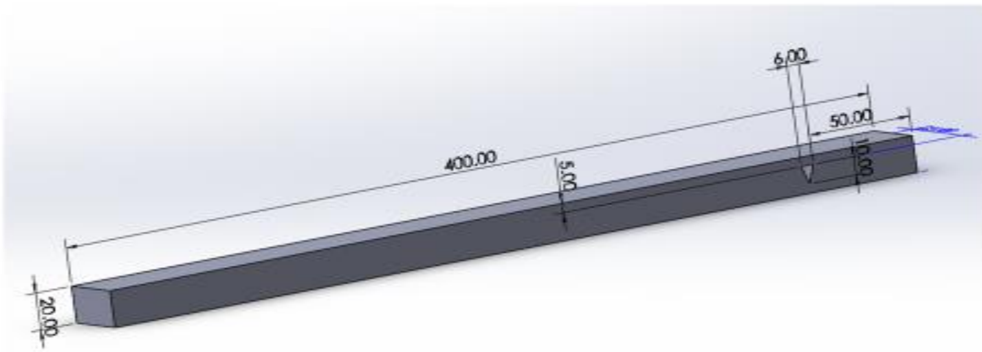

Fig 26: Both end fixed beam with triangular crack in the middle where the crack location varies

In fig 26, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, an triangular crack of base 6mm, height 10 mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm in the middle of the beam. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 19: Natural Frequencies found for different location of triangular crack in the middle for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with triangular crack at 50mm from right	Two end fixed beam of 400mm with triangular crack at 100mm from right in middle	Two end fixed beam of 400mm with triangular crack at 150mm from rightin middle	Two end fixed beam of 400mm with triangular crack at 200mm from right in middle	Two end fixed beam of 400mm with triangular crack at 244mm from right in middle	Two end fixed beam of 400mm with triangular crack at 294mm from right in middle	Two end fixed beam of 400mm with triangular crack at 344mm from right in middle
644.2	643.2	644.74	645.68	646.03	645.52	644.76	643.24
797.14	790.87	797.42	790.76	786.8	790.5	797.44	790.97
1738.1	1736.3	1741.5	1739.2	1736	1738.7	1741.6	1736.4
2127.3	2126.5	2099.8	2103.5	2125.5	2102.8	2100.4	2126.5
3316.1	3315.2	3319.8	3309.1	3320.2	3308.3	3319.9	3315.5
3493.7	3460.2	3478.4	3496.2	3503	3495.6	3478.9	3461.2

4.20. Both end fixed beam with semi-circular crack:

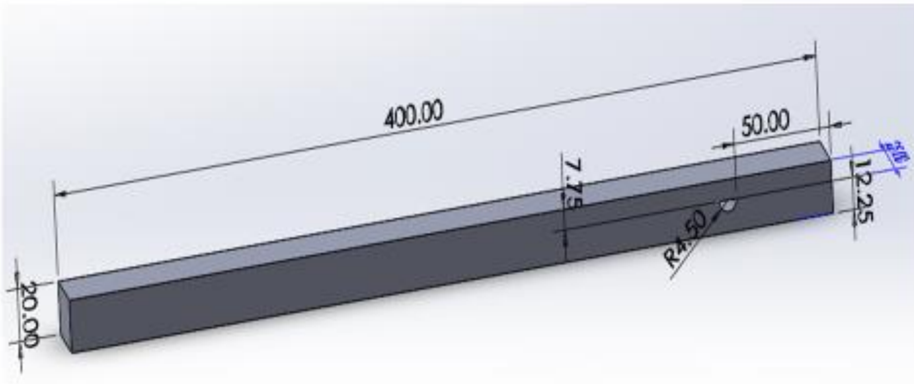

Fig 27: Both end fixed beam with semi-circular crack where the crack location varies

In fig 27, beam of length 400mm, breadth 20mm and height 25mm is considered. Both end of the beam is kept fixed. Modal analysis is used to find the Eigen natural frequencies. Initially, the beam is taken without any defect(crack). A minimum of the first six mode shapes and natural frequencies are obtained and shown in Table. Block Lanczos method which is generally used in the case of symmetric structures is used to find the fundamental frequencies. Then, a semi-circular crack of radius 4.5mm and thickness 25 mm is placed at 50mm. The location of crack is varied between 50 mm to 347.5 mm with an increment of 50 mm in the middle of the beam. Similarly, fundamental frequencies and mode shapes for the step by step crack locations are obtained.

Table. 20: Natural Frequencies found for different location of semi-circular in the middle crack for Both end fixed beam

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with semi-circular crack at 50mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 100mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 150mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 200mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 241mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 291mm from right in middle	Two end fixed beam of 400mm with semi-circular crack at 341mm from right in middle
644.2	643.8	644.96	646.64	647.36	646.64	644.96	643.81
797.14	795.35	798.21	797.24	796.42	797.24	798.2	795.37
1738.1	1737.1	1745.1	1741.8	1736.3	1741.8	1745.1	1737.1
2127.3	2128.8	2124.4	2123.5	2126.9	2123.5	2124.2	2128.9
3316.1	3319	3325.1	3311.3	3328.1	3311.3	3325	3319.1
3493.7	3466.5	3482.4	3497.3	3502.6	3497.3	3482.4	3466.8

5. Discritized Model of Beams:

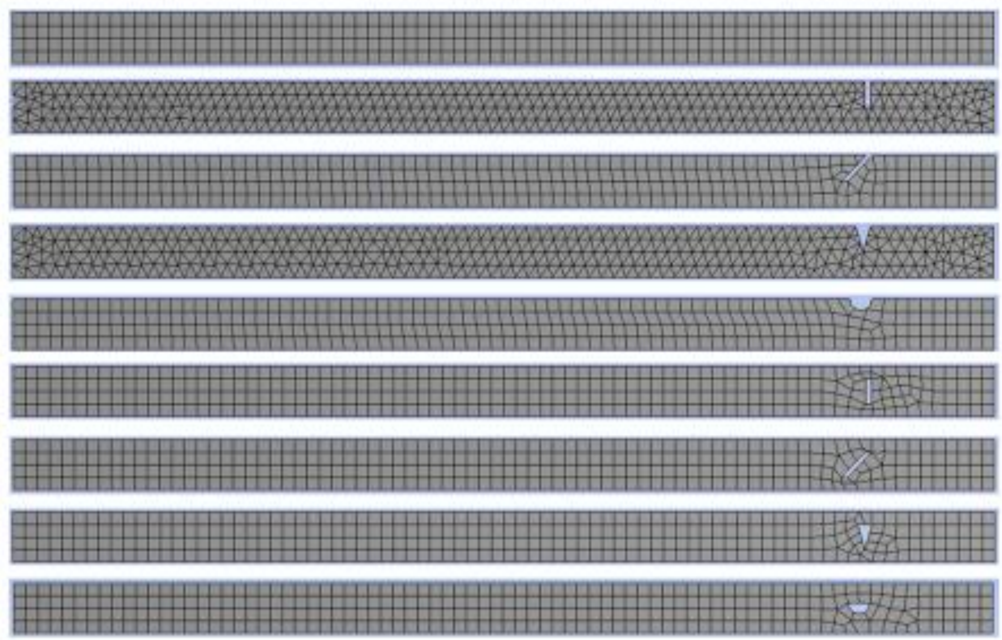

Fig 28: Discritized model of beam with different types of cracks

The FE model of the beam without any crack consists of 1600 elements and 8799 nodes. The beams model with crack are also discretized in a similar way of the beam discretized without crack. The model of the simply supported beam with slit crack has 7211 elements and 13188 nodes. The model of the beam with angular crack has 1565 elements and 8644 nodes. The model of the simply supported beam with triangular crack has 7170 elements and 13149 nodes. The model of the beam with semi-circular crack has 1511 elements and 8631 nodes. The model of the simply supported beam with slit crack in the middle of the width of the beam has 1533 elements and 8701 nodes. The model of the beam with angular crack in the middle has 1541 elements and 8690 nodes. The model of the simply supported beam with triangular crack in the middle of the width of the beam has 1529 elements and 8656 nodes. The model of the beam with semi-circular crack in the middle of the width of the beam has 1543 elements and 8613 nodes. The discretized beam models are shown in Fig 21.

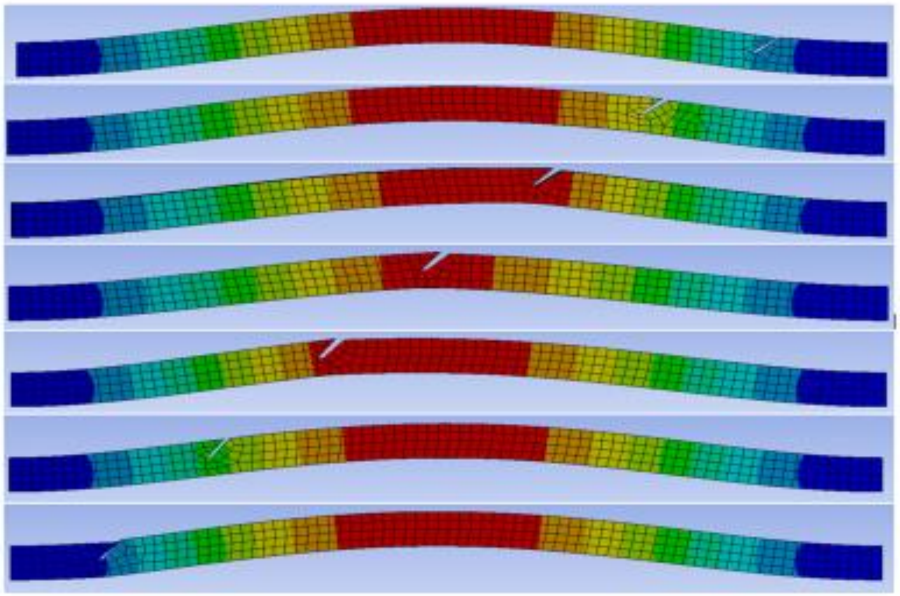

Fig 29: Mode 1 of beam with angular crack for diff. location of cracks

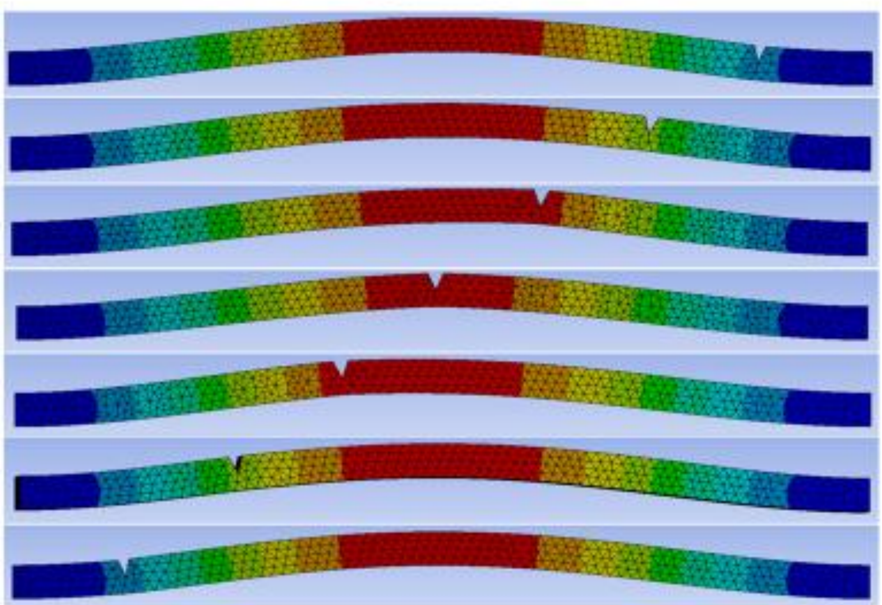

Fig 30: Mode 1 of beam with triangular crack for diff. location of cracks

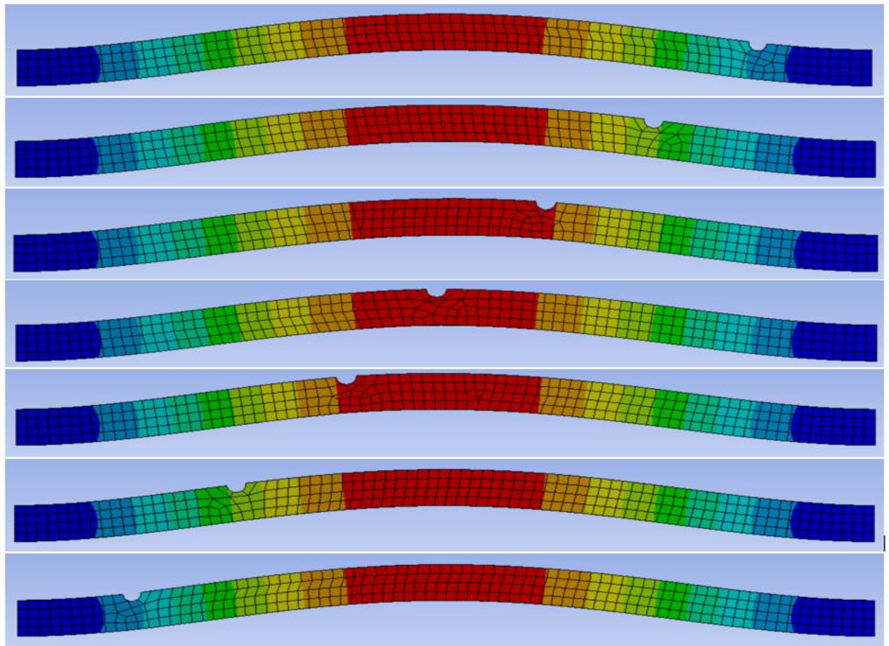

Fig 31: Mode 1 of beam with semi-circular crack for diff. location of cracks

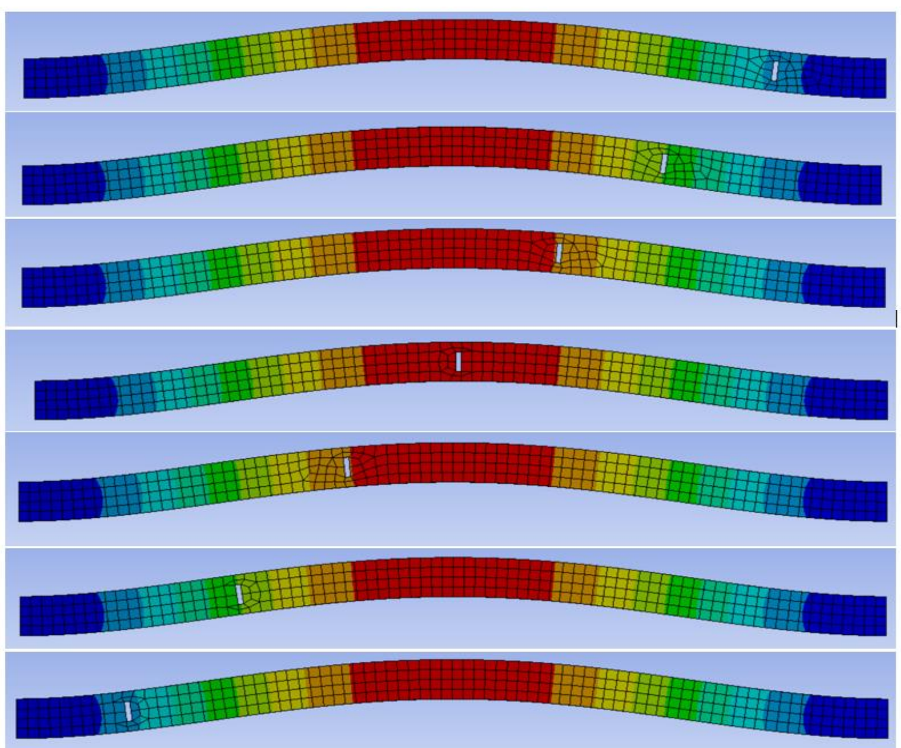

Fig 32: Mode 1 of beam with slit crack in the middle for diff. location of cracks

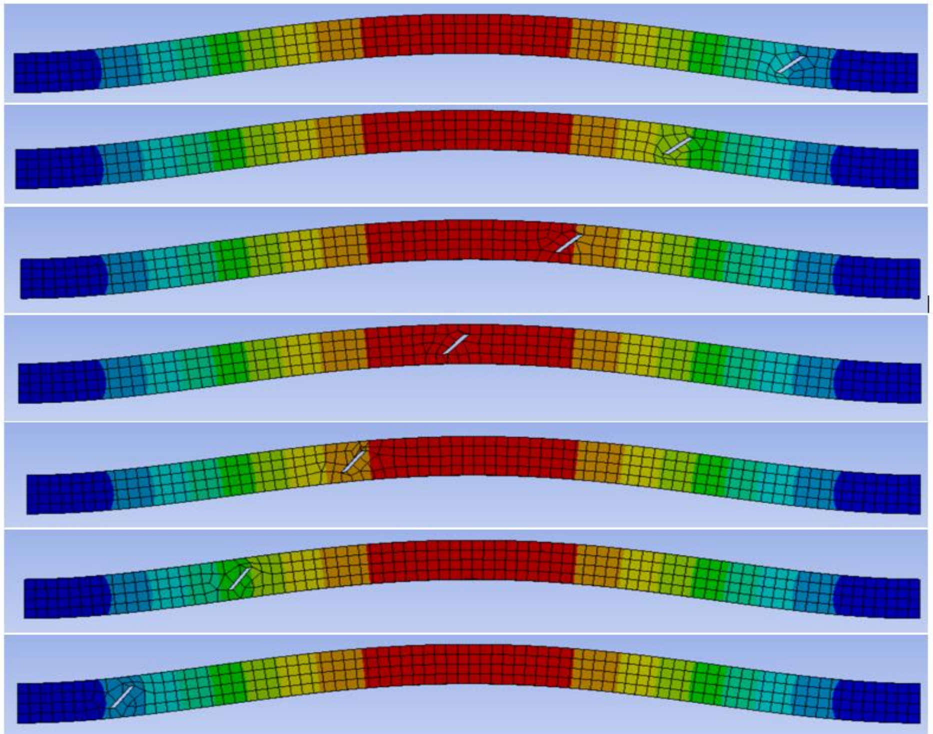

Fig 33: Mode 1 of beam with angular crack in the middle for diff. location of cracks

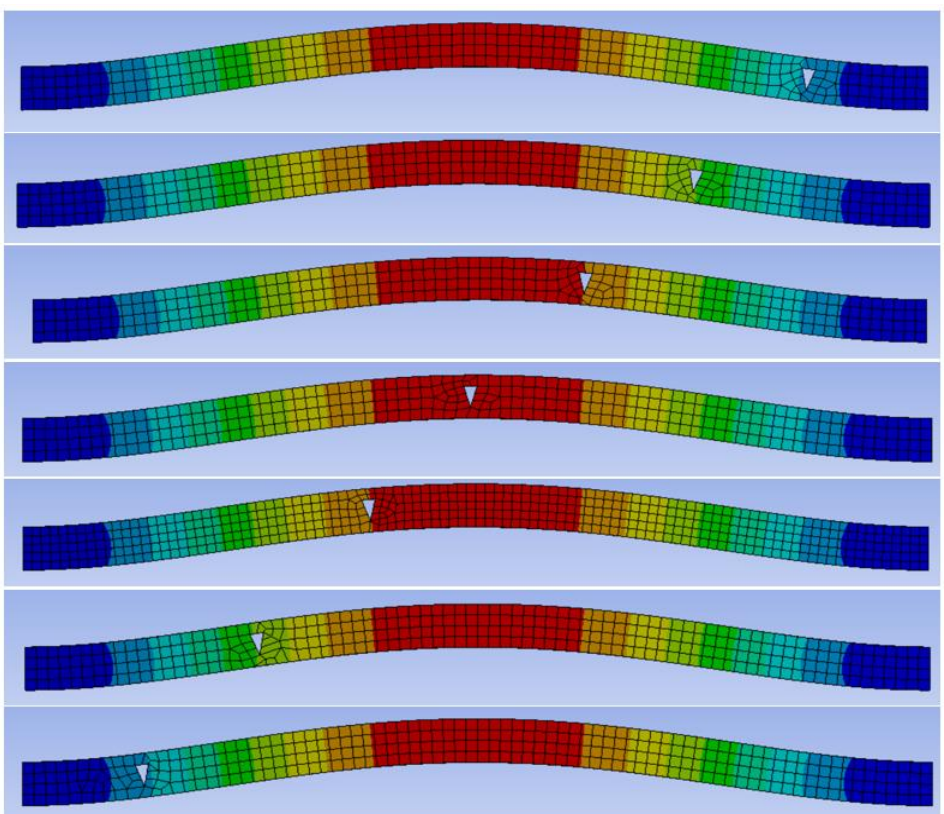

Fig 34: Mode 1 of beam with triangular crack in the middle for diff. location of cracks

6. Different Types of Comparisons:

Comparison of 1st mode frequency between beams without any crack and with diff. types of cracks

Crack locatiom	Frequency of lowest mode for beam without crack	Frequency of lowest mode for beam with slit crack	Frequency of lowest mode for beam with angular crack	Frequency of lowest mode for beam with triangular crack	Frequency of lowest mode for beam with semi-circular crack
50	644.2	620.9	629.3	626.05	638.86
100	644.2	642.99	638.41	643.09	644.52
150	644.2	610.54	601.32	615.02	637.23
200	644.2	591.39	588.51	600.15	633.17
247.5	644.2	610.35	601.37	615.26	637.23
297.5	644.2	642.93	638.39	643.07	644.52
347.5	644.2	621	629.43	625.86	638.86

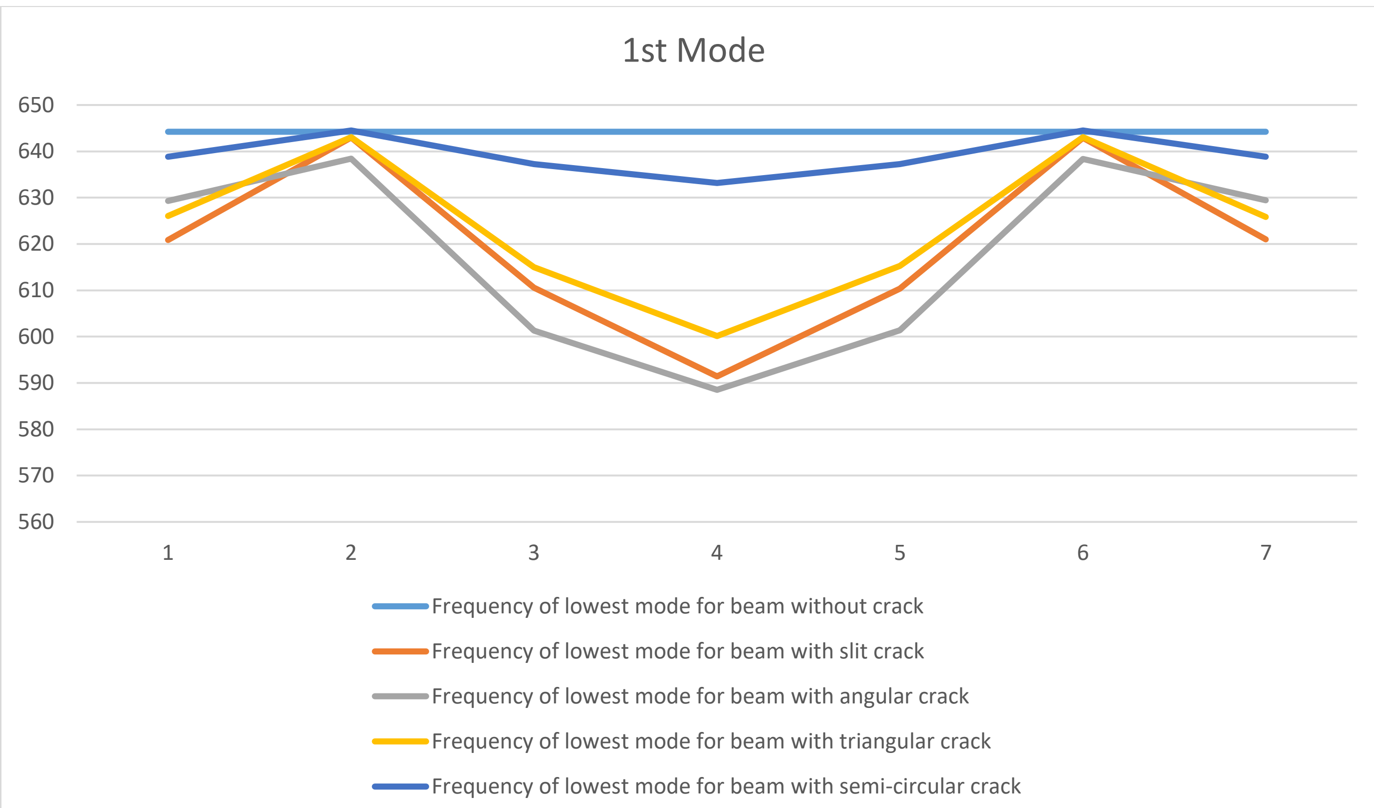

Figure: 35

In the following graph, a comparison is made between the frequency of 1st mode of beam with slit crack, angular crack, triangular crack and semi-circular crack with frequency of 1st mode without any crack. In all of the cases, similar behavior can be seen. Frequency of cracked beams first increases and reaches the frequency of un-cracked beams and starts to decrease and becomes minimum at the middle and after that it shows a similar behavior. Symmetric behavior can be seen as both ends are fixed.

Comparison of 1st mode frequency between beams without any crack and with diff. types of cracks in the middle

Crack locatiom	Frequency of lowest mode for beam without crack	Frequency of lowest mode for beam with slit crack in middle	Frequency of lowest mode for beam with angular crack in middle	Frequency of lowest mode for beam with triangular crack in middle	Frequency of lowest mode for beam with semi-circular crack in middle
50	644.2	643.19	642.91	643.2	643.8
100	644.2	644.57	644.39	644.74	644.96
150	644.2	645.32	645.26	645.68	646.64
200	644.2	645.57	645.53	646.03	647.36
247.5	644.2	645.32	645.27	645.52	646.64
297.5	644.2	644.57	644.39	644.76	644.96
347.5	644.2	643.17	642.88	643.24	643.81

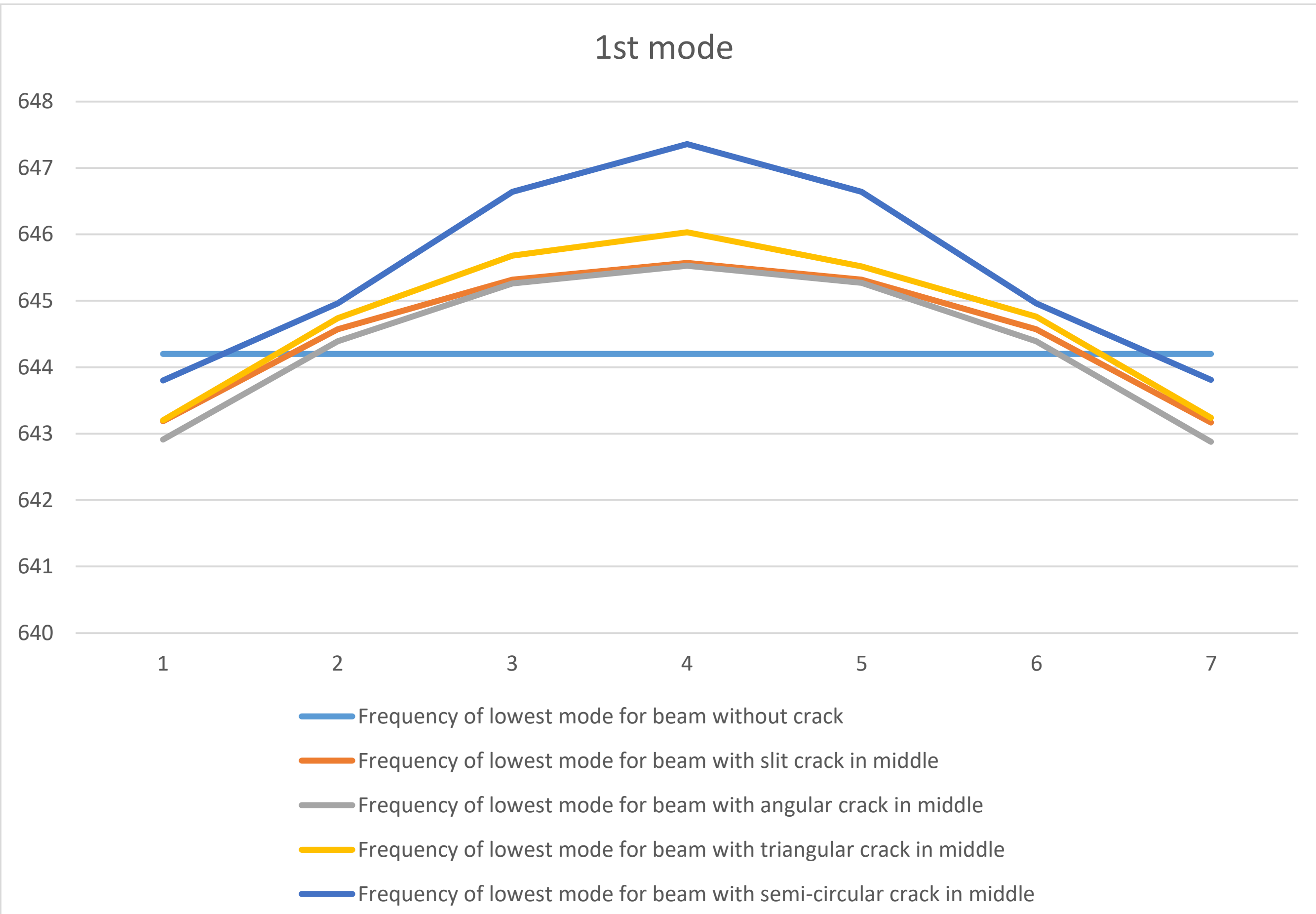

Figure: 36

In the following graph, a comparison is made between the frequency of 1st mode of beam with slit crack, angular crack, triangular crack and semi-circular crack in the middle of the width of the beam with frequency of 1st mode without any crack. In all of the cases, similar behavior can be seen. Frequency of cracked beams first increases and crosses the frequency of un-cracked beams and becomes minimum at the middle and then, it starts to decrease and after that it shows a similar behavior. Symmetric behavior can be seen as both ends are fixed.

7. Different Boundary Conditions:

Three different boundary conditions are considered:

7.1. One end fixed Beam i.e. Cantiliver Beam:

Table. 21: Natural frequencies of 1st six modes for Cantiliver beam without crack and with crack

Beam Without any crack	Beam with crack at 390mm	Beam with crack at 380mm	Beam with crack at 370mm	Beam with crack at 360mm	Beam with crack at 350mm	Beam with crack at 340mm	Beam with crack at 330mm	Beam with crack at 320mm	Beam with crack at 310mm	Beam with crack at 300mm	Beam with crack at 290mm
102.4	103.17	103.11	103.07	103.01	102.95	102.9	102.83	102.75	102.66	102.55	102.41
127.81	128.76	128.7	128.63	128.56	128.51	128.45	128.39	128.33	128.26	128.19	128.11
634.31	638.8	637.7	636.67	635.26	633.31	630.64	626.81	621.81	616.01	608.96	601.74
786.62	792.16	790.85	789.64	788.39	787.22	785.84	784.23	782.39	780.29	777.91	775.43
1743.5	1751.4	1751.2	1744.4	1731.5	1709.5	1679.4	1642.9	1605.8	1576.6	1553.3	1543.2
1744.4	1756.2	1754.2	1753.6	1753.1	1752.3	1751.2	1749.8	1748.5	1746.7	1744.9	1742.8

Beam Without any crack	Beam with crack at 280mm	Beam with crack at 270mm	Beam with crack at 260mm	Beam with crack at 250mm	Beam with crack at 240mm	Beam with crack at 230mm	Beam with crack at 220mm	Beam with crack at 210mm	Beam with crack at 200mm	Beam with crack at 190mm	Beam with crack at 180mm
102.4	102.25	102.05	101.83	101.57	101.23	100.92	100.54	100.07	99.554	99.046	98.445
127.81	128.02	127.93	127.82	127.71	127.57	127.44	127.28	127.12	126.91	126.71	126.47
634.31	594.08	586.54	579.71	574.22	568.1	565.8	563.87	562.61	562.78	565.79	569.45
786.62	772.87	770.23	767.61	765.49	763.2	762.22	761.3	760.86	760.82	761.93	763.19
1743.5	1542.2	1551	1569.2	1595.6	1623.9	1658.9	1691.8	1719.1	1720.1	1717.6	1714.7
1744.4	1740.9	1738.3	1736.3	1733.8	1730.9	1728.3	1726	1722.9	1737.9	1743.7	1735.5

Beam Without any crack	Beam with crack at 170mm	Beam with crack at 160mm	Beam with crack at 150mm	Beam with crack at 140mm	Beam with crack at 130mm	Beam with crack at 120mm	Beam with crack at 110mm	Beam with crack at 100mm	Beam with crack at 90mm	Beam with crack at 80mm	Beam with crack at 70mm
102.4	97.852	97.114	96.451	95.687	94.769	94.102	93.204	92.189	91.361	90.404	89.401
127.81	126.25	125.99	125.71	125.38	125.05	124.73	124.38	124	123.62	123.16	122.7
634.31	575.23	581.27	589.46	597.92	606.36	615.47	623.21	629.37	633.42	634.7	632.94
786.62	765.51	768.06	771	773.91	777.06	780.17	782.79	784.85	786.08	786.29	785.43
1743.5	1712.6	1689	1660.9	1634.6	1612.3	1602.8	1600.9	1609.5	1632.5	1663.5	1690.5
1744.4	1716.1	1709.4	1707	1704.6	1702.2	1700.5	1698.2	1695.7	1694.2	1692.6	1698.1

Beam Without any crack	Beam with crack at 60mm	Beam with crack at 50mm	Beam with crack at 40mm	Beam with crack at 30mm	Beam with crack at 20mm	Beam with crack at 10mm	Beam with crack at 0mm
102.4	88.413	87.508	86.325	85.53	84.227	83.252	83.751
127.81	122.25	121.88	121.36	120.87	120.33	120.04	121.05
634.31	628.15	620.59	610.27	598.36	583.86	568.73	557.55
786.62	783.29	779.95	775.32	769.42	762.29	755.17	754.23
1743.5	1689	1688	1686.9	1686.4	1681.7	1634.8	1590.1
1744.4	1728	1743.8	1740.5	1718.8	1685.3	1686.7	1693.3

7.2. Both end Fixed Beam:

Table. 22: Natural frequencies of 1st six modes for Both end beam without crack and with crack

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 0mm from right	Two end fixed beam of 400mm with crack at 10mm from right	Two end fixed beam of 400mm with crack at 20mm from right	Two end fixed beam of 400mm with crack at 30mm from right	Two end fixed beam of 400mm with crack at 40mm from right	Two end fixed beam of 400mm with crack at 50mm from right	Two end fixed beam of 400mm with crack at 60mm from right	Two end fixed beam of 400mm with crack at 70mm from right	Two end fixed beam of 400mm with crack at 80mm from right	Two end fixed beam of 400mm with crack at 90mm from right
644.2	575.33	574.68	589.73	603.15	615.2	626.05	634.38	640.43	643.91	644.71
797.14	768.79	762.77	769.02	776.25	782.73	788.01	792.06	794.91	796.61	797.17
1738.1	1595.8	1619.6	1666.5	1703.7	1728.7	1738.7	1731.6	1709.9	1680.7	1650.6
2127.3	2064.9	2060.9	2083.7	2103.8	2117.2	2122.8	2121.4	2114.8	2105.1	2094.2
3316.1	3101	3168.6	3255.9	3307.6	3308.9	3258.6	3183.8	3122.8	3099.3	3108.6
3493.7	3404	3383.8	3382.4	3385.4	3391.5	3400.1	3408.4	3417.8	3427.8	3437

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 100mm from right	Two end fixed beam of 400mm with crack at 110mm from right	Two end fixed beam of 400mm with crack at 120mm from right	Two end fixed beam of 400mm with crack at 130mm from right	Two end fixed beam of 400mm with crack at 140mm from right	Two end fixed beam of 400mm with crack at 150mm from right	Two end fixed beam of 400mm with crack at 160mm from right	Two end fixed beam of 400mm with crack at 170mm from right	Two end fixed beam of 400mm with crack at 180mm from right	Two end fixed beam of 400mm with crack at 190mm from right
644.2	643.09	639.38	634.12	627.95	621.71	615.02	610.07	605.73	602.27	600.1
797.14	796.81	795.62	793.81	791.54	789.26	786.68	784.86	783.1	781.76	780.84
1738.1	1627.4	1612.8	1608.6	1614.7	1630.3	1649.9	1675.9	1700.9	1721.5	1734.8
2127.3	2085.3	2079.3	2076.9	2078.1	2083.9	2091	2101.2	2110.3	2117.9	2122.4
3316.1	3147.1	3200.1	3255.9	3299.6	3313.9	3288.1	3231.2	3161.3	3098.7	3058
3493.7	3446.8	3457.6	3467.2	3477.8	3487.4	3495.3	3502.6	3508.1	3512	3513.6

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 200mm from right	Two end fixed beam of 400mm with crack at 210mm from right	Two end fixed beam of 400mm with crack at 220mm from right	Two end fixed beam of 400mm with crack at 230mm from right	Two end fixed beam of 400mm with crack at 240mm from right	Two end fixed beam of 400mm with crack at 250mm from right	Two end fixed beam of 400mm with crack at 260mm from right	Two end fixed beam of 400mm with crack at 270mm from right	Two end fixed beam of 400mm with crack at 280mm from right	Two end fixed beam of 400mm with crack at 290mm from right
644.2	600.15	601.2	604.14	607.76	613.2	619.11	625.25	631.85	637.49	641.84
797.14	780.86	781.19	782.47	784.07	786.11	788.21	790.59	792.98	794.95	796.46
1738.1	1737.1	1728	1709.9	1685.6	1660.9	1637.8	1619	1611.2	1610.6	1619.7
2127.3	2123.1	2120	2113.6	2104.8	2095.6	2086.3	2079.7	2077	2077.9	2082.7
3316.1	3052.9	3078.6	3134.1	3202.1	3268.6	3308.2	3309.7	3276.6	3223.7	3166.6
3493.7	3514.3	3512.6	3510.2	3504.9	3498.4	3490.5	3481.5	3471.9	3461.5	3450.5

Two end fixed beam of 400mm without ant crack	Two end fixed beam of 400mm with crack at 300mm from right	Two end fixed beam of 400mm with crack at 310mm from right	Two end fixed beam of 400mm with crack at 320mm from right	Two end fixed beam of 400mm with crack at 330mm from right	Two end fixed beam of 400mm with crack at 340mm from right	Two end fixed beam of 400mm with crack at 350mm from right	Two end fixed beam of 400mm with crack at 360mm from right	Two end fixed beam of 400mm with crack at 370mm from right	Two end fixed beam of 400mm with crack at 380mm from right	Two end fixed beam of 400mm with crack at 390mm from right
644.2	644.34	644.55	642.12	637.11	629.68	619.78	608.46	594.76	580.39	569.61
797.14	797.18	796.92	795.72	793.29	789.74	784.96	779.01	771.87	764.82	764.09
1738.1	1640.8	1668.6	1698.8	1724.4	1737.7	1734.7	1715.5	1682	1638.7	1595.9
2127.3	2090.9	2100.6	2111.2	2119.3	2122.9	2120.2	2110.2	2092.3	2069.4	2057.2
3316.1	3122.2	3099.6	3109.1	3156.3	3229.4	3294.2	3314.8	3281.1	3206	3115.9
3493.7	3440.7	3431	3420.8	3411.5	3403.2	3395	3388.6	3381.7	3380.6	3392.1

7.3. Both end Hinged Beam:

Table. 23: Natural frequencies of 1st six modes for Both end hinged without crack and with crack

Remote displacement at two ends of beam of 400mm without ant crack	Remote displacement at two ends of beam of 400mm with crack at 0mm	Remote displacement at two ends of beam of 400mm with crack at 10mm	Remote displacement at two ends of beam of 400mm with crack at 20mm	Remote displacement at two ends of beam of 400mm with crack at 30mm	Remote displacement at two ends of beam of 400mm with crack at 40mm	Remote displacement at two ends of beam of 400mm with crack at 50mm	Remote displacement at two ends of beam of 400mm with crack at 60mm	Remote displacement at two ends of beam of 400mm with crack at 70mm	Remote displacement at two ends of beam of 400mm with crack at 80mm
2.85E-03	0	1.32E-03	1.34E-04	0.00E+00	0	0	0	0	0
5.44E-03	0	1.55E-03	6.69E-04	1.12E-03	1.03E-03	1.37E-03	1.05E-03	0	2.93E-04
797.14	768.79	762.77	769.02	776.25	782.73	788.01	792.06	794.91	796.61
1133.6	1067.1	1051.2	1056.8	1065	1075.9	1089.3	1102.5	1115.1	1125.5
2127.3	2064.9	2060.9	2083.7	2103.8	2117.2	2122.8	2121.4	2114.8	2105.1
2274.3	2165.2	2158.7	2187	2218.4	2247.5	2268.2	2272.3	2255.3	2218.7
Remote displacement at two ends of beam of 400mm without ant crack	Remote displacement at two ends of beam of 400mm with crack at 90mm	Remote displacement at two ends of beam of 400mm with crack at 100mm	Remote displacement at two ends of beam of 400mm with crack at 110mm	Remote displacement at two ends of beam of 400mm with crack at 120mm	Remote displacement at two ends of beam of 400mm with crack at 130mm	Remote displacement at two ends of beam of 400mm with crack at 140mm	Remote displacement at two ends of beam of 400mm with crack at 150mm	Remote displacement at two ends of beam of 400mm with crack at 160mm	Remote displacement at two ends of beam of 400mm with crack at 170mm
2.85E-03	0	5.49E-04	0	0	1.11E-03	0	0	0	0
5.44E-03	1.95E-03	2.41E-03	7.39E-04	1.68E-03	2.61E-03	1.15E-03	1.69E-03	9.78E-04	1.37E-03
797.14	797.17	796.81	795.62	793.81	791.54	789.26	786.68	784.86	783.1
1133.6	1131.9	1133.2	1128.6	1118.5	1104.2	1088	1070.4	1056.4	1044.2
2127.3	2094.2	2085.3	2079.3	2072.7	2075.2	2083.9	2091	2101.2	2110.3
2274.3	2170.3	2124.3	2089.7	2077	2078.2	2095.8	2125.7	2167.6	2209.6
Remote displacement at two ends of beam of 400mm without ant crack	Remote displacement at two ends of beam of 400mm with crack at 180mm	Remote displacement at two ends of beam of 400mm with crack at 190mm	Remote displacement at two ends of beam of 400mm with crack at 200mm	Remote displacement at two ends of beam of 400mm with crack at 210mm	Remote displacement at two ends of beam of 400mm with crack at 220mm	Remote displacement at two ends of beam of 400mm with crack at 230mm	Remote displacement at two ends of beam of 400mm with crack at 240mm	Remote displacement at two ends of beam of 400mm with crack at 250mm	Remote displacement at two ends of beam of 400mm with crack at 260mm
2.85E-03	0	0	0	3.18E-04	0	0	0	0	0
5.44E-03	0	8.54E-04	1.72E-03	1.03E-03	1.60E-03	2.05E-03	1.27E-03	0	6.02E-04
797.14	781.76	780.84	780.86	781.19	782.47	784.07	786.11	788.21	790.59
1133.6	1034.7	1028.7	1028.7	1031.8	1039.8	1050.2	1065	1081.2	1097.4
2127.3	2117.9	2122.4	2123.1	2120	2113.6	2104.8	2095.6	2086.3	2079.6
2274.3	2245	2268.1	2272.2	2256.3	2224.9	2183.9	2143	2107	2080
Remote displacement at two ends of beam of 400mm without ant crack	Remote displacement at two ends of beam of 400mm with crack at 270mm	Remote displacement at two ends of beam of 400mm with crack at 280mm	Remote displacement at two ends of beam of 400mm with crack at 290mm	Remote displacement at two ends of beam of 400mm with crack at 300mm	Remote displacement at two ends of beam of 400mm with crack at 310mm	Remote displacement at two ends of beam of 400mm with crack at 320mm	Remote displacement at two ends of beam of 400mm with crack at 330mm	Remote displacement at two ends of beam of 400mm with crack at 340mm	Remote displacement at two ends of beam of 400mm with crack at 350mm
2.85E-03	0	9.02E-04	0	0	0	0	8.37E-04	0	0
5.44E-03	1.49E-03	2.39E-03	0	1.27E-03	0.00E+00	8.33E-04	2.20E-03	9.72E-04	1.84E-03
797.14	792.98	794.95	796.46	797.18	796.92	795.72	793.29	789.74	784.96
1133.6	1113.3	1125.2	1132	1133.1	1128.6	1119.6	1107.7	1094.7	1081.1
2127.3	2073.5	2077.9	2082.7	2090.9	2100.6	2111.2	2119.3	2122.9	2120.2
2274.3	2077	2081.7	2107.6	2151.5	2200.2	2242.8	2268.2	2272.1	2257.3
Remote displacement at two ends of beam of 400mm without ant crack	Remote displacement at two ends of beam of 400mm with crack at 360mm	Remote displacement at two ends of beam of 400mm with crack at 370mm	Remote displacement at two ends of beam of 400mm with crack at 380mm	Remote displacement at two ends of beam of 400mm with crack at 390mm					
2.85E-03	0	4.59E-04	0	0					
5.44E-03	2.06E-03	1.29E-03	1.53E-03	1.67E-03					
797.14	779.01	771.87	764.82	764.09					
1133.6	1069.7	1059	1052.4	1054.6					
2127.3	2110.2	2092.3	2069.4	2057.2					
2274.3	2231	2198.7	2168.5	2152.7					

8. Bending mode shapes for different boundary conditions:

First 3 mode shapes of bending for Cantiliver Beam:

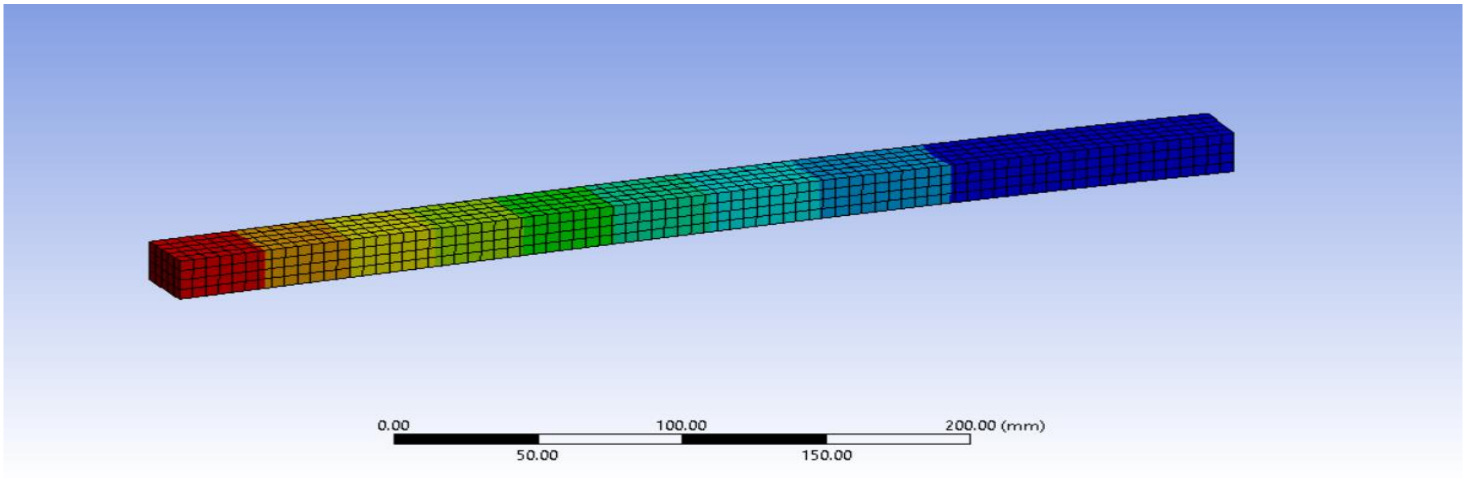

Fig 37: Bending mode shape 1 of Cantiliver Beam

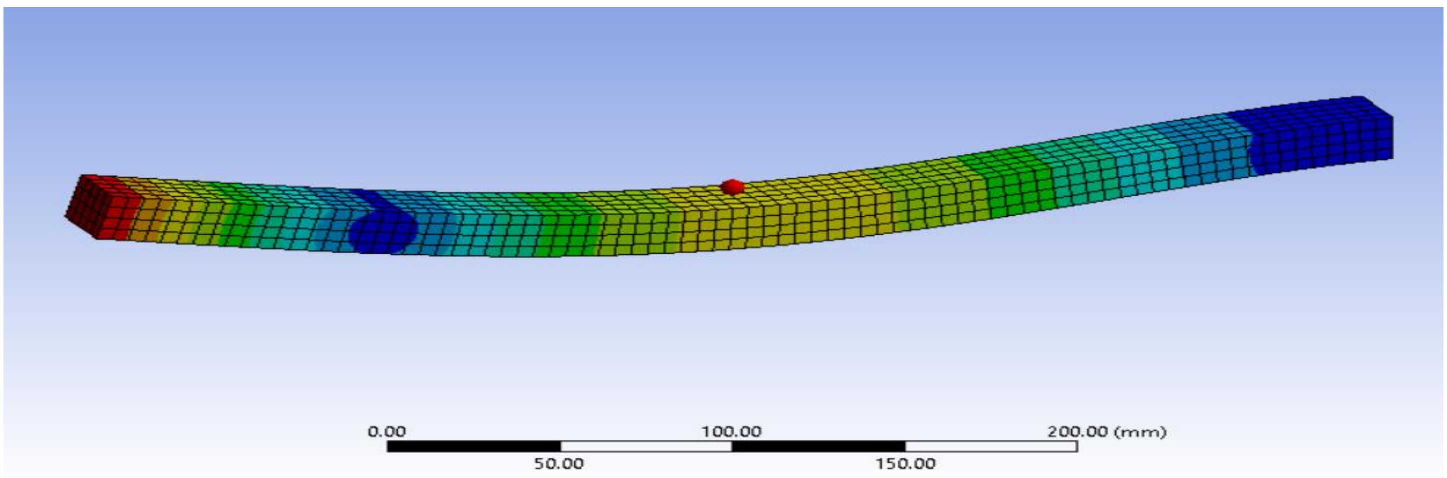

Fig 38: Bending mode shape 2 of Cantiliver Beam

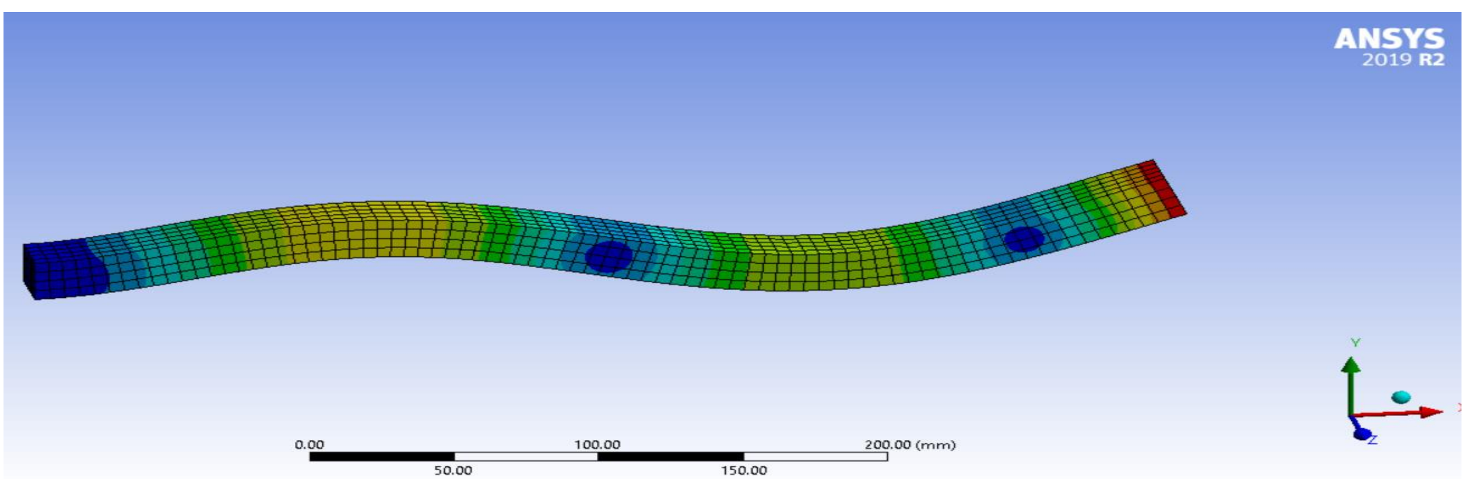

Fig 39: Bending mode shape 3 of Cantiliver Beam

First 3 mode shapes of bending for Both end fixed Beam:

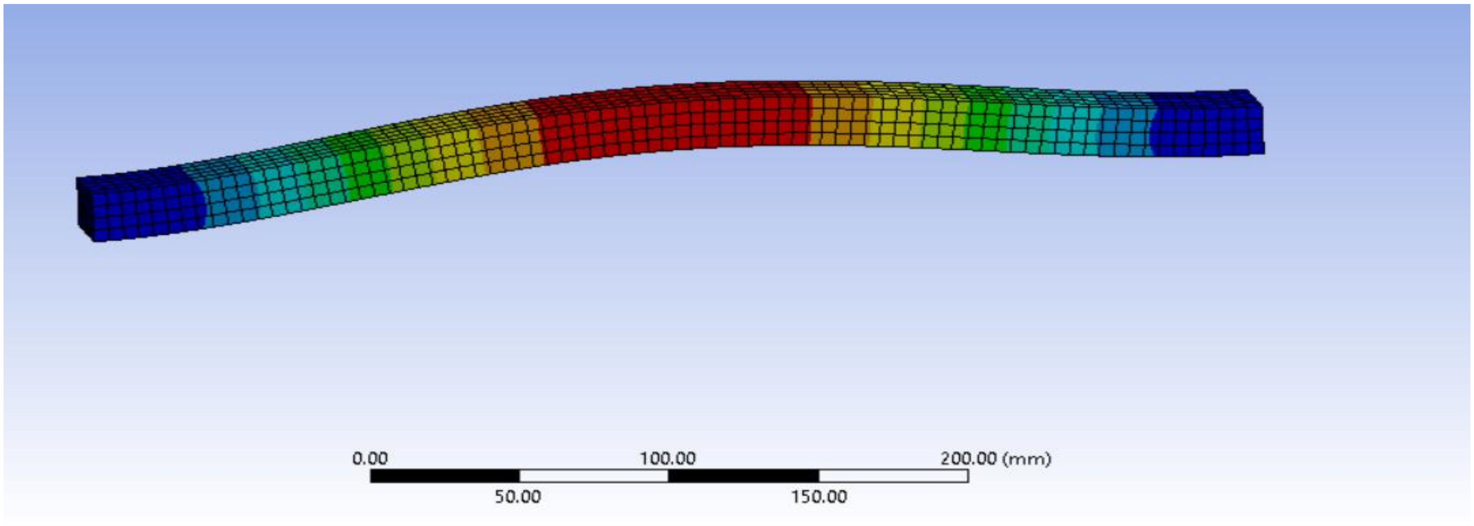

Fig 40: Bending mode shape 1 of both end fixed Beam

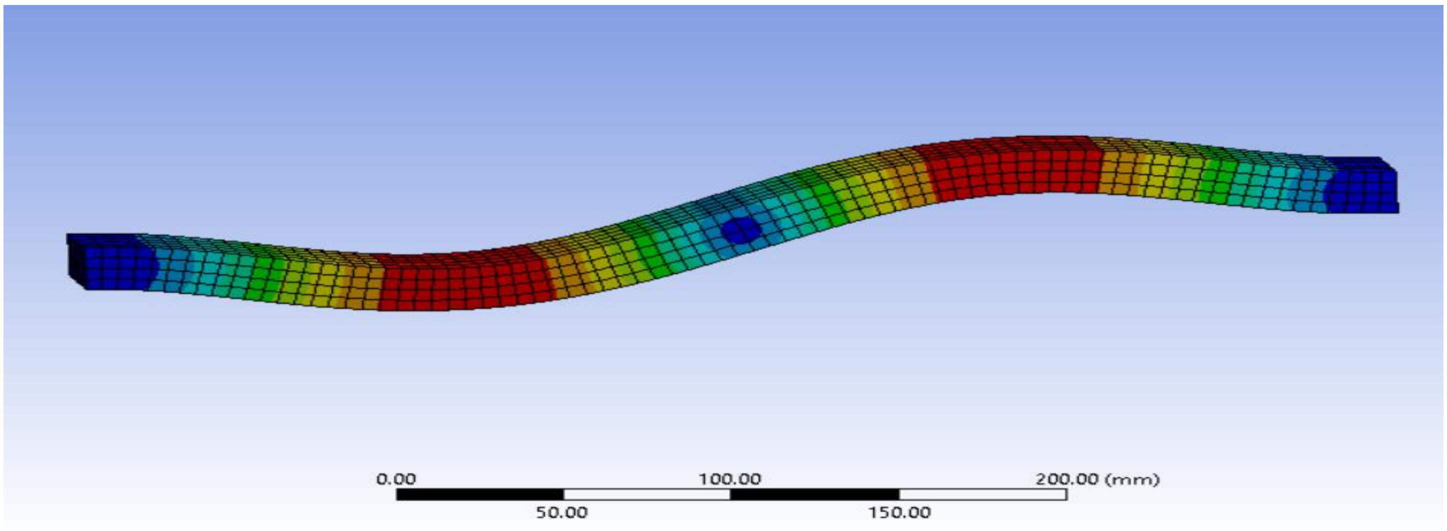

Fig 41: Bending mode shape 2 of both end fixed Beam

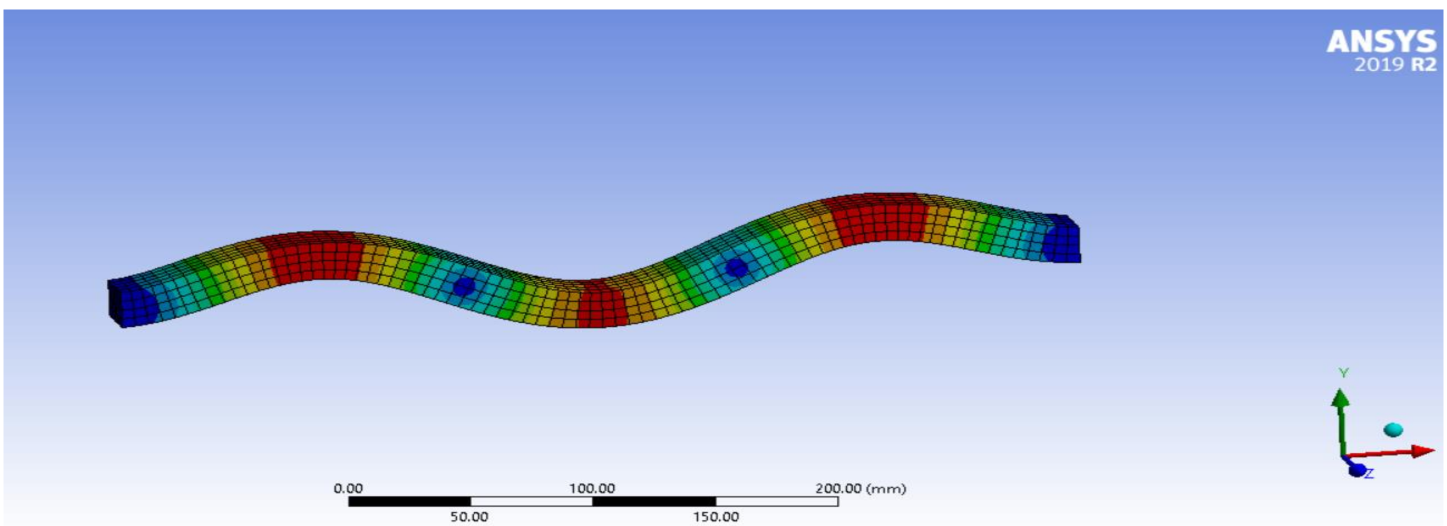

Fig 42: Bending mode shape 3 of both end fixed Beam

First 3 mode shapes of bending for Simply Supported Beam:

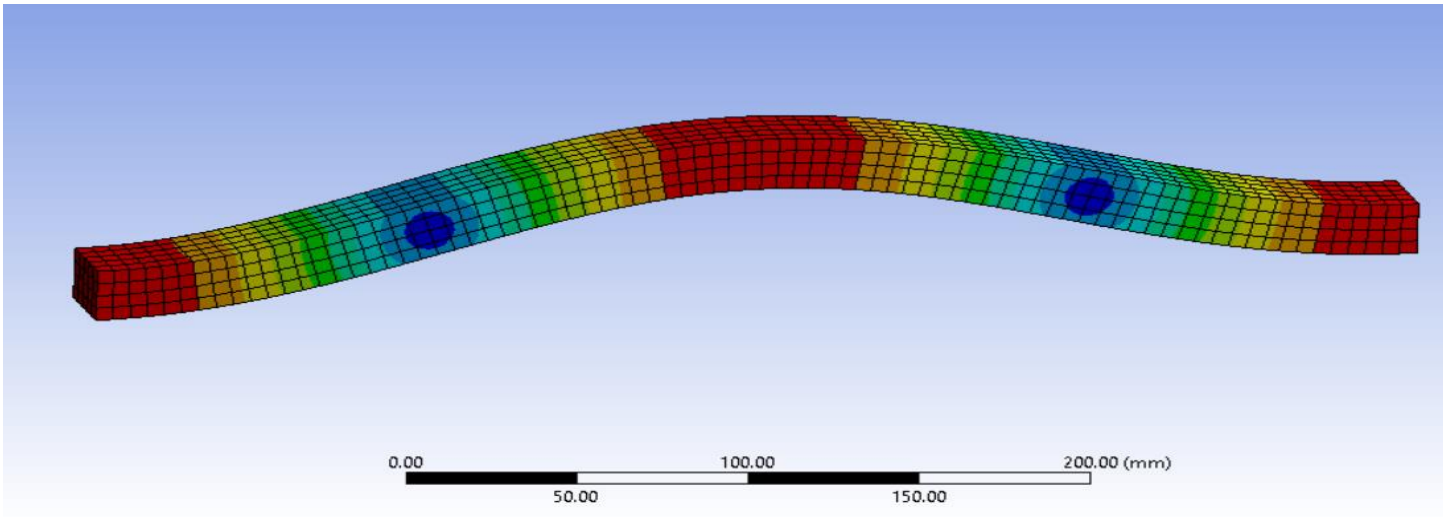

Fig 43: Bending mode shape 1 of simply supported Beam

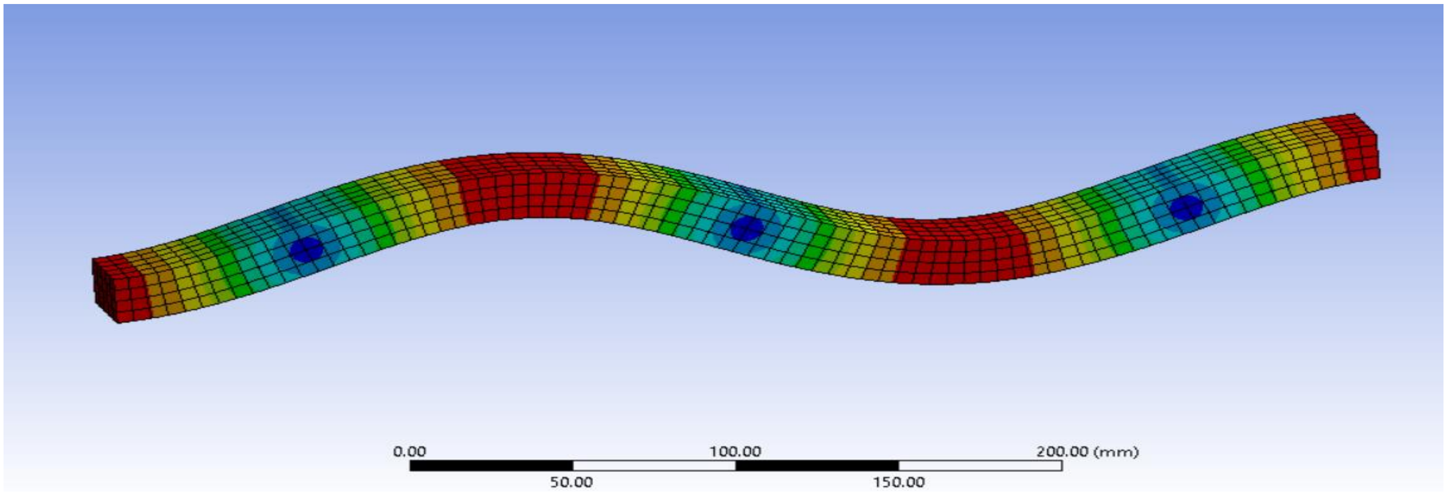

Fig 44: Bending mode shape 2 of simply supported Beam

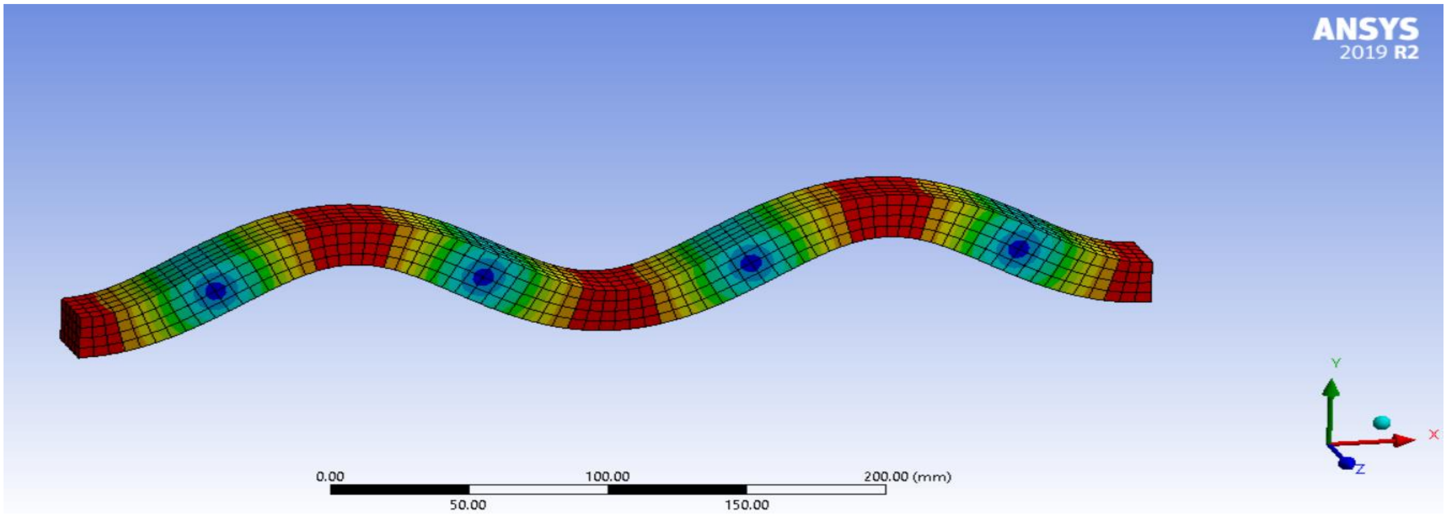

Fig 45: Bending mode shape 3 of simply supported Beam

9. Results and Discussion for Different Boundary Conditions:

Comparison of Natural Frequency between beams without and with Cracks for 1ST Mode of Bending

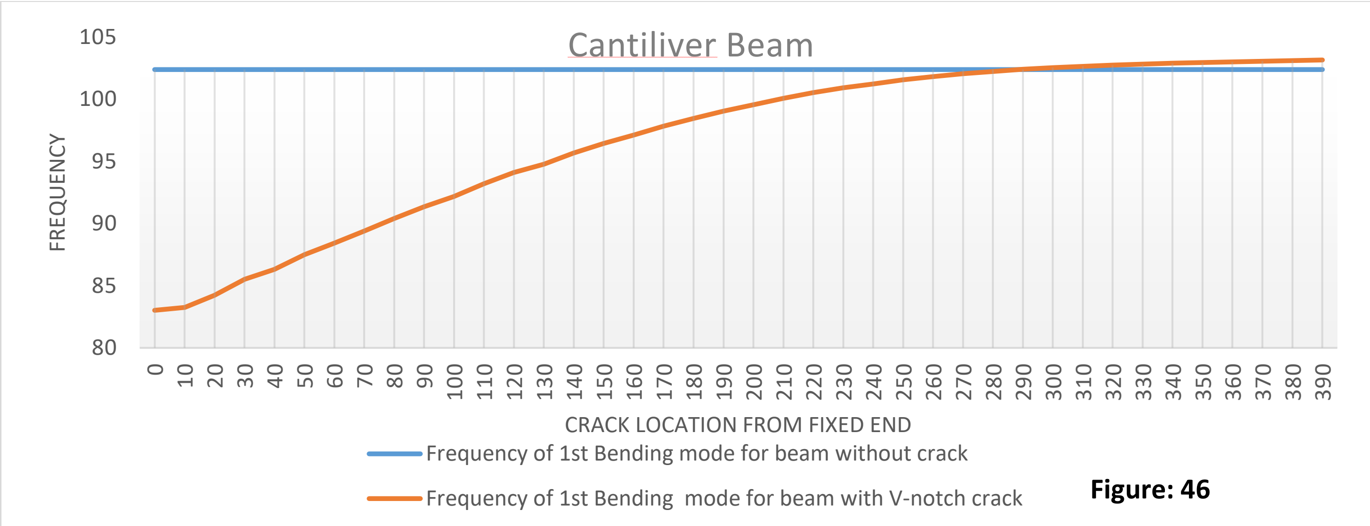

Figure: 46

At 1st we took the crack at 10mm from the fixed end and found natural frequencies in the same way as was found in case of un-cracked beam. Then, the location of crack is varied between 0 mm to 390 mm with an increment of 10 mm and found six mode shapes and natural frequencies. Among the six, 3 were bending modes and we have considered those three in every case. Finally we made a curve comparing the natural frequencies of cracked and un-cracked beam in all the three boundary conditions. In figure: 46, comparison of natural frequency for cracked and un-cracked condition for cantilever beam is shown. Here, frequency is minimum near the fixed end and increases continuously as the crack moves away the fixed end and at 300mm it becomes same as for un-cracked beam and after that increases a bit nearest the free end.

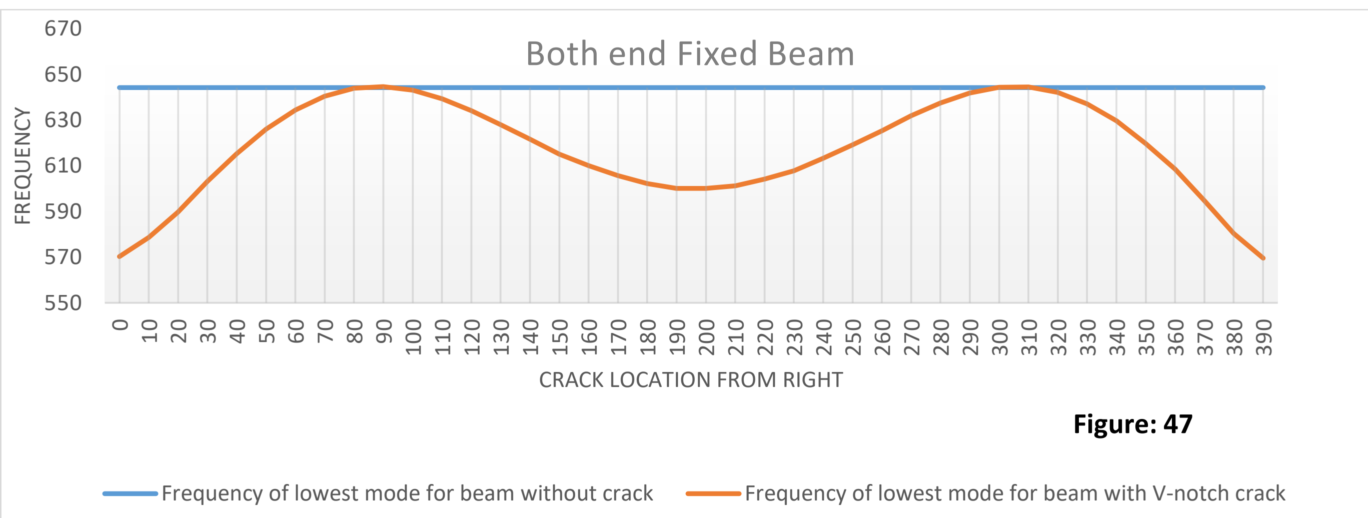

Figure: 47

In figure 47, in case of both end fixed beam, frequency is also minimum near the fixed end and increases continuously as the crack moves away the fixed end and at 100mm it becomes same as for un-cracked beam and then reduces and increases in the same way but never crosses the frequency of un-cracked beam.

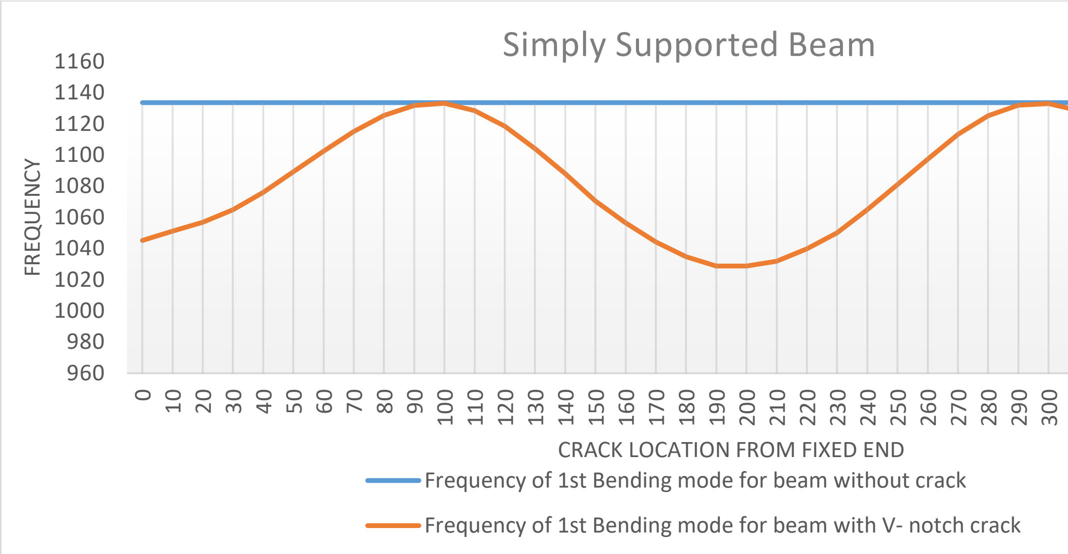

Figure: 48

In figure: 48, in case of simply supported beam, behavior is almost same as like the two end fixed condition but near the two ends curve is a bit different.

Comparison of Frequency between beams without and with Cracks for 2nd Mode of Bending

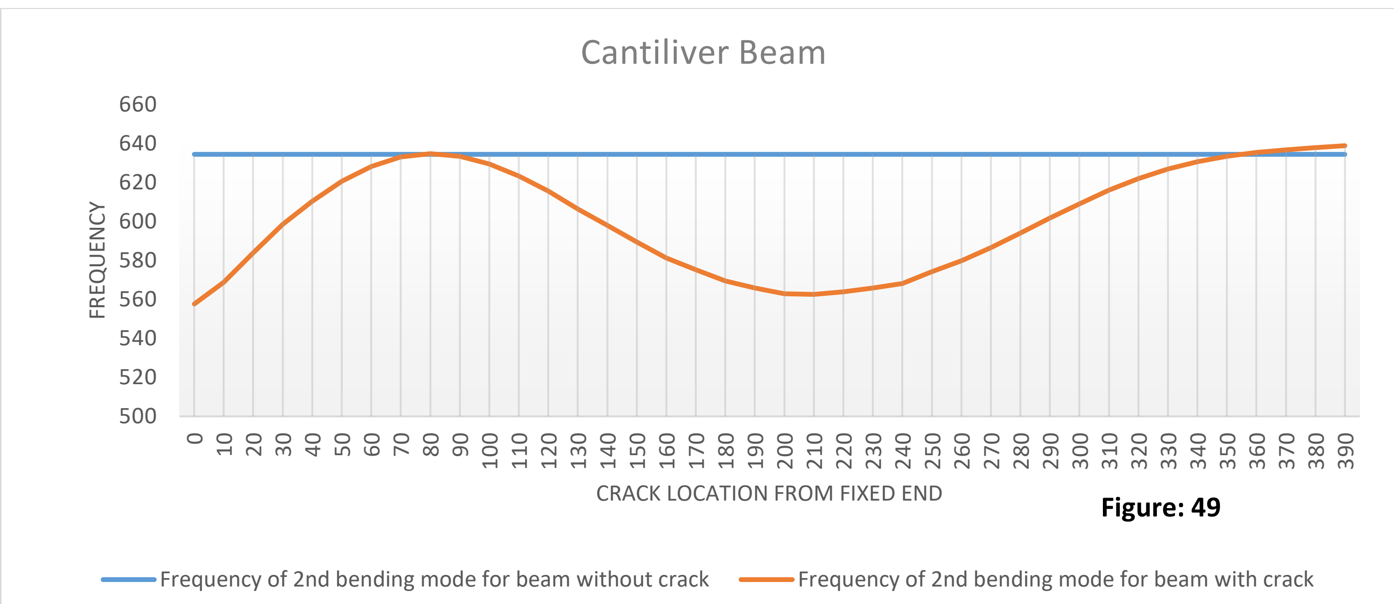

Figure: 49

In the similar way, we have drawn the curve for 2nd mode of bending for all the boundary conditions.

In figure: 49, in case of Cantiliver beam, Frequency increases as it moves away from the fixed end and reaches the frequency of un-cracked beam and starts to decline and becomes minimum at 210mm, afterwards, it continued to increase. frequency of cracked beam crosses the un-cracked beam frequency at 360mm.

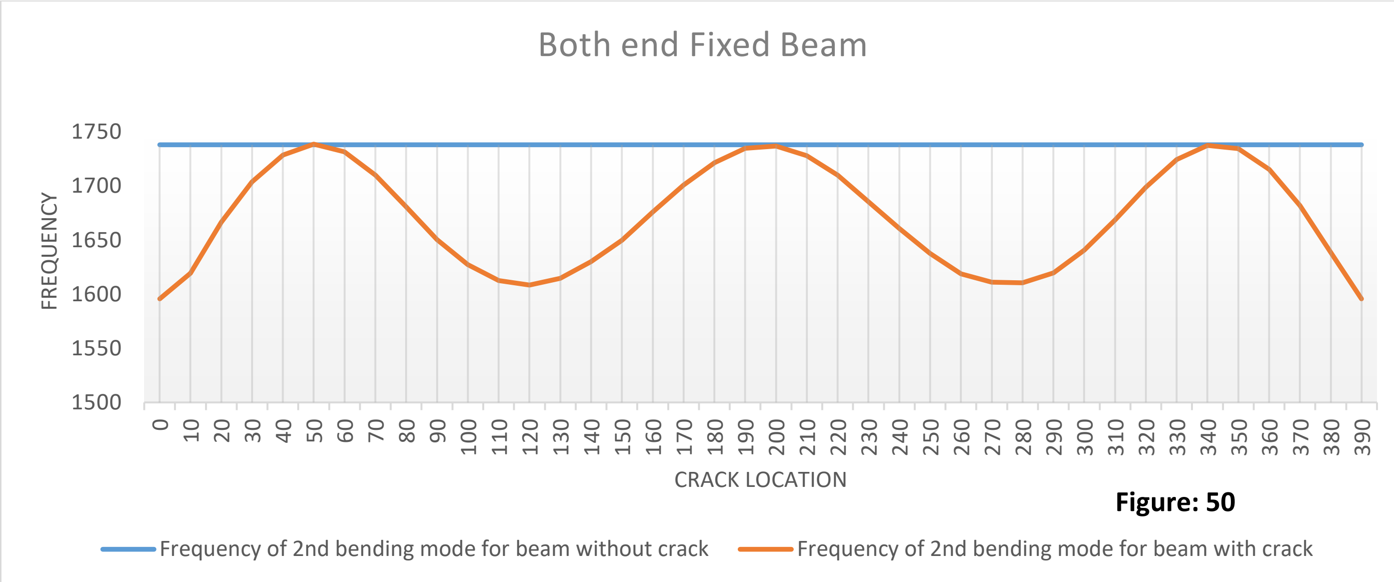

Figure: 50

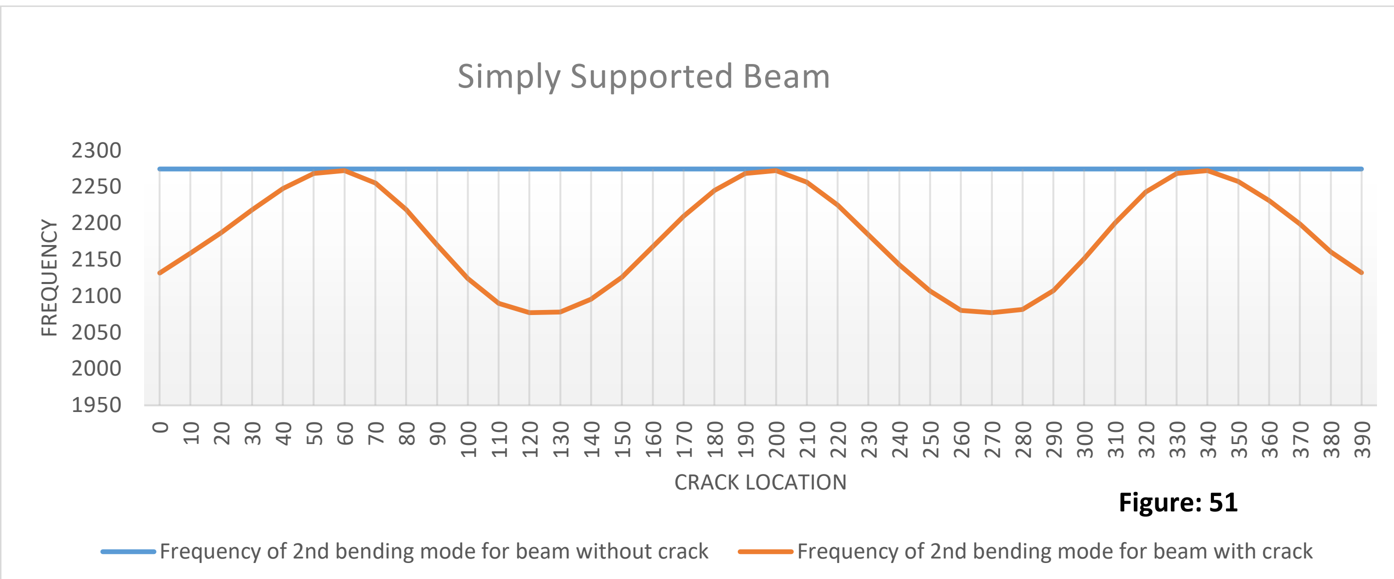

Figure: 51

In figure: 50 & 51, in case of both end fixed beam and simply supported beam, it increases in the similar way and reaches the un-cracked beam natural frequency but never goes beyond it and shows a symmetric behavior. In figure: 14, The frequency of un-cracked beam crosses the frequency of cracked beam at 40mm, 200 mm and 360mm and it remains minimum at 110mm, 280 mm and 390mm. In case of simply supported beam, almost similar behavior can be seen.

Comparison of Frequency between beams without and with Cracks for 3rd Mode of Bending:

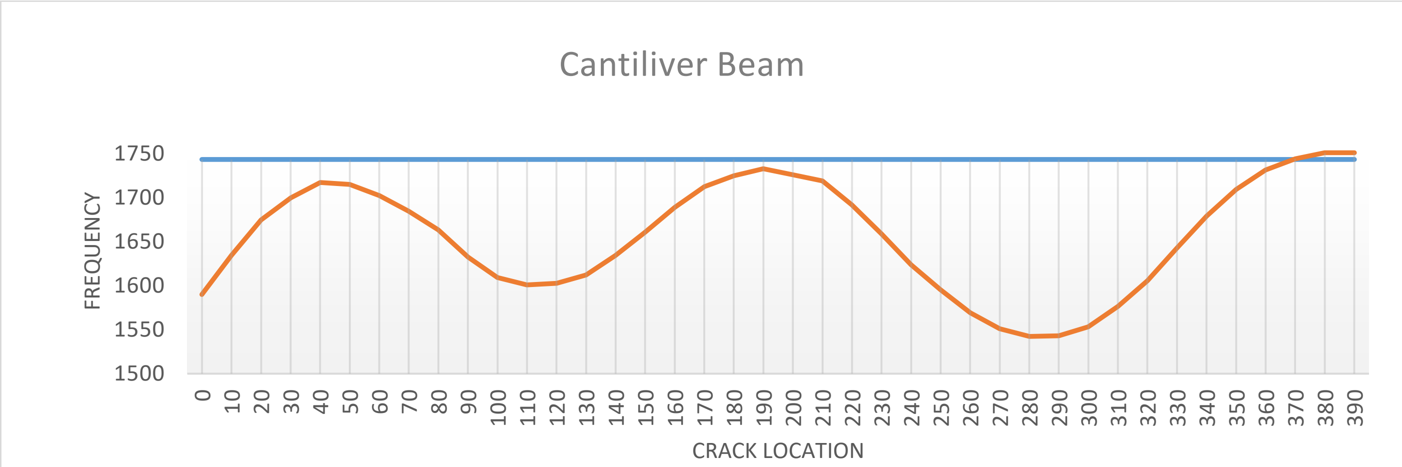

Figure: 52

Here, comparison of Frequency between beams without and with Cracks for 3rd Mode of Bending

In figure: 52, in case of Cantiliver beam, Natural frequency remains minimum near the fixed end. As the crack moves away from the fixed end, frequency increases and decreases and it goes in a similar fashion and finally, natural frequency of cracked beam crosses the un-cracked beam at 370mm

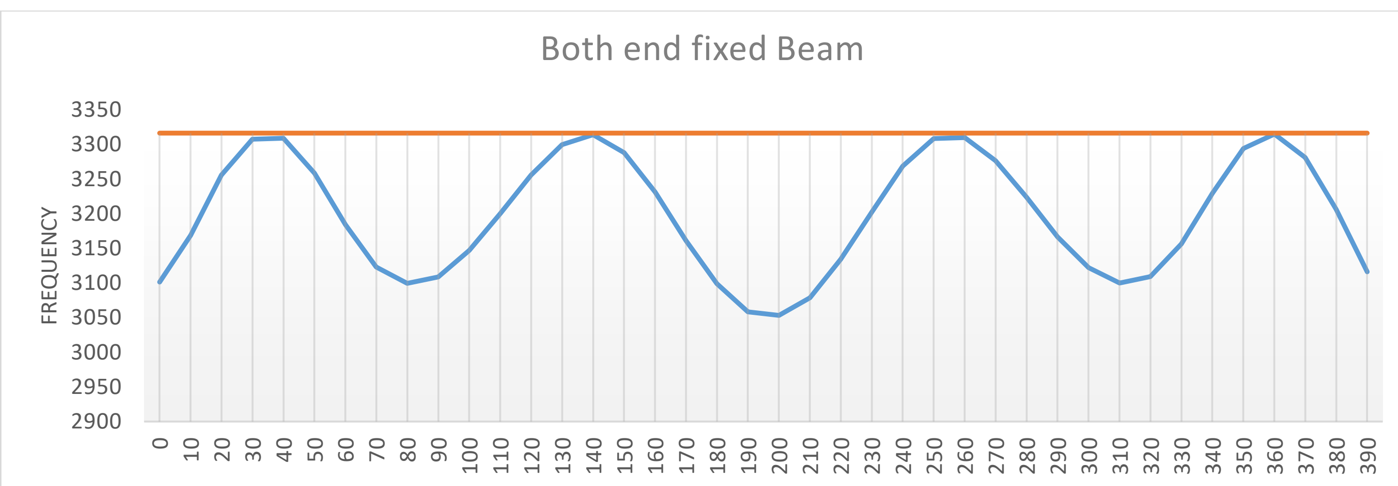

Figure: 53

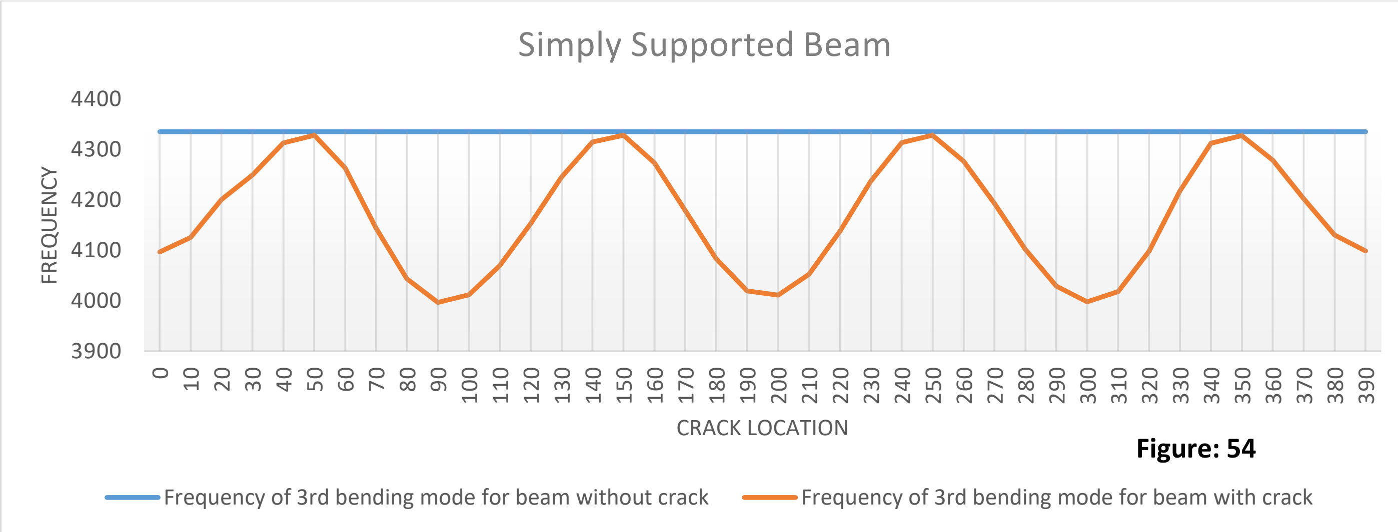

Figure: 54

In figure: 53 & 54, in case of 3rd mode for both end fixed beam and simply supported beam, more Symmetric behavior can be seen but never goes beyond the frequency of un-cracked beam. In Graph: 17, we can see, frequency of cracked beam reaches the frequency of un-cracked beam at 40 mm, 140 mm, 260 mm and 360mm and remains minimum at 80 mm, 210 mm, 300 mm and at 390 mm. In graph: 20, we can see, frequency of cracked beam reaches the frequency of un-cracked beam at 50 mm, 150 mm, 250 mm and 350mm and remains minimum at 90 mm, 200 mm, and 300 mm.

Comparison of 1st, 2nd and 3rd mode for 3 different boundary conditions of beams with cracks

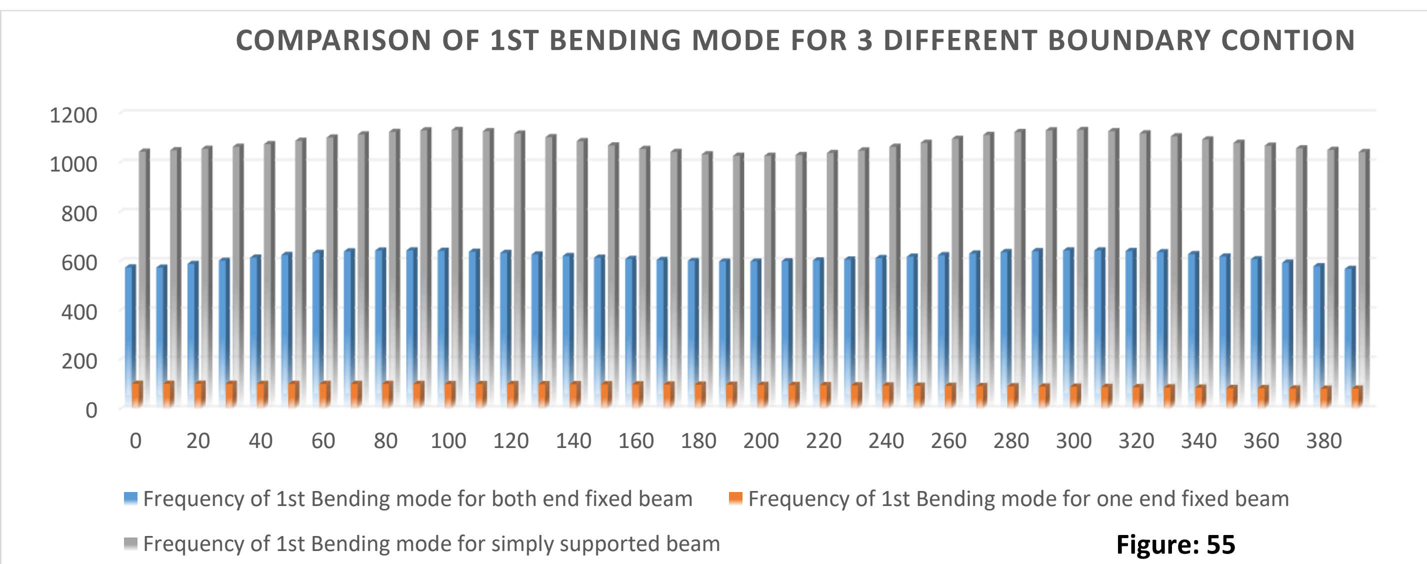

Figure: 55

In the following figure, a comparison is made between the natural frequencies of 1st mode for caltilever beam, both end fixed beam and simply supported beam. We can see, natural frequencies for cantiliver beam are much smaller than the natural frequencies for both end fixed beam and simply supported beam. Natural frequencies of simply supported beam are the highest among the three boundary conditions. Besides, symmetric behaviour can be seen in case of both end fixed beam and simply supported beam.

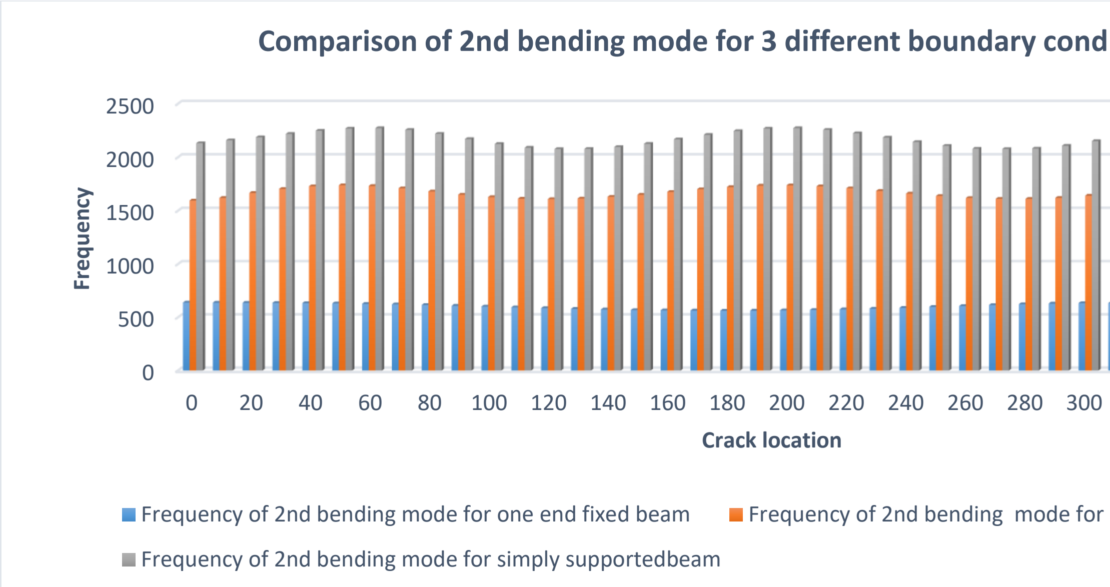

Figure: 56

In the following figure, a comparison is made between the natural frequencies of 2nd mode for caltilever beam, both end fixed beam and simply supported beam. We can see, natural frequencies for cantiliver beam are much smaller than the natural frequencies for both end fixed beam and simply supported beam. Natural frequencies of simply supported beam are the highest among the three boundary conditions. Besides, symmetric behaviour can be seen in case of both end fixed beam and simply supported beam. Here, the difference between the frequency of cantilever beam and simply supported beam decreases than in case of 1st mode.

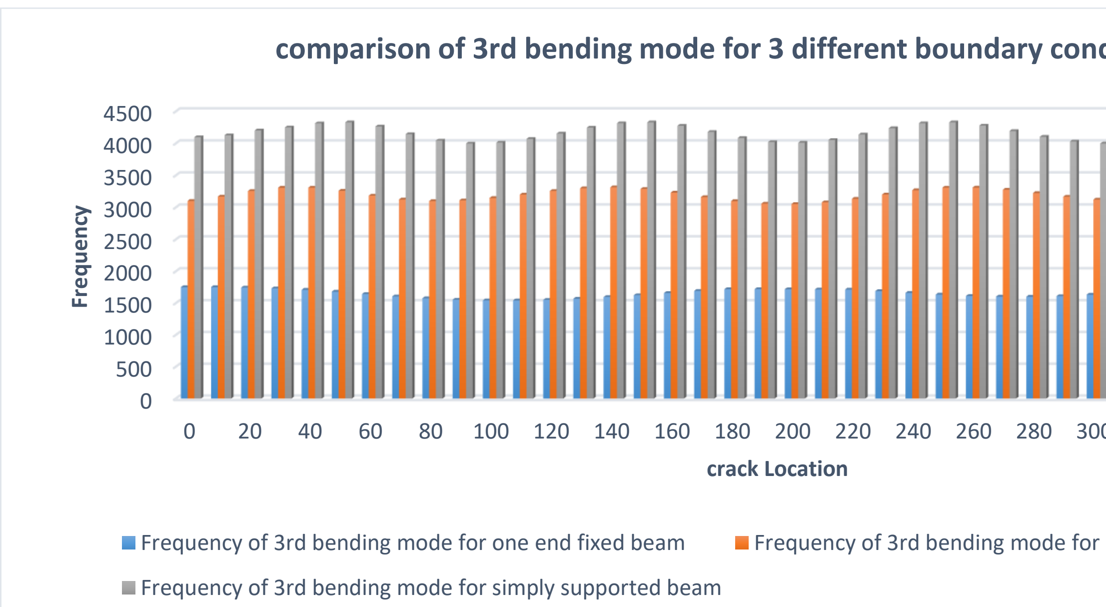

Figure: 57

In the following figure, a comparison is made between the natural frequencies of 3rd mode for caltilever beam, both end fixed beam and simply supported beam. We can see, natural frequencies for cantiliver beam are much smaller than the natural frequencies for both end fixed beam and simply supported beam. Natural frequencies of simply supported beam are the highest among the three boundary conditions. Besides, symmetric behaviour can be seen in case of both end fixed beam and simply supported beam. Here, the difference between the frequency of cantilever beam and simply supported beam decreases much more than in case of 2nd mode.

10. Harmonic Analysis:

Harmonic analysis is used to predict the steady state dynamic response of a structure subjected to sinusoidally varying loads. The structure is excited harmonically at the fixed degrees of freedom. The excitation is defined by a direction vector of displacement, velocity or acceleration. Ansys Mechanical APDL and Mechanical Workbench can perform harmonic analysis on a structure, determining the steady-state sinusoidal response to sinusoidal varying loads all acting at a specified frequency. Some load types can be applied with a phase offset.

Specifications sometimes call for products to be subject to harmonic acceleration loading applied to the base of the product. If absolute accelerations are to be measured at points on the product, a harmonic finite element analysis would be best run with non-zero acceleration inputs at a base. Ansys supports only non-zero displacement loading at nodes in harmonic models. The desired non-zero harmonic acceleration loading can be converted to displacement as a function of frequency, and the desired load applied in Ansys via a Table Array that is a function of frequency (referred to as TIME in the table array).

APDL coding can apply the desired loading on faces indicated via Named Selections in an Ansys Workbench analysis.

Frequency response plots at the base where the loads are applied can be used to confirm that the desired displacement and acceleration loads were input. Frequency response plots at other points in the model can show the absolute (relative to the global origin, not relative to the base) displacement and acceleration results elsewhere in the model.

The usual displacement, stress and strain plots can be generated. we should note that chosen frequencies and phase angles must be manually entered in many of the results plot object details.

In our analysis, we have used harmonic analysis to find the deflection curve in case of cracked and un-cracked beams. Three different boundary condition are considered here. These are- one end fixed beam i.e. cantilever beam, both end fixed beam and both end hinged beam. In all the three boundary conditions, we have found out the deflection curve in case of no crack, crack at 10mm, crack at 200mm and crack at 390mm. In all of the cases, we have used a force of 5000 N for excitation.

10.1. Deflection Curve:

Deflection Curve for beams without crack and with crack for Cantiliver beam:

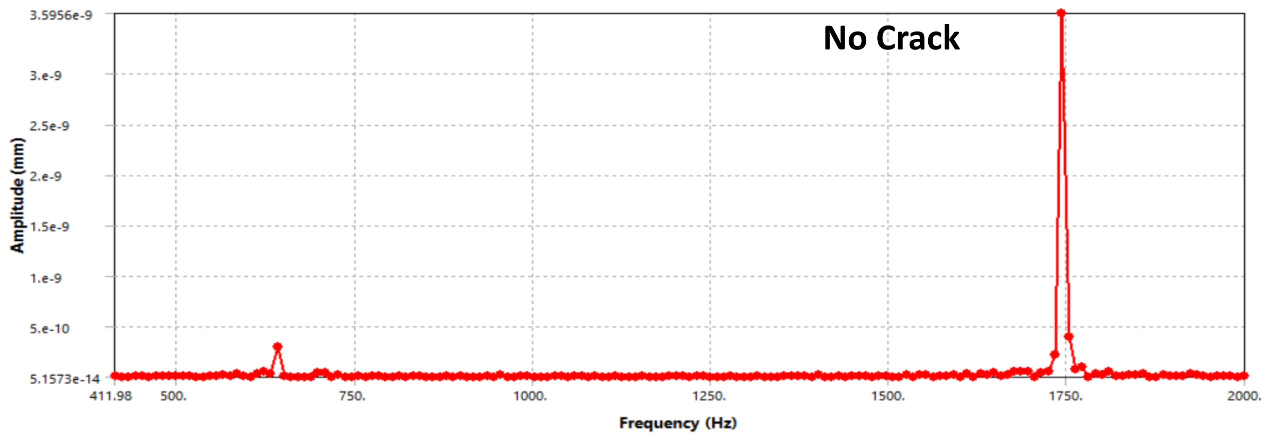

Figure: 58

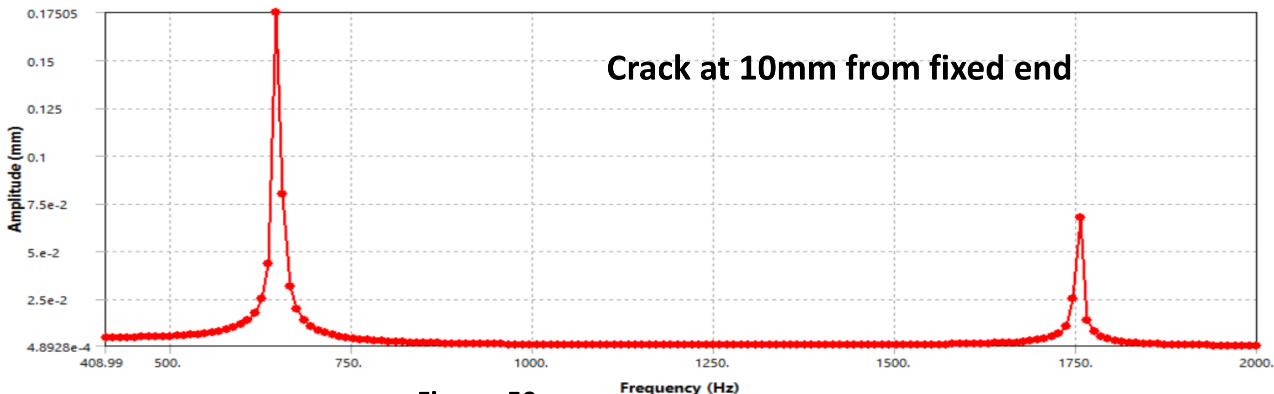

Figure: 59

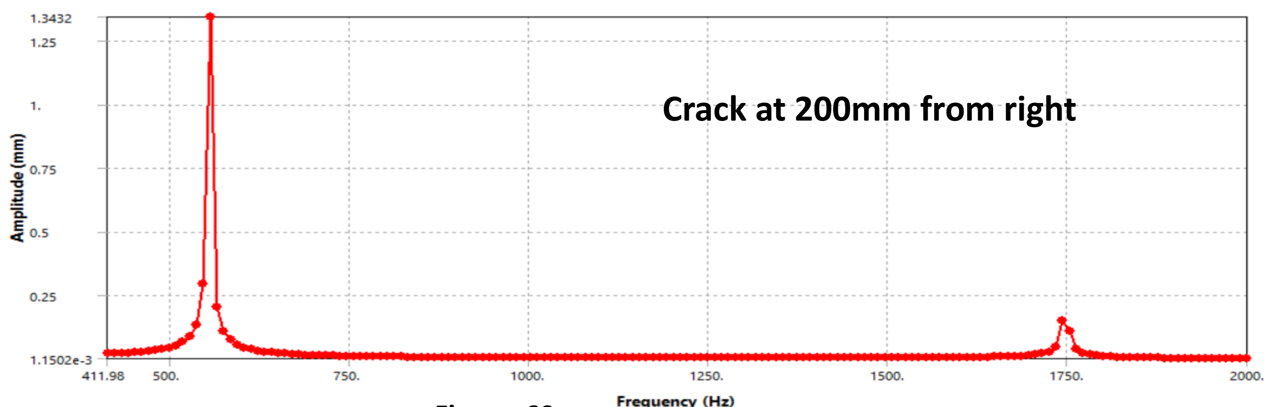

Figure: 60

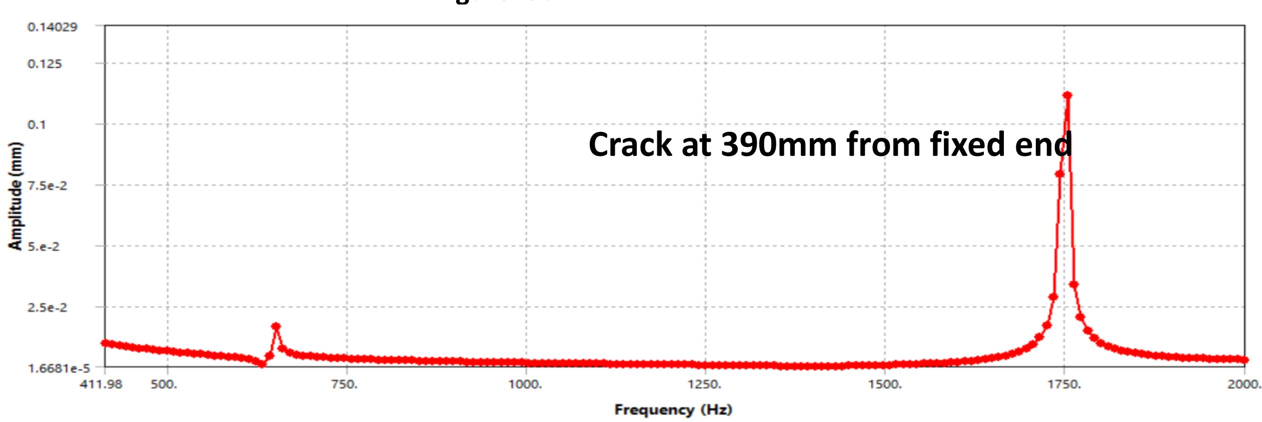

Figure: 61

We have determined deflection curve applying a force 5000N using harmonic analysis in Ansys workbench. The frequency range is taken as 500Hz to 3500Hz. The deflection curve for beam without crack and with crack are quite different.

Here, in case of cantilever beam, we can see the differences in deflection curve due to presence of crack.

When there is no crack (figure: 58) maximum deflection is at 1750 Hz but in case of crack at 20mm (figure: 59), maximum deflection is at 700 Hz. In case of cracks at 200mm and 390mm (figure: 60-61), the deflection curves are also quite different. And we can also see amplitude increases immensely due to presence of crack.

Deflection Curve for beams without crack and with crack for Both end fixed beam:

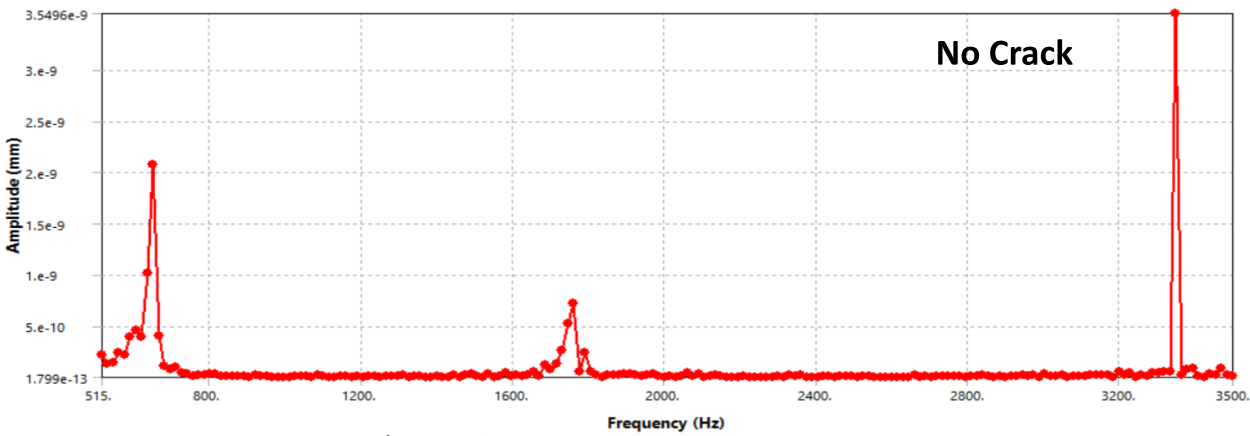

Figure: 62

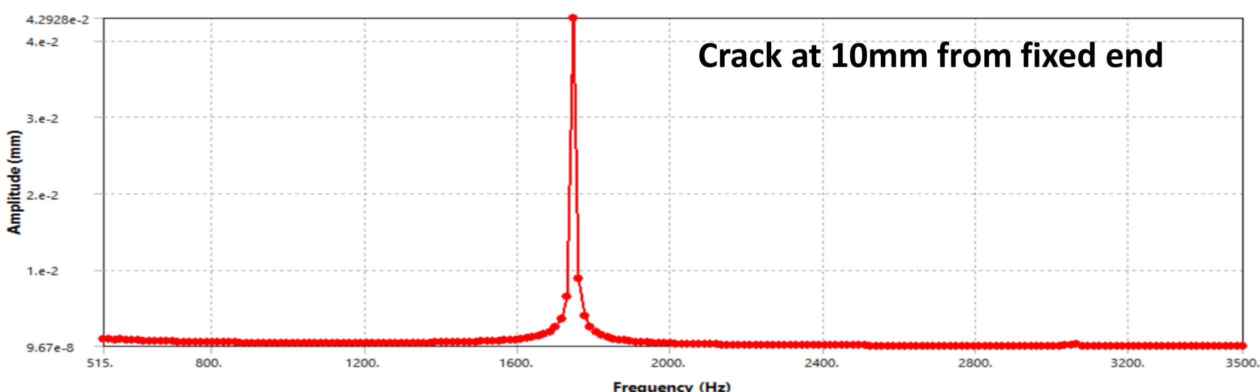

Figure: 63

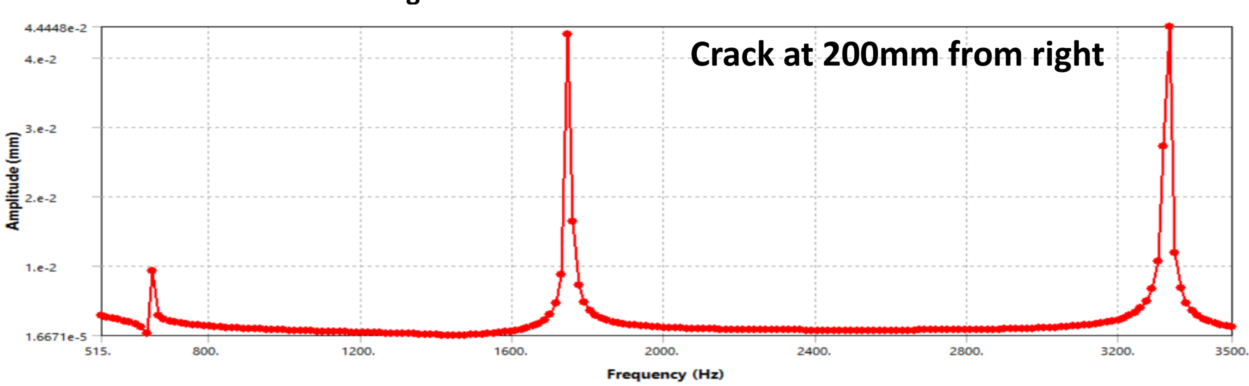

Figure: 64

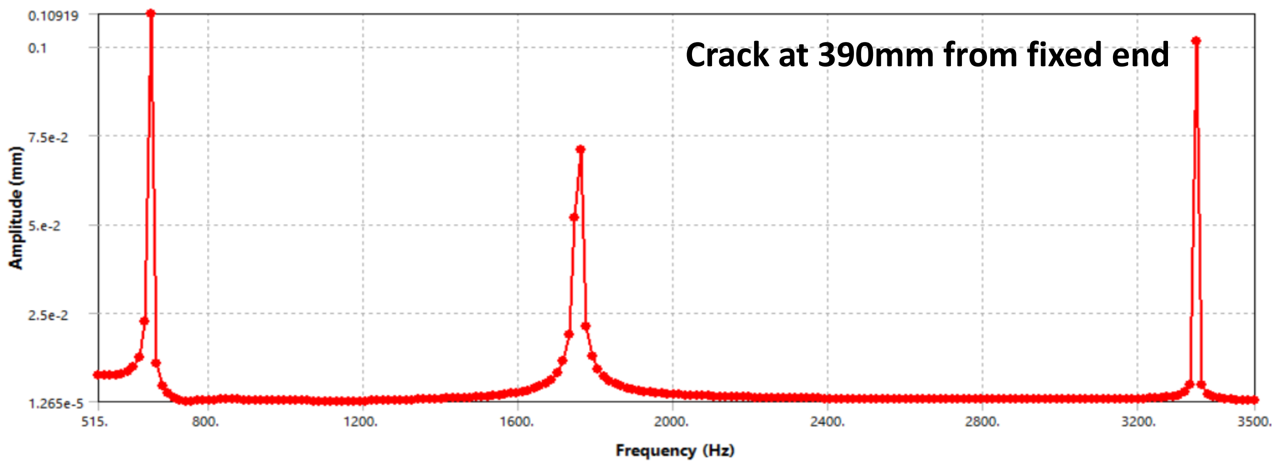

Figure: 65

Then, We have determined deflection curve applying a force 5000N using harmonic analysis in Ansys workbench. The frequency range is taken as 500Hz to 3500Hz. The deflection curve for beam without crack and with crack are quite different.

Here, in case of both end fixed beam, we can see the differences in delection curve due to presence of crack.

When there is no crack (figure: 62) maximum deflection is at 3400 Hz but in case of crack at 20mm (figure: 63), maximum deflection is at 1750 Hz. In case of cracks at 200mm and 390mm (figure:64-65), the deflection curves are also quite different. The maximum deflection in case of figure 64, maximum deflection is at 3300Hz and in case of figure 65, maximum deflection is at 700 Hz. We can also see amplitude increases immensely due to presence of crack.

Deflection Curve for beams without crack and with crack for Both end fixed beam:

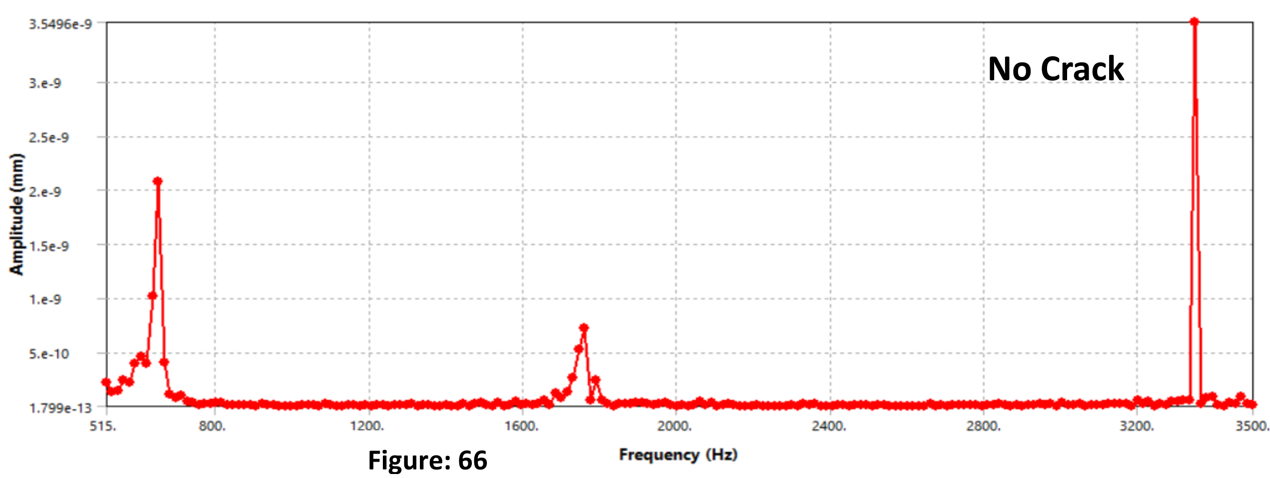

Figure: 66

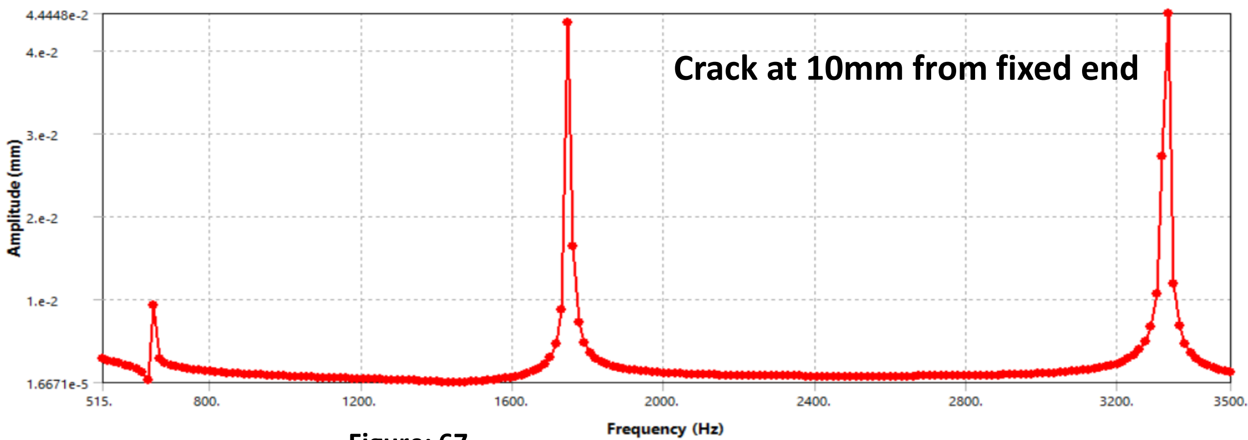

Figure: 67

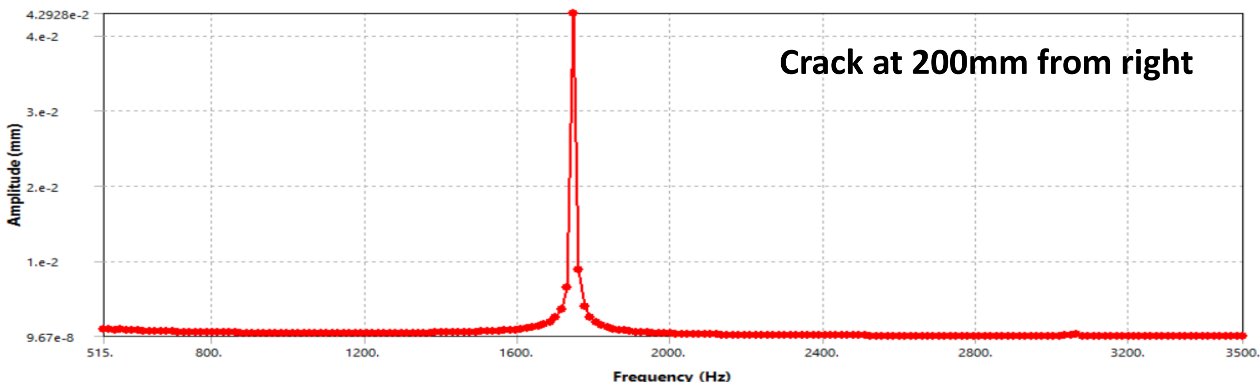

Figure: 68

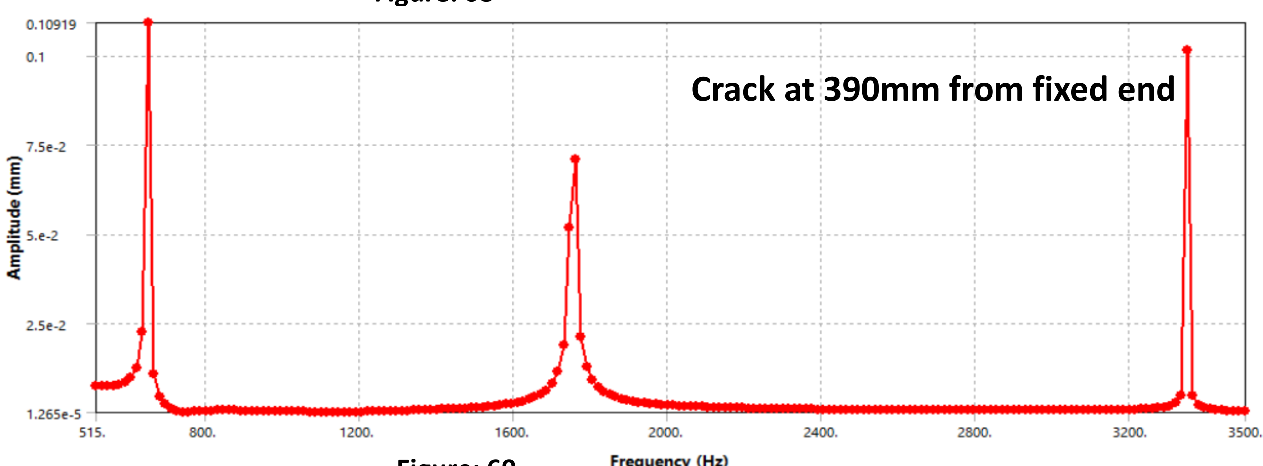

Figure: 69

Then, We have determined deflection curve applying a force 5000N using harmonic analysis in Ansys workbench. The frequency range is taken as 500Hz to 3500Hz. The deflection curve for beam without crack and with crack are quite different.

Here, in case of simply supported beam, we can see the differences in delection curve due to presence of crack.

When there is no crack (figure: 66) maximum deflection is at 3400 Hz but in case of crack at 20mm (figure: 67), maximum deflection is at 3200 Hz. In case of cracks at 200mm and 390mm (figure: 68-69), the deflection curves are also quite different. The maximum deflection in case of figure: 68, maximum deflection is at 1750 Hz and in case of figure: 69, maximum deflection is at 700 Hz. We can also see amplitude increases immensely due to presence of crack.

Discussion:

At First, we have started our work with plates which have slit cracks and found their natural frequency and compared our result with a renowned research journal for validation purpose and our result matched with their results. So, then we studied different types of cracks like angular, triangular, semi-circular. Afterwards, we modelled our plates with those cracks and found their natural frequency and the results were quite satisfactory for identification of cracks. We changed the location of these cracks and size of the cracks as well on the plates and found different results and made comparisons to analyze the results.

Then we started working with beams. First, we modelled the beam with v-shaped crack and compared the result with another published research work for validation purpose and the results matched. So, we modelled the beam with other cracks and found natural frequencies and mode shapes which helped us to make different comparison between non-cracked and cracked beams. And the results were sufficiently good. Subsequently, we started to work with different boundary conditions of beam like one end fixed beam, both end fixed beam, both end hinged beam etc. Besides, we have made different comparison to evaluate the results.

Later, we used harmonic analysis to find out the deflection curves for different boundary conditions and compared the deflection curves of un-cracked beams with cracked beams which also provided us with results that can be used for identification of cracks on beams.

Presence of crack in a single cantilever beam may cause the failure of a vast structure. A crack makes the structure more vulnerable to external effects, accelerates the ageing process and can immediately reduce the mechanical resistance of the structure. Cracks reduce the ability of a structure to absorb stress and may lead to collapse. So, It is of enormous importance to identify cracks in a plate or in a beam. And, we believe our research work can contribute well for the identification of beams.

Besides, experimental based testing has been widely used as a means to analyze cracks on beams and plates. While this is a method that produces real life response, it is extremely time consuming and the use of materials can be quite costly. The use of finite element analysis to study these components has also been used. In recent years, however, the use of finite element analysis has increased due to progressing knowledge and capabilities of computer software and hardware. Researchers are presently focusing on various methods for analysis and diagnosis of crack detection in a beam structure like artificial neural network, genetic algorithm and continuous wavelet analysis etc. So, we aim to research with those methods in the near future.

11. Validation:

For validation of the plates with cracks we have used a research paper:

> **N. Tao, Y. Ma, H. Jiang, M. Dai, and F. Yang, "Investigation on Non-Linear Vibration Response of Cantilevered Thin Plates with Crack Using Electronic Speckle Pattern Interferometry," *Proceedings*, vol. 2, no. 8, p. 539, 2018.**

<table>
<tr><td>Data from paper</td><td>Our data</td><td rowspan="8">Length of plate - 210 mm
Breadth of plate - 80 mm
Thickness of plate - 1.43 mm

Length of crack - 20 mm
Breadth of crack – 1 mm
Thickness of crack – 1.43 mm</td></tr>
<tr><td>Natural frequency when the crack is at fixed end</td><td>Natural frequency when the crack is at fixed end</td></tr>
<tr><td>42.698</td><td>42.33</td></tr>
<tr><td>182.52</td><td>182.01</td></tr>
<tr><td>266.34</td><td>265.94</td></tr>
<tr><td>589.87</td><td>589.45</td></tr>
<tr><td>748.34</td><td>747.211</td></tr>
<tr><td>1113.4</td><td>1113.03</td></tr>
</table>

For validation of the beams with cracks we have used another research paper:

> **P. Yamuna and K. Sambasivarao, "Vibration analysis of beam with varying crack location," Int. J. Eng. Res. Gen. Sci., vol. 2, no. 6, pp. 1008–17, 2014.**

<table>
<tr><td>Data from paper</td><td>Our data</td><td rowspan="7">Length of beam - 500 mm
Breadth of beam - 25 mm
Thickness of beam - 15 mm

Length of crack - 10 mm
Breadth of crack – 5 mm
Thickness of crack – 15 mm</td></tr>
<tr><td>Natural frequency when the crack is at 50mm from the fixed end</td><td>Natural frequency when the crack is at 50 mm from the fixed end</td></tr>
<tr><td>230.74</td><td>230.72</td></tr>
<tr><td>316.79</td><td>316.79</td></tr>
<tr><td>867.35</td><td>867.99</td></tr>
<tr><td>893.09</td><td>893.593</td></tr>
<tr><td>1682.3</td><td>1683.23</td></tr>
</table>

12. Conclusion:

The following conclusions can be drawn from the present study-

- Due to existence of crack, natural frequency changes. The amount of change varies depending on crack location, depth and crack opening size.
- For a certain crack location, the natural frequencies of a cracked beam are inversely proportional to the crack depth
- For a certain crack depth, change in natural frequency is less as the crack position moves away from fixed end
- Frequency is minimum near the fixed ends
- It is possible to identify the cracks since natural frequency changes
- Effect of crack opening size on frequency becomes significant as crack opening size decreases.
- Modelling of cracked structure requires fine meshing otherwise accuracy decrease.
- Accuracy increases due to reduction of element size with increase of analysis time
- Deflection curve changes due to presence of crack. So, this curve also can be used for crack identification
- All mode of vibrations do not have same type of effect due to presence of cracks

13. References:

[1] Crack identification in Beams by Vibration based analysis techniques – A Review Tajammul Riaz Sial, Yan Jin, Zhang Juan.

[2]Vibration Analysis of Beam With Varying Crack Location P.Yamuna , K.Sambasivarao.

[3] Silva, J. M. M., & Gomes, A. J. M. A. (1990). Experimental dynamic analysis of cracked free-free beams. Experimental Mechanics, 30, 20-25.

[4] Doebling, S. W., Farrar, C. R., Prime, M. B., & Shevitz, D. W. (1998). A summary review of vibration based damage identification methods. The Shock and Vibration Digest, 30, 91-105.

[5] Christides, S., & Barr, A. D. S. (1984). One-dimensional theory of cracked Bernoulli_Euler beams. International Journal of Mechanical Sciences, 26, 639-648.

[6] Dimarogonas, A. D. (1976). Vibration Engineering, West Publishers, St Paul, Minesota.

[7] Chondros, T. G., & Dimarogonas, A. D. (1980). Identification of cracks in welded joints of complex structures. Journal of Sound and Vibration, 69, 531-538.

[8] Krawczuk, M., Żak, A., & Ostachowicz, W. (2000). Elastic beam finite element with a transverse elasto-plastic crack. Finite Elements in Analysis and Design, 34, 61-73.

[9] Bouboulas, A. S., & Anifantis, N. K. (2008). Formulation of cracked beam element for analysis of fractured skeletal structures. Engineering Structures, 30, 894-901.

[10] Gudmundson, P. (1983). The dynamic behavior of slender structures with cross-sectional cracks. Journal of the Mechanics and Physics of Solids, 31, 329-345.

[11] Cacciola, P., & Muscolino, G. (2002). Dynamic response of a rectangular beam with a known non-propagating crack of certain or uncertain depth. Computers and Structures, 80, 2387-2396.

[12] Benfratello, S., Cacciola, P., Impollonia, N., Masnata, A., & Muscolino, G. (2007). Numerical and experimental verification of a technique for locating a fatigue crack on beams vibrating under Gaussian excitation. Engineering Fracture Mechanics, 74, 2992-3001.

[13] Sholeh, K., Vafai, A., & Kaveh, A. (2007). Online detection of the breathing crack using an adaptive tracking technique. Acta Mechanica, 188, 139-154.

[14] Clark, R., Dover, W. D., & Bond, L. J. (1987). The effect of crack closure on the reliability of NDT predictions of crack size. NDT International, 20, 269-275.

[15] Abraham, O. N. L., & Brandon, J. A. (1995). The modelling of the opening and closure of a crack. Journal of Vibration and Acoustics, 117, 370-377.

[16] Douka, E., & Hadjileontiadis, L. J. (2005). Time-frequency analysis of the free vibration response of a beam with a breathing crack. NDT&E International, 38, 3-10.

[17] Loutridis, S., Douka, E., & Hadjileontiadis, L. J. (2005). Forced vibration behaviour and crack detection of cracked beams using instantaneous frequency. NDT&E International, 38, 411-419.

[18] Andreaus, U.; Casini, P. Identification of Multiple Open and Fatigue Cracks in Beam-like Structures Using Wavelets

on Deflection Signals. Contin. Mech. Thermodyn. 2016, 28, 361–378.

[19] Al-Shudeifat, M.A.; Butcher, E.A. On the Dynamics of a Beam with Switching Crack and Damaged Boundaries. J. Vib. Control 2013, 19, 30–46.

[20] Cole, D.P.; Habtour, E.M.; Sano, T.; Fudger, S.J.; Grendahl, S.M.; Dasgupta, A. Local Mechanical Behavior of Steel Exposed to Nonlinear Harmonic Oscillation. Exp. Mech. 2017, 57, 1027–1035.

[21] Neves, A.C.; Simões, F.M.F.; Pinto da Costa, A. Vibrations of Cracked Beams: Discrete Mass and Stiffness Models. Comput. Struct. 2016, 168, 68–77.

[22] H. S. Rane, R.B. Barjibhe and A.V. Patil , Free Vibration Analysis of Cracked Structure, Bhusawal, India. IJERT, 3 (2) (2014).

[23] M. Quila , S. C. Mondal and S. Sarkar, Free Vibration Analysis of an Un-cracked & Cracked Fixed Beam , Jadavpur University, India, Journal of Mechanical and Civil Engineering, 11 (2014) 76-83.

[24] M. J. Prathamesh and M. A. Chakrabarti, Free Vibration Analysis of Cracked Beam, VJTI, Mumbai, Journal of Engineering Research and Applications, 3 (6) (2013) 1172-1176.

[25] M. Behzad, A. Meghdari, and A. Ebrahimi, A New Approach For Vibration Analysis Of A Cracked Beam, Mechanical Engineering Department, Sharif University of Technology, Tehran, IRAN, 2005.

[26] Shen MHH and Chu YC. Vibrations of beams with a fatigue crack. Comp Struct 1992; 45(1): 79–93.

[27] Cacciola P and Muscolino G. Dynamic response of a rectan_gular beam with a known non-propagating crack of certain or uncertain depth. Comp Struct 2002; 80(27–30): 2387–2396.

[28] Sholeh K, Vafai A and Kaveh A. Online detection of the breathing crack using an adaptive tracking technique. Acta Mech 2007; 188(3–4): 139–154

[29] Clark R, Dover WD and Bond LJ. The effect of crack closure on the reliability of NDT predictions of crack size. NDT Int 1987; 20(5): 269–275.

[30] Abraham ONL and Brandon JA. The modelling of the opening and closure of a crack. J Vib Acoust 1995; 117(3A): 370–377.

[31] Cheng SM, Wu XJ, Wallace W and Swamidas ASJ. Vibrational response of a beam with a breathing crack. J Sound Vib 1999; 225(1): 201–208.

[32] Douka E and Hadjileontiadis LJ. Time–frequency analy_sis of the free vibration response of a beam with a breath_ing crack. NDT&E Int 2005; 38(1): 3–10

[33] Kisa M and Brandon J. The effects of closure of cracks on the dynamics of a cracked cantilever beam. J Sound Vib 2000; 238(1): 1–18

[34] Kapania RK, Raciti S. Recent advances in analysis of laminated beams and plates: Part I. Shear effects and buckling; Part II. Vibrations and wave propagation. AIAA Journal, 27 (1989): 923–46.

[35] Yuan, F.G. and R.E. Miller. A higher order finite element for laminated composite beams. Computers & Structures, 14 (1990): 125-150.

[36] Dipak Kr. Maiti& P. K. Sinha. Bending and free vibration analysis of shear deformable laminated composite beams

by finite element method. Composite Structures, 29 (1994): 421- 431

[37] Teboub Y, Hajela P. Free vibration of generally layered composite beams using symbolic computations. Composite Structures, 33 (1995): 123–34.

[38] Banerjee, J.R. Free vibration of axially loaded composite Timoshenko beams using the dynamic stiffness matrix method. Computers & Structures, 69 (1998): 197-208

[39] Bassiouni AS, Gad-Elrab RM, Elmahdy TH. Dynamic analysis for laminated composite beams. Composite Structures, 44 (1999): 81–7.

[40] Kisa “Free vibration analysis of a cantilever composite beam with multiple cracks”. Composites Science and Technology 64, 1391–1402. 2003.

[41] P. G. KIRMSHER 1944 Proceedings of the American Society of Testing and Materials 44, 897-904. The effect of discontinuities on the natural frequency of beams.

[42] T. G. Chondros, A. D. Dimarogonas, and J. Yao, “Vibration of a beam with a breathing crack,” Journal of Sound and Vibration, vol. 239, no. 1, pp. 57–67, 2001.

[43] O. N. L. Abraham and J. A. Brandon, “The modelling of the opening and closure of a crack,” Journal of Vibration & Acoustics, vol. 117, no. 3, pp. 370–377, 1995.

[44] S. M. Cheng, A. S. J. Swamidas, X. J. Wu, and W. Wallace, “Vibrational response of a beam with a breathing crack,” Journal of Sound & Vibration, vol. 225, no. 1, pp. 201–208, 1999.

[45] N. Wu, “Study of forced vibration response of a beam with a breathing crack using iteration method,” Journal of Mechanical Science and Technology, vol. 29, no. 7, pp. 2827–2835, 2015.

[46] J.M. Silva, A.J.L. Gomes, Experimental dynamic analysis of cracked free–free beams, Experimental Mechanics 30 (1) (1990) 20–25.

[47] X.F. Yang, Vibration Based Crack Analysis and Detection in Beams Using Energy Method, PhD Thesis, Faculty of Engineering and Applied Science Memorial University of Newfoundland, 2001.

[48] P.F. Rizos, N. Aspragathos, A.D. Dimarogonas, Identification of crack location and magnitude in a cantilever beam from the vibration modes, Journal of Sound and Vibration 138 (3) (1990) 381–388.

[49] B.P. Nandwana, S.K. Maiti, Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies, Journal of Sound and Vibration 203 (3) (1997) 435–446.

[50] T.D. Chaudhari, S.K. Maiti, A study of vibration of geometrically segmented beams with and without crack, International Journal of Solids and Structures 37 (2000) 761–779.

[51] D.K.L. Tsang, S.O. Oyadiji, A.Y.T. Leung, Dynamic analysis of a penny-shaped crack by the fractal-like finite element method, Proceedings of the Fifth International Conference on Vibration, Nanjing, China, September 2002, pp. 59–65.

[52] S. Chinchalkar, Determination of crack location in beams using natural frequencies, Journal of Sound and Vibration 247 (3) (2001) 417–429.

[53] W.M. Ostachowicz, M. Krawczuk, Vibration analysis of a cracked beam, Computers and Structures 77 (2) (1990) 327–342.

[54] S.P. Lele, S.K. Maiti, Modeling of transverse vibration of short beams for crack detection and measurement of crack extension, Journal of Sound and Vibration 257 (3) (2002) 559–583.

[55] C.G. Go, Y.S. Lin, Infinitely small element for dynamic problems of cracked beam, Engineering Fracture Mechanics 48 (4) (1994) 475–482.

[56] W. Dahmen, Wavelet methods for PDES, some recent develops, Journal of Computational Applied Mathematics

128 (2001) 133–185.

[57] Albert Cohen, Numerical Analysis of Wavelet Methods, North-Holland Press, Amsterdam, 2003.

[58] S. Jaffard, P. Laurengot, Orthonormal wavelets, analysis of operators, and applications to numerical analysis, in: C.K. Chui (Ed.), Wavelets—A Tutorial in Theory and Applications, Academic Press, New York, 1992, pp. 543–601.

[59] J. Ko, A.J. Kurdila, M.S. Pilant, A class of finite element methods based on orthonormal, compactly supported wavelets, Computational Mechanics 16 (1995) 235–244.

[60] G. Beylkin, R. Coifman, V. Rokhlin, Fast wavelet transforms and numerical algorithms I, Communications on Pure and Applied Mathematics 44 (1991) 141–183.

[61] C. Canuto, A. Tabacco, K. Urban, The wavelet element method: part I. Construction and analysis, Applied and Computational Harmonic Analysis 6 (1999) 1–52.

[62] C. Canuto, A. Tabacco, K. Urban, The wavelet element method: part II. Realization and additional features in 2D and 3D, Applied and Computational Harmonic Analysis 8 (2000) 123–165.

[63] J.X. Ma, J.J. Xue, S.J. Yang, Z.J. He, A study of the construction and application of a Daubechies wavelet-based beam element, Finite Elements in Analysis and Design 39 (10) (2003) 965–975.

[64] X.F. Chen, S.J. Yang, J.X. Ma, Z.J. He, The construction of wavelet finite element and its application, Finite Elements in Analysis and Design 40 (5–6) (2004) 541–554.

[65] M.I. Friswell, J.E.T. Penny, The practical limits of damage detection and location using vibration data, Proceedings of the 11th VPI and SU Symposium on Structural Dynamics and Control, Blacksburg, 1997, pp. 31–40.

[66] S. R. Shuvo, “Detection of Cracks in a Cantilever Metal Plate Using Vibrational Techniques,” September, 2020.