Performance Comparison of Different Types of Beam Forming Algorithms for Smart Antenna

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Declaration of Candidate

This is to certify that the work presented in this thesis paper is the outcome of research carried out by the candidate under the supervision of Dr. Ruhul Amin, Professor Electrical and Electronic Engineering (EEE). It is also declared that neither this thesis paper nor any part thereof has been submitted anywhere else for the reward of any degree or any judgment.

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Abstract

In recent years there has been a rapid growth in the number of wireless users, particularly in the area of mobile communications. This rapid growth in mobile communications demands for more system capacity through efficient utilization of frequency spectrum and also keeping the Interference as low as possible. In today’s Radio resource management, Adaptive array antennas have an important role in increasing the system capacity and controlling the interference in mobile communications.

Adaptive array antenna is an antenna system, which uses spatially separated antennas called array antennas and processes the received signals with a digital signal processor. Simply speaking these array antennas can reduce the co-channel interference and effectively utilize the bandwidth by steering a high gain in the direction of interest and low gains in the undesired directions, technically this is called adaptive beamforming. This adaptive beamforming enables the base station to form narrow beam towards desired user and nulls towards the interfering user, hence improving the signal quality.

Chapter 1

Introduction

Telecommunication is the transmission of signs, signals, messages, writings, images and sounds or intelligence of any nature by wire , radio, optical or other electromagnetic systems. Telecommunication occurs when the exchange of information between communication participants includes the use of technology. It is transmitted either electrically over physical media, such as cables, or via electromagnetic radiation. Such transmission paths are often divided into communication channels which afford the advantages of multiplexing. The term is often used in its plural form, telecommunications, because it involves many different technologies.

Early means of communicating over a distance included visual signals, such as beacons, smoke signals, semaphore telegraphs, signal flags, and optical heliographs. Other examples of pre-modern long-distance communication included audio messages such as coded drumbeats, lung-blown horns, and loud whistles. 20th and 21st century technologies for long-distance communication usually involve electrical and electromagnetic technologies, such as telegraph, telephone, mobile networks, radio, microwave transmission, fiber optics, and communications satellites.

Now-a-days communication via mobile network is popular for short and long distance. Mobile technology communication technology largely depends on Antenna through which UE communicates with other via different communication equipment. A revolution in wireless communication began in the first decade of the 20th century with the pioneering developments in radio communications.

Generally, Mobile communication systems with the help of wireless technology achieve great performance not only in calling area but also the data transfer as well. With the evolution of wireless system Antenna technology has also been upgraded. In modern world wireless system has introduced us 3G, LTE (long term evolution) etc. and the antenna has also been introduced us as Smart antenna MIMO antenna from OMNI directional patch antenna microstrip antenna Yagi antenna etc.

Smart antennas (also known as adaptive array antennas, multiple antennas and, recently, MIMO) are antenna arrays with smart signal processing algorithms used

to identify spatial signal signature such as the direction of arrival (DOA) of the signal, and use it to calculate beamforming vectors, to track and locate the antenna beam on the mobile/target. Smart antennas should not be confused with reconfigurable antennas, which have similar capabilities but are single element antennas and not antenna arrays.

Smart antenna techniques are used notably in acoustic signal processing, track and scan radar, radio astronomy and radio telescopes, and mostly in cellular systems like W-CDMA, UMTS, and LTE.

1.1 Antenna Basic

An antenna is an electrical device which converts electric power into radio waves, and vice versa. Smart antennas are composed of a collection of two or more antennas working in concert to establish a unique radiation pattern for the electromagnetic environment at hand. The antenna elements are allowed to work in concert by means of array element phasing, which is accomplished with hardware or is performed digitally.

Arrays of antennas can assume any geometric form. The various array geometries of common interest are linear arrays, circular arrays, planar arrays, and conformal arrays.

1.1.1 Infinitesimal Dipole

An infinitesimal linear wire ($l \ll \lambda$) is positioned symmetrically at the origin of the coordinate system and oriented along the z axis. The wire, in addition to being very small ($l \ll \lambda$),is very thin ($a \ll \lambda$). The spatial variation of the current is assumed to be constant and given by

$I(z') = \hat{a}_z I_o$

Where $I_0 =$ constant

1.1.2 Radiated Fields

To find the fields radiated by the current element, the two-step procedure is used. It will be required to determine first A and F and then find the E and H.

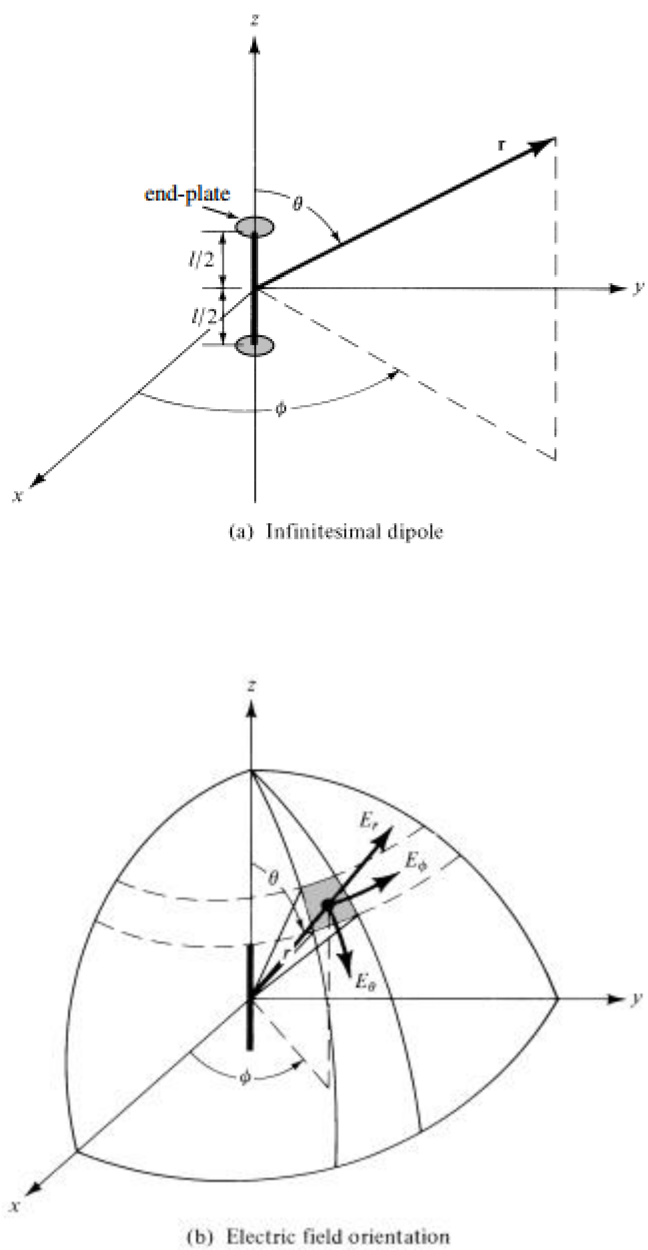


Figure 1.2.2: Geometrical arrangement of an infinitesimal dipole and its associated electric-field Components on a spherical surface.

Where

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2}$$
$$= r = \text{constant}$$
$$dl' = dz'$$

E and H can be found from the following equations

$$H_r = H_\theta = 0$$
$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

And

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$
$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$
$$E_\phi = 0$$

The E-and H-field components are valid everywhere, except on the source itself.

1.1.3 Power Density and Radiation Resistance

The reactive power density, which is most dominant for small values of kr, has both radial and transverse components. It merely changes between outward and inward directions to form a standing wave at a rate of twice per cycle. It also moves in the transverse direction.

Equation which gives the real and imaginary power that is moving outwardly, Canalso be written as,

$$P = \frac{1}{2}\iint_S \mathbf{E}\times\mathbf{H}^*\cdot d\mathbf{s} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2\left[1 - j\frac{1}{(kr)^3}\right]$$
$$= P_{\text{rad}} + j2\omega(\tilde{W}_m - \tilde{W}_e)$$

where

P = power (in radial direction)
P_{rad} = time-average power radiated
$\tilde{W}_m$ = time-average magnetic energy density (in radial direction)
$\tilde{W}_e$ = time-average electric energy density (in radial direction)
$2\omega(\tilde{W}_m - \tilde{W}_e)$ = time-average imaginary (reactive) power (in radial direction)

And also,

$$P_{\text{rad}} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

$$2\omega(\tilde{W}_m - \tilde{W}_e) = -\eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2\frac{1}{(kr)^3}$$ ---------------------(ii)

It is clear from (i) that the radial electric energy must be larger than the radial magnetic energy. For large values of k_r (k_r>>1 or r>>λ), the reactive power diminishes and vanishes when k_r=∞.Since the antenna radiates its real power through the radiation resistance, for the infinitesimal dipole it is found by equation,

$$P_{\text{rad}} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 = \frac{1}{2}|I_0|^2 R_r$$

Where R_r is called the radiation Resistance, can be found by the equation,

$$R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

CHAPTER 2

Fundamentals of Antenna Array

2.1 TWO-ELEMENT ARRAY

Let us assume that the antenna under investigation is an array of two infinitesimal horizontal dipoles positioned along the z-axis, as shown in Figures,

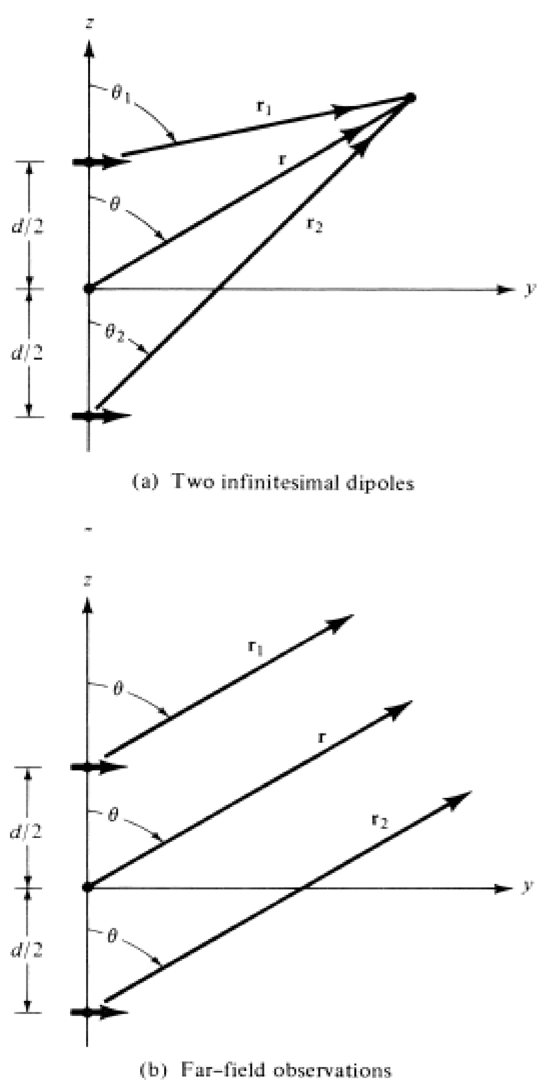


Figure 2.1: Geometry of a two-element array positioned along the z-axis.

The total field radiated by the two elements, assuming no coupling between the elements, is equal to the sum of the two and in they-z plane it is given by,

$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi} \left\{ \frac{e^{-j[kr_1-(\beta/2)]}}{r_1} \cos\theta_1 + \frac{e^{-j[kr_2+(\beta/2)]}}{r_2} \cos\theta_2 \right\}$$

Where β is the difference in phase excitation between the elements. The magnitude excitation of the radiators is identical, assuming far field observation,

$$\begin{aligned} &\theta_1 \simeq \theta_2 \simeq \theta \\ &\left.\begin{aligned} r_1 &\simeq r - \frac{d}{2}\cos\theta \\ r_2 &\simeq r + \frac{d}{2}\cos\theta \end{aligned}\right\} \text{ for phase variations} \\ &r_1 \simeq r_2 \simeq r \qquad \text{for amplitude variations} \end{aligned}$$

The equation E_t reduces to,

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta [e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}]$$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta \left\{ 2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]\right\}$$

It is apparent that the total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is widely referred to as the array factor. Thus for the two-element array of constant amplitude, the array factor is given by

$$\mathrm{AF} = 2\cos[\tfrac{1}{2}(kd\cos\theta + \beta)]$$

Which in normalized form can be written as

$$(\mathrm{AF})_n = \cos[\tfrac{1}{2}(kd\cos\theta + \beta)]$$

The array factor is a function of the geometry of the array and the excitation phase. By varying the separation and/or the phase β between the elements, the characteristics of the array factor and of the total field of the array can be controlled.

It has been illustrated that the far-zone field of a uniform two-element array of identical elements is equal to the product of the field of a single element, at a selected reference point (usually the origin), and the array factor of that array. That is,

E (total)= [E(single element at reference point)]×[array factor]

2.2 N-ELEMENT LINEAR ARRAY: UNIFORM AMPLITUDE AND SPACING

An array of identical elements all of identical magnitude and each with a progressive phase is referred to as a uniform array. The array factor can be obtained by considering the elements to be point sources. If the actual elements are not isotropic sources, the total field can be formed by multiplying the array factor of the isotropic sources by the field of a single element.This is the pattern multiplication rule, and it applies only for arrays of identical elements.

The array factor is given by,

$$\mathrm{AF} = 1 + e^{+j(kd\cos\theta+\beta)} + e^{+j2(kd\cos\theta+\beta)} + \cdots + e^{j(N-1)(kd\cos\theta+\beta)}$$

$$\mathrm{AF} = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)}$$

This can be written as,

$$\mathrm{AF} = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

$$\text{where} \quad \psi = kd\cos\theta + \beta \qquad \text{.............(1)}$$

Since the total array factor for the uniform array is a summation of exponentials, it can be represented by the vector sum of N phasors each of unit amplitude and progressive phase ψ relative to the previous one. This is illustrated graphically,

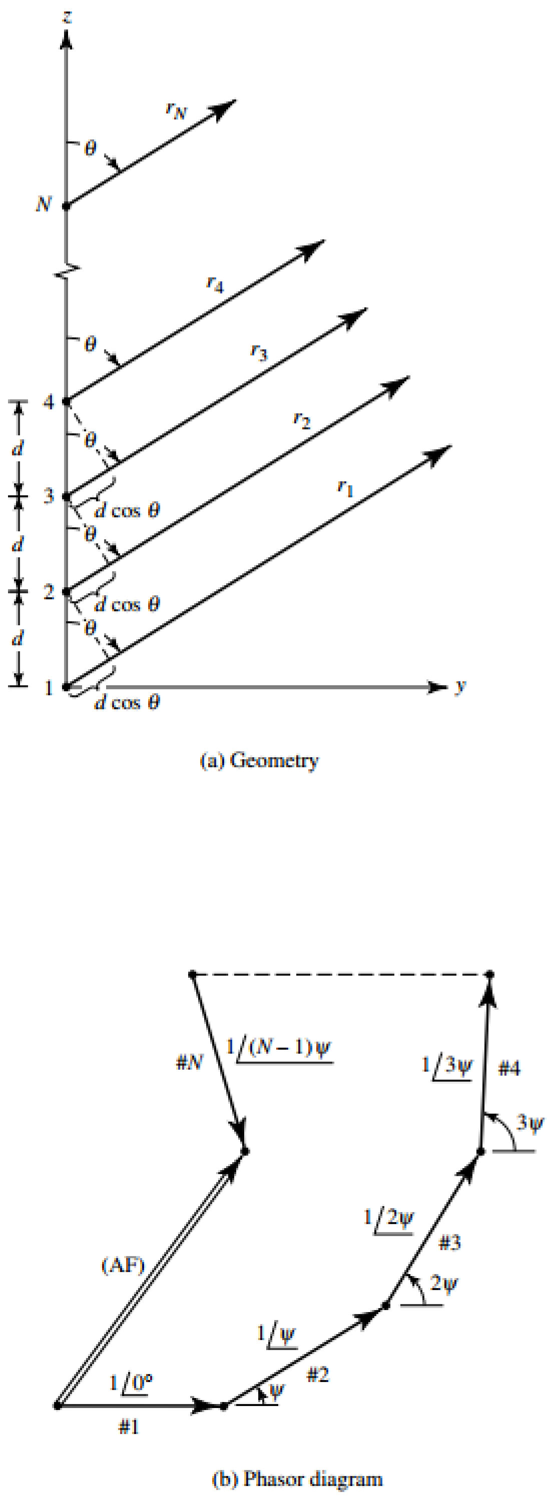


Figure 2.2: Far-field geometry and phasor diagram of N-element array of isotropic sources positioned along the z-axis.

It is apparent from the phasor diagram that the amplitude and phase of the AF can be controlled in uniform arrays by properly selecting the relative phase ψ between the elements; in non-uniform arrays, the amplitude as well as the phase can be used to control the formation and distribution of the total array factor.

The array factor can also be expressed in an alternate, compact and closed form whose functions and their distributions are more recognizable. This is accomplished as follows. Multiplying both side by $e^{j\psi}$, it can be written as,

$$(\mathrm{AF})e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(N-1)\psi} + e^{jN\psi}$$

Which is subtracting from (1) gives,

$$\mathrm{AF}(e^{j\psi} - 1) = (-1 + e^{jN\psi})$$

This can also be written as

$$\mathrm{AF} = \left[\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}\right] = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right]$$

$$= e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

……………(2)

If the reference point is the physical center of the array, the array factor of (2)

Reduced to,

$$\mathrm{AF} = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

………….(3a)

For small values of ψ, the above expression can be approximated by,

$$\mathrm{AF} \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}}\right]$$ ....(3b)

Maximum value of (3a) and (3b) is equal to N. To normalize the array factors so that the maximum value of each is equal to unity, (3a) and (3b) are written in normalized form as,

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

And

$$(\mathrm{AF})_n \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

To find the nulls of the array,

$$\sin\left(\frac{N}{2}\psi\right) = 0 \Rightarrow \frac{N}{2}\psi|_{\theta=\theta_n} = \pm n\pi \Rightarrow \theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

For n=N,2N, 3N,...etc. attains its maximum values. The values of n determine the order of the nulls. For a zero to exist, the argument of the arccosine cannot exceed unity. Thus the number of nulls that can exist will be a function of the element separation and the phase excitation difference β.

The maximum values occur when,

$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta + \beta)|_{\theta=\theta_m} = \pm m\pi \Rightarrow \theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$
$$m = 0, 1, 2, \ldots$$

The array factor has only one maximum and occurs when m=0

That is,

$$\theta_m = \cos^{-1}\left(\frac{\lambda\beta}{2\pi d}\right)$$

Which is the observation angle that makes ψ=0.

2.3 Array Weighting

The array factor has sidelobes. For a uniformly weighted linear array, the largest side lobes are down approximately 24 percent from the peak value. The presence of sidelobes means that the array is radiating energy in untended directions. Additionally, due to reciprocity, the array is receiving energy from unintended directions. In a multipath environment, the side lobes can receive the same signal from multiple angles. This is the basis for fading experienced in communications. If the direct transmission angle is known, it is best to steer the beam toward the desired direction and to shape the side lobes to suppress unwanted signals.

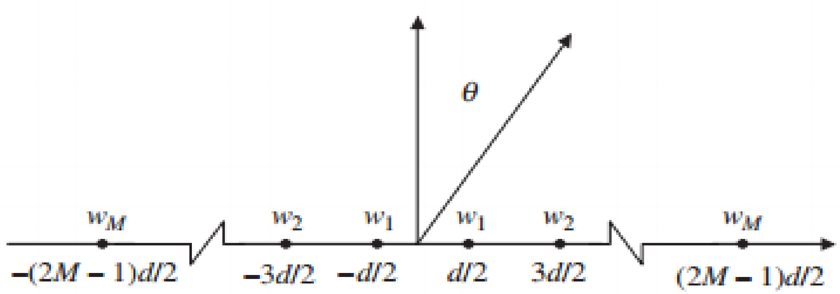


Fig 2.3: Even array with weights

The side lobes can be suppressed by weighting, shading , or windowing the array elements. These terms are taken from the EM, underwater acoustics, and array signal processing communities, respectively. Array element weighting has numerous applications in areas such as digital signal processing (DSP), radio astronomy, radar, sonar, and communications.

Figure shows a symmetric linear array with an even number of elements N. The array is symmetrically weighted with weights as indicated:

The array factor is found by summing the weighted outputs of each element such that,

$$\begin{aligned}\mathrm{AF}_{\text{even}} = {} & w_M e^{-j\frac{(2M-1)}{2}kd\sin\theta} + \cdots + w_1 e^{-j\frac{1}{2}kd\sin\theta} \\ & + w_1 e^{j\frac{1}{2}kd\sin\theta} + \cdots + w_M e^{j\frac{(2M-1)}{2}kd\sin\theta}\end{aligned}$$

Where 2M=N=total number of array elements. Each apposing pair of exponential terms in forms complex conjugates. We can invoke the Euler's identity for the cosine to recast the even array factor given as follows:

$$\mathrm{AF}_{\text{even}} = 2\sum_{n=1}^{M} w_n \cos\left(\frac{(2n-1)}{2}kd\sin\theta\right)$$

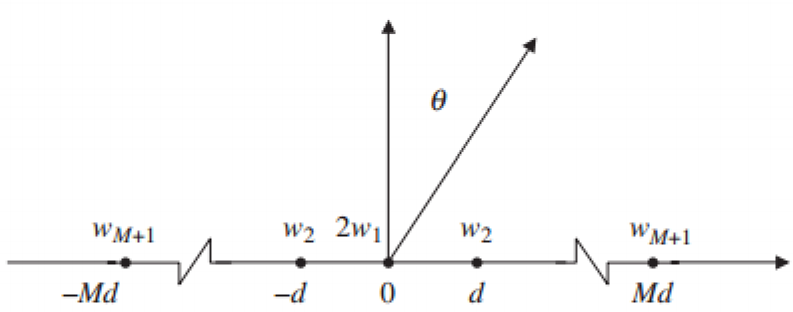
Without loss of generality, the 2 can be eliminated from the expression to produce a quasi-normalization.

$$\mathrm{AF}_{\text{even}} = \sum_{n=1}^{M} w_n \cos((2n-1)u)$$

Where $u=(\pi d/\lambda)\sin\theta$.

The array factor is maximum when the argument is zero, implying

$\theta=0$. The maximum is then the sum of all of the array weights. Thus



We may completely normalize AF_{even} to be

$$\mathrm{AF}_{\mathrm{even}} = \frac{\sum_{n=1}^{M} w_n \cos((2n-1)u)}{\sum_{n=1}^{M} w_n}$$

We may again sum all of the exponential contributions from each array element to get the quasi-normalized odd array factor.

$$\mathrm{AF}_{\mathrm{odd}} = \sum_{n=1}^{M+1} w_n \cos(2(n-1)u)$$

Where 2M+1=N

In order to normalize, we must again divide by the sum of the array weights to get

$$\mathrm{AF}_{\mathrm{odd}} = \frac{\sum_{n=1}^{M+1} w_n \cos(2(n-1)u)}{\sum_{n=1}^{M+1} w_n}$$

The array factor can be expressed in vector terms as

$$\mathrm{AF} = \bar{w}^T \cdot \bar{a}(\theta)$$

$$\begin{aligned} &\text{where } \bar{a}(\theta) = \text{array vector} \\ &\qquad \bar{w}^T = [w_M \quad w_{M-1} \quad \ldots \quad w_1 \quad \ldots \quad w_{M-1} \quad w_M] \end{aligned}$$

The weights w_n can be chosen to meet any specific criteria. Generally the criterion is to minimize the sidelobes or possibly to place nulls at certain angles.

Chapter-3

Smart Antennas

3.1 Introduction

Over the last decade, wireless technology has grown at a formidable rate, thereby creating new and improved services at lower costs. This has resulted in an increase in airtime usage and in the number of subscribers. The most practical solution to this problem is to use spatial processing. Spatial processing is the central idea of adaptive antennas or smart-antenna systems. As the number of users and the demand for wireless services increases at an exponential rate, the need for wider coverage area and higher transmission quality rises. Smart-antenna systems provide a solution to this problem.

3.2 SMART-ANTENNA ANALOGY

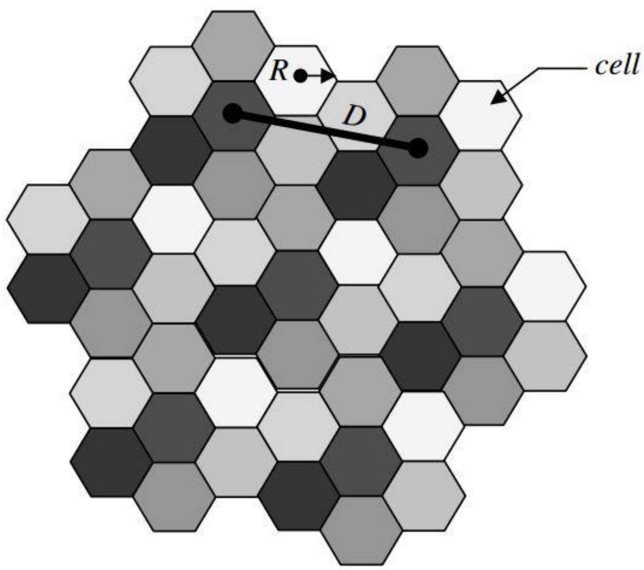
Electrical smart-antenna systems work using two antennas and a digital signal processor .Therefore, after the digital signal processor measures the time delays from each antenna element, it computes the direction of arrival (DOA) of the signal-of-interest (SOI), and then it adjusts the excitations (gains and phases of the signals) to produce a radiation pattern that focuses on the SOI while, ideally, tuning out any signal-not-of interest (SNOI).

3.3 CELLULAR RADIO SYSTEMS EVOLUTION

Maintaining capacity has always been a challenge as the number of services and subscribers increased. To achieve the capacity demand required by the growing number of subscribers, cellular radio systems had to evolve throughout the years. To justify the need for smart-antenna systems in the current cellular system structure, a brief history on the evolution of the cellular radio systems is presented.

3.3.1 Omnidirectional Systems

Since the early days, system designers knew that capacity was going to be a problem. Therefore, to achieve the capacity required for thousands of subscribers, a suitable cellular structure had to be designed; an example of the resulting structure is depicted in figure below



Each shaded hexagonal area in Figure represents a small geographical area named cell with maximum radius *R* .At the center of each cell resides a base station equipped with an omnidirectional antenna with a given band of frequencies .Base stations in adjacent cells are assigned frequency bands that contain different frequencies compared to the neighboring cells. The design process of selecting and allocating the same bands of frequencies to different cells of cellular base stations within a system is referred to as frequency reuse. In the first cellular radio systems deployed, each base station was equipped with an omnidirectional antenna with a typical amplitude pattern. As the number of users increased, so did the interference, thereby reducing capacity. An immediate solution to this problem was to subdivide a cell into smaller cells; this technique is referred to as cell splitting**.**

3.3.1A.Cell Splitting:

Cell splitting as shown in Figure below subdivides a congested cell into smaller cells called *microcells*, each with its own base station and a corresponding reduction in antenna height and transmitter power. The disadvantages of cell splitting are costs incurred from the installation of new base stations, the increase in the number of *handoffs* and a higher processing load per subscriber.

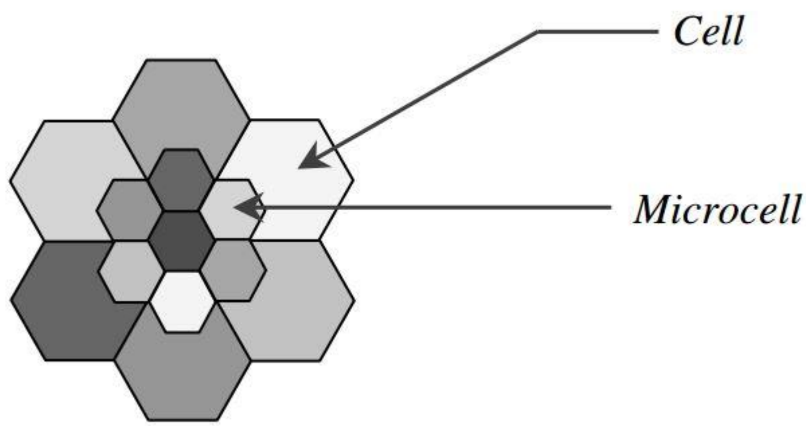


Figure 3.3.1A: Cell Splitting

3.3.1B. Sectorized Systems:

As the demand for wireless service grew even higher, the number of frequencies assigned to a cell eventually became insufficient to support the required number of subscribers. Thus, a cellular design technique was needed to provide more frequencies per coverage area. This technique is referred to as cell sectoring where a single omnidirectional antenna is replaced at the base station with several directional antennas. Typically, a cell is sectorized into three sectors of 120◦.

3.3.2 Smart-Antenna Systems

Despite its benefits, cell sectoring did not provide the solution needed for the capacity problem. Therefore, the system designers began to look into a system that could dynamically sectorize a cell. Hence, they began to examine smart antennas. Many refer to smart-antenna systems as smart antennas, but in reality antennas are not smart; it is the digital signal processing, along with the antennas, which makes the system smart. Smart-antenna systems are basically an extension of *cell sectoring* in which the sector coverage is composed of multiple beams. This is achieved by the use of antenna arrays, and the number of beams in the sector (e.g., 120◦) is a function of the array geometry. These systems can generally be classified as either Switched-Beam or Adaptive Array.

3.3.2A. Switched –Beam Systems

A switched-beam system is a system that can choose from one of many predefined patterns in order to enhance the received signal and it is obviously an extension of cell sectoring as each sector is subdivided into smaller sectors. As the mobile unit moves throughout the cell, the switched-beam system detects the signal strength, chooses the appropriate predefined beam pattern, and continually switches the beams as necessary. The overall goal of the switched-beam system is to increase the gain according to the location of the user. However, since the beams are fixed, the intended user may not be in the center of any given main beam. If there is an interferer near the center of the active beam, it may be enhanced more than the desired user.

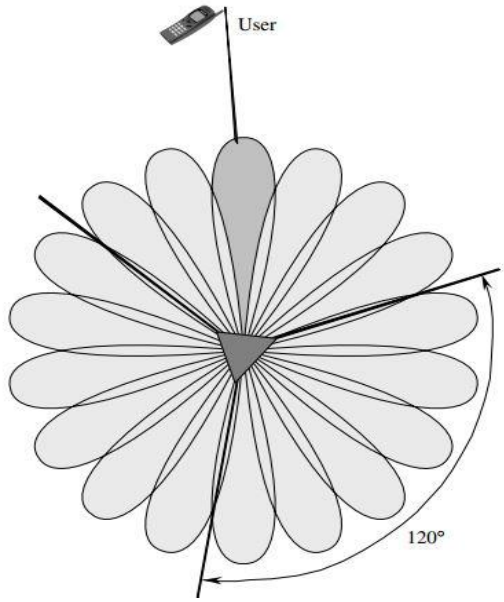


Fig 3.3.2A: Switched-beam system

3.3.2B. Adaptive Array Systems:

Adaptive array systems provide more degrees of freedom since they have the ability to adapt in real time the radiation pattern to the RF signal environment. In other words, they can direct the main beam toward the pilot signal or SOI while suppressing the antenna pattern in the direction of the interferers or SNOIs. To put it simply, adaptive array systems can customize an appropriate radiation pattern for each individual user. This is far superior to the performance of a switched-beam system, as shown in Figure below. This figure shows that not only the switched-beam system may not able to place the desired signal at the maximum of the main lobe but also it exhibits the inability to fully reject the interferers. Because of the ability to control the overall radiation pattern in a greater coverage area for each cell site, adaptive array systems greatly increase capacity.

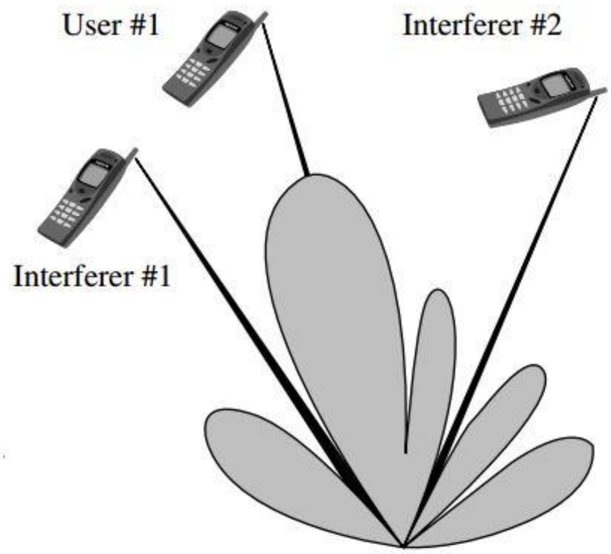


Fig 3.3.2B: Adaptive Array Systems

3.4 SMART ANTENNAS' BENEFITS:

The primary reason for the growing interest in smart-antenna systems is the *capacity* increase. In densely populated areas, mobile systems are usually interference-limited, meaning that the interference from other users is the main source of noise in the system. This means that the signal-to-interference ratio (SIR) is much smaller than the signal-to-noise ratio (SNR). In general, smart antennas will, by simultaneously increasing the useful received signal level and lowering the interference level, increase the SIR.

Another benefit that smart-antenna systems provide is *range* increase. Because smart antennas are more directional than omnidirectional and sectorized antennas, a range increase potential is available.

Another added advantage of smart-antenna systems is *security*. In a society that becomes more dependent on conducting business and transmitting personal information, security is an important issue. Smart antennas make it more difficult to tap a connection because the intruder must be positioned in the same direction as the user as seen from the base station to successfully tap a connection.

3.5 SMART ANTENNAS' DRAWBACKS:

While smart antennas provide many benefits, they do suffer from certain drawbacks. For example, their transceivers are much more complex than traditional base station transceivers. The antenna needs separate transceiver chains for each array antenna element and accurate real-time calibration for each of them. Moreover, the antenna beamforming is computationally intensive, which means that smart-antenna base stations must be equipped with very powerful digital signal processors. This tends to increase the system costs in the short term, but since the benefits outweigh the costs, it will be less expensive in the long run. For a smart antenna to have pattern-adaptive capabilities and reasonable gain, an array of antenna elements is necessary.

Chapter 4

Beamforming Basics:

Beamforming or spatial filtering is a signal processing technique used in sensor arrays for directional signal transmission or reception. This is achieved by combining elements in a phased array in such a way that signals at particular angles experience constructive interference while others experience destructive interference. Beamforming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity. The improvement compared with omnidirectional reception/transmission is known as the directivity of the element.

To change the directionality of the array when transmitting, a beamformer controls the phase and relative amplitude of the signal at each transmitter, in order to create a pattern of constructive and destructive interference in the wavefront. When receiving, information from different sensors is combined in a way where the expected pattern of radiation is preferentially observed.

For example, in sonar, to send a sharp pulse of underwater sound towards a ship in the distance, simply transmitting that sharp pulse from every sonar projector in an array simultaneously fails because the ship will first hear the pulse from the speaker that happens to be nearest the ship, then later pulses from speakers that happen to be the further from the ship. The beamforming technique involves sending the pulse from each projector at slightly different times (the projector closest to the ship last), so that every pulse hits the ship at exactly the same time, producing the effect of a single strong pulse from a single powerful projector. The same thing can be carried out in air using loudspeakers, or in radar/radio using antennas.

In passive sonar, and in reception in active sonar, the beamforming technique involves combining delayed signals from each hydrophone at slightly different times (the hydrophone closest to the target will be combined after the longest delay), so that every signal reaches the output at exactly the same time, making one loud signal, as if the signal came from a single, very sensitive hydrophone. Receive beamforming can also be used with microphones or radar antennas.

With narrow-band systems the time delay is equivalent to a "phase shift", so in this case the array of antennas, each one shifted a slightly different amount, is called a phased array. A narrow band system, typical of radars, is one where the bandwidth is only a small fraction of the center frequency. With wide band systems this approximation no longer holds, which is typical in sonars.

In the receive beamformer the signal from each antenna may be amplified by a different "weight." Different weighting patterns (e.g., Dolph-Chebyshev) can be used to achieve the desired sensitivity patterns. A main lobe is produced together with nulls and sidelobes. As well as controlling the main lobe width (the beam) and the sidelobe levels, the position of a null can be controlled. This is useful to ignore noise or jammers in one particular direction, while listening for events in other directions. A similar result can be obtained on transmission.

Conventional beamformers use a fixed set of weightings and time-delays (or phasings) to combine the signals from the sensors in the array, primarily using only information about the location of the sensors in space and the wave directions of interest. In contrast, adaptive beamforming techniques generally combine this information with properties of the signals actually received by the array, typically to improve rejection of unwanted signals from other directions. This process may be carried out in either the time or the frequency domain.

As the name indicates, an adaptive beamformer is able to automatically adapt its response to different situations. Some criterion has to be set up to allow the adaption to proceed such as minimizing the total noise output. Because of the variation of noise with frequency, in wide band systems it may be desirable to carry out the process in the frequency domain.

Beamforming can be computationally intensive. Sonar phased array has a data rate low enough that it can be processed in real-time in software, which is flexible enough to transmit and/or receive in several directions at once. In contrast, radar phased array has a data rate so high that it usually requires dedicated hardware processing, which is hard-wired to transmit and/or receive in only one direction at a time. However, newer field programmable gate arrays are fast enough to handle radar data in real-time, and can be quickly re-programmed like software, blurring the hardware/software distinction

Chapter 5

DOA (Direction of Arrival) Estimation

In order to analyze and derive more conveniently, now assume the following conditions for the ideal mathematical model of DOA problems.

1) Each test signal source has the same but unrelated polarization. Generally consider that the signal sources are narrow bands, and each source has the same centre frequency ω_0. The number of testing signal source is D.

2) Antenna array is a spaced linear array which consists of M (M>D) array elements; each element has the same characteristics, and it is isotropic in each direction.

3) The spacing is d, and the array element interval is not larger than half the wavelength of the highest signal frequency.

4) Each antenna element in the far field source, namely, an antenna array receiving the signals coming from the signal source is a plane wave.

5) Both array elements and test signals are uncorrelated; variance σ^2 is zero-mean Gaussian noise $n_m(t)$.

6) Each receiving branch has the same characteristics

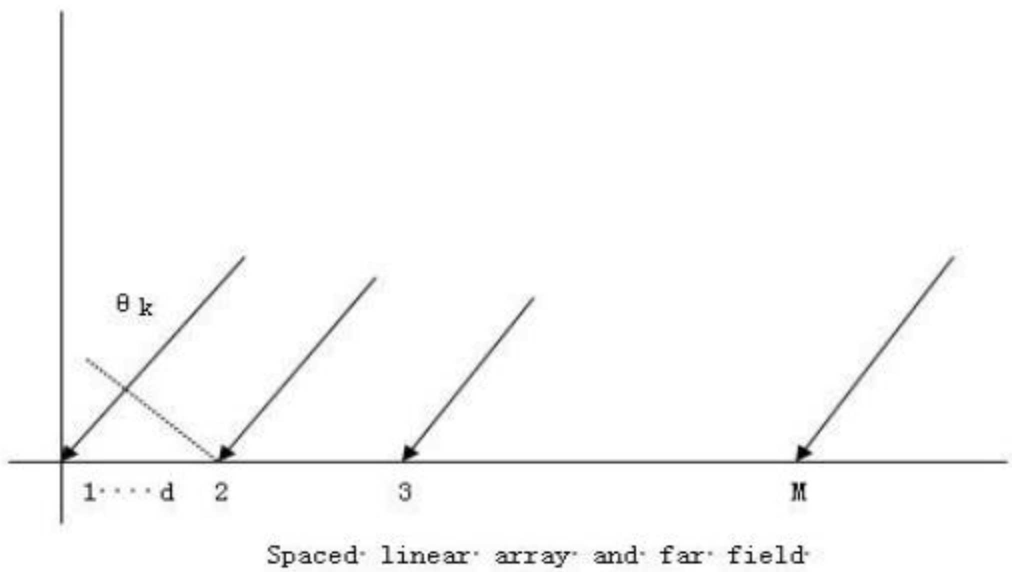


Fig 5.1 : The model of DOA estimation

Let the number of signal sources k (k=1,2,…,D) to the antenna array, the wavefront signal be $S_k(t)$, as previously assumed, $S_k(t)$ is a narrowband signal, and $S_k(t)$ can be expressed in the following form

$$S_k(t) = s_k(t)\exp\{j\omega_k(t)\} \quad \text{……………………………… (5.1)}$$

Where $s_k(t)$ is the complex envelope of $S_k(t)$ and $\omega_k(t)$ is the angular frequency of $S_k(t)$.

As assumed before, all signals have the same centre frequency. So

$$\omega_k=\omega_0=2\pi c/\lambda \quad \text{……………………………………… (5.2)}$$

Let the time required by the electromagnetic antenna array dimension be t_1. According to the narrowband assumption, the following approximation is valid

$$S_k(t- t_1)\approx S_k(t) \quad \text{………………………………………(5.3)}$$

Therefore, the delayed wave front signal is

$$\tilde{s}_k(t-t_1) = s_k(t-t_1)\exp[j\omega_0(t-t_1)] = s_k(t)exp[j\omega_0(t-t_1)].$$

……………………………(5.4)

So use the first array element as reference points. At the moment t, the induction signal of the array element m (m=1,2,…M) to the k-th signal source in the spaced linear array is

$$a_k s_k(t)\exp[-j(m-1)\frac{2\pi \, \mathrm{d} \sin\theta_k}{\lambda}],$$

…………………………………(5.5)

Where a_k is the impact of array element m on the signal source k-th. As assumed before, each array element has no direction, so let a_k=1. θ_k is direction angle of signal source k, $(m-1)d \sin\theta_k/\lambda$ is signal phase difference which is caused by the path difference between m-th array element and the first array element.

Record and measure the noise and all waves from the source, the output signal of the mth element is

$$x_m(t) = \sum_{k=1}^{D} s_k(t)\exp\left[-j(m-1)\frac{2\pi \, \mathrm{d} \sin\theta_k}{\lambda}\right] + n_m(t)$$

…………(5.6)

where $n_m(t)$ is measurement noise; all quantities of labeled m belong to the m-th array element; all quantities of label k belong to the signal source k. Let

$$a_m(\theta_k) = \exp[-j(m-1)\frac{2\pi \, \mathrm{d} \sin\theta_k}{\lambda}]$$

……………………………(5.7)

be the response function of array element m to signal source k.

Then the output signal of array element m is

$$x_m(t) = \sum_{k=1}^{D} a_m(\theta_k)s_k(t) + n_m(t)$$

………………………………(5.8)

Where $s_k(t)$ is signal strength of signal source k.

This expression can be described by matrices:

$$\mathbf{X=AS+N}$$

where

$$X=[x_1(t),x_2(t),\ldots,x_M(t)]^T,$$

$$S=[S_1(t),S_2(t),\ldots,S_D(t)]^T,$$

$$A=[a(\theta_1), a(\theta_2), \ldots, a(\theta_D)]^T$$

$$=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & \cdots & e^{-j\varphi_D} \\ \cdots & \cdots & \cdots & \cdots \\ e^{-j(M-1)\varphi_1} & e^{-j(M-1)\varphi_2} & \cdots & e^{-j(M-1)\varphi_D} \end{bmatrix},$$

$$\text{with } \varphi_k=\frac{2\pi d}{\lambda}\sin\theta_k,$$

$$N=[n_1(t),n_2(t),\ldots,n_M(t)]^T.$$

To conduct N sampling point for $x_m(t)$, the issue becomes to sample the output signal $x_m(t)$, then estimate the angle $\theta_1, \theta_2, \ldots, \theta_D$ of signal source DOA from

$\{x_m(i), i=1,2,\ldots,M\}$.

Chapter 6

Comparison in performance of different beamforming and DOA estimation algorithms

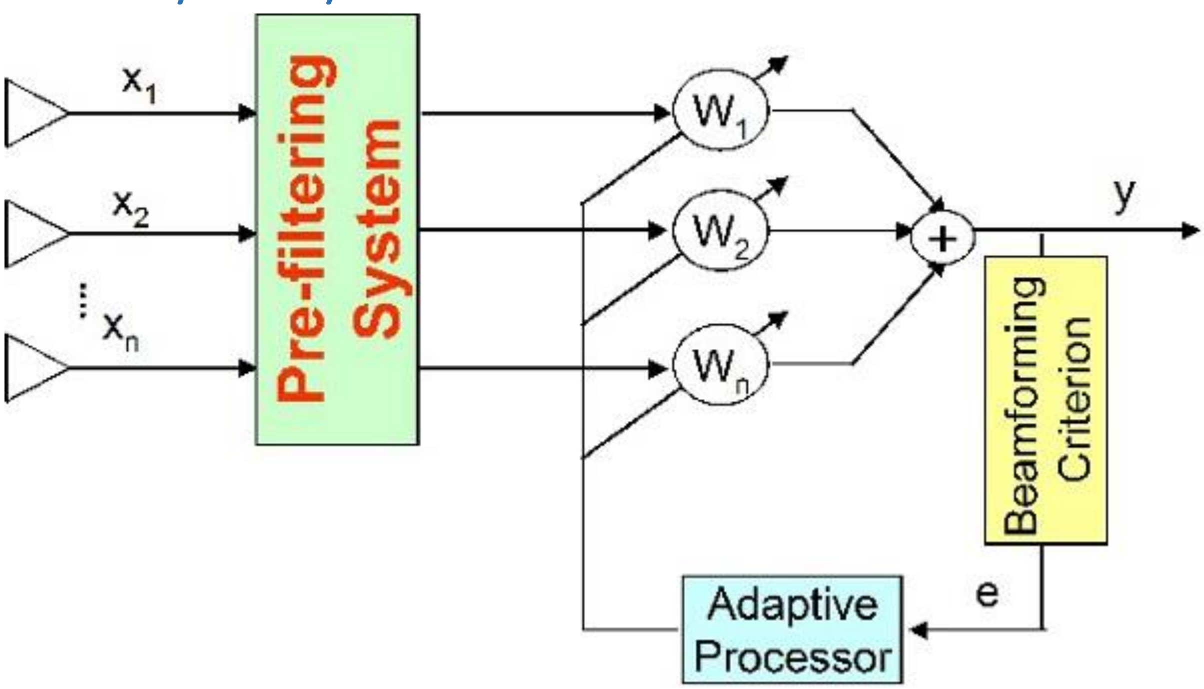
6.1 Non-blind adaptive algorithms

1) Least Mean Square (LMS)

2) Sample Matrix Inversion (SMI)

3) Recursive Least Square (RLS)

6.1.0 Physical System



6.1.1 Least Mean Squares

The least mean squares algorithm is a gradient based approach. Gradient based algorithms assume an established quadratic performance surface. When the performance surface is a quadratic function of the array weights, the performance surfaceis in the shape of an elliptic paraboloid having one minimum. The error is calculated by the equation,

$$\varepsilon(k) = d(k) - \bar{w}^H(k)\bar{x}(k)$$

The squared error is given as,

$$|\varepsilon(k)|^2 = \left|d(k) - \bar{w}^H(k)\bar{x}(k)\right|^2$$

We can employ an iterative technique called the method of steepest descentto approximate the gradient of the cost function. The direction of steepest descent is in the opposite direction as the gradient vector. The steepest descent iterative approximation is given as,

$$\bar{w}(k+1) = \bar{w}(k) - \frac{1}{2}\mu\nabla_{\bar{w}}\left(J\left(\bar{w}(k)\right)\right)$$

The LMS solution is given by,

$$\begin{aligned}\bar{w}(k+1) &= \bar{w}(k) - \mu[\hat{R}_{xx}\bar{w} - \hat{r}] \\ &= \bar{w}(k) + \mu e^*(k)\bar{x}(k)\end{aligned}$$

Where the error signal is calculated from the following equation,

$$e(k) = d(k) - \bar{w}^H(k)\bar{x}(k) = \text{error signal}$$

The convergence of the LMS algorithm is directly proportional to the step-size parameter µ. Stability is insured provided that the following condition is met,

$$0 \leq \mu \leq \frac{1}{2\lambda_{\max}}$$

Where λ_m axis the largest eigen value of correlation matrix. Since the correlation matrix is positive definite, all eigenvalues are positive. If all the interfering signals are noise and there is only one signal of interest, we can approximate the condition,

$$0 \leq \mu \leq \frac{1}{2\ \mathrm{trace}[\hat{R}_{xx}]}$$

$\hat{R}_{xx}$ = is the correlation matrix.

Problem statement:

- 8 element array
- Signal of interest(SOI) at 15 degree
- Signal not of interest(SNOI) at -15 degree
- Iteration number 100
- step size .02

Results: Without LMS algorithm,

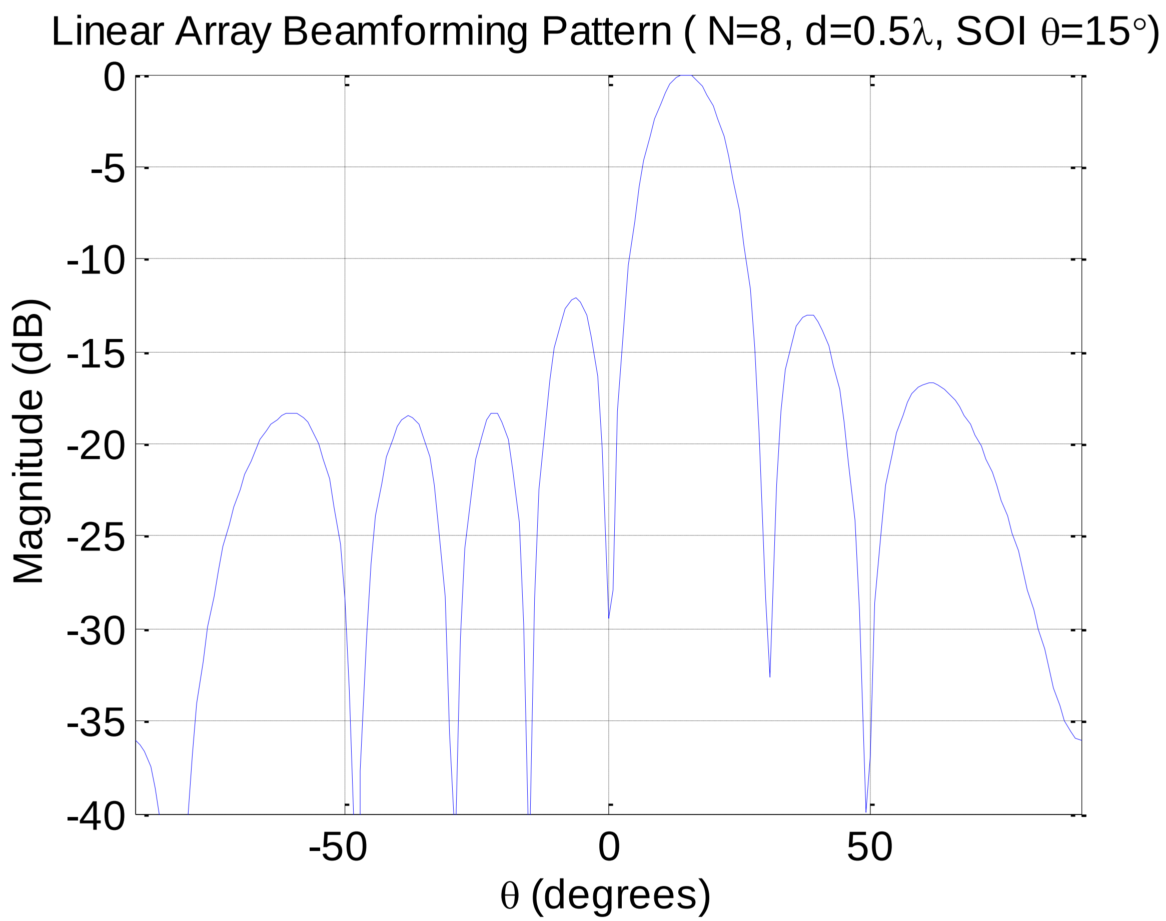


Figure 6.1.1(a): linear beamforming pattern without LMS adaptation

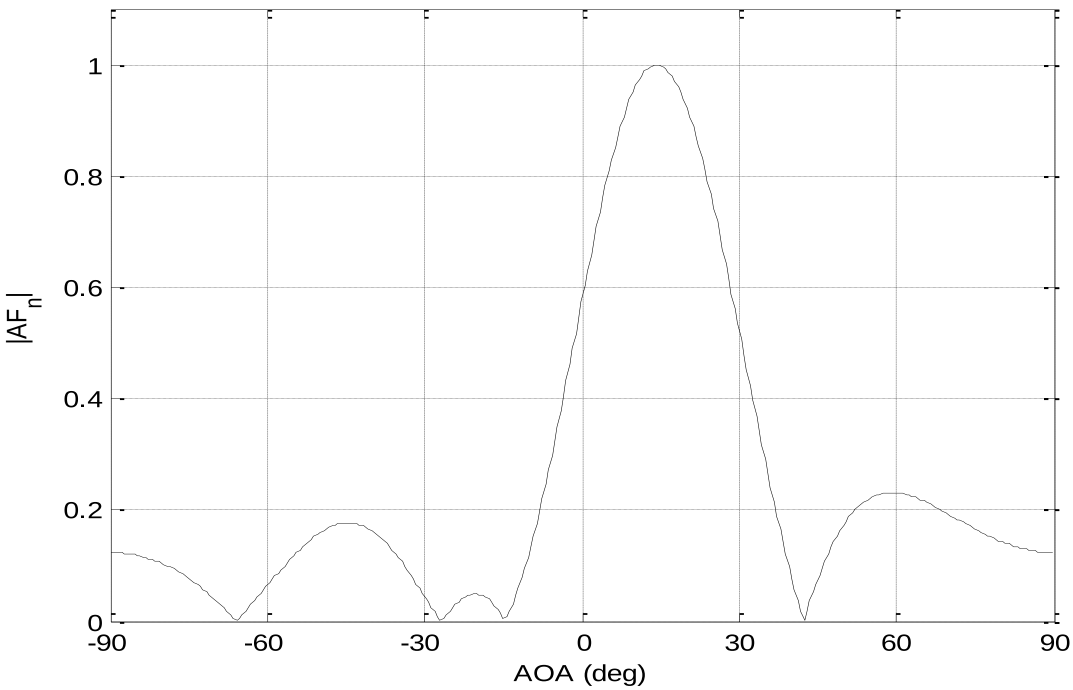


Figure 6.1.1(b): Array factor vs. angle of arrival with LMS algorithm

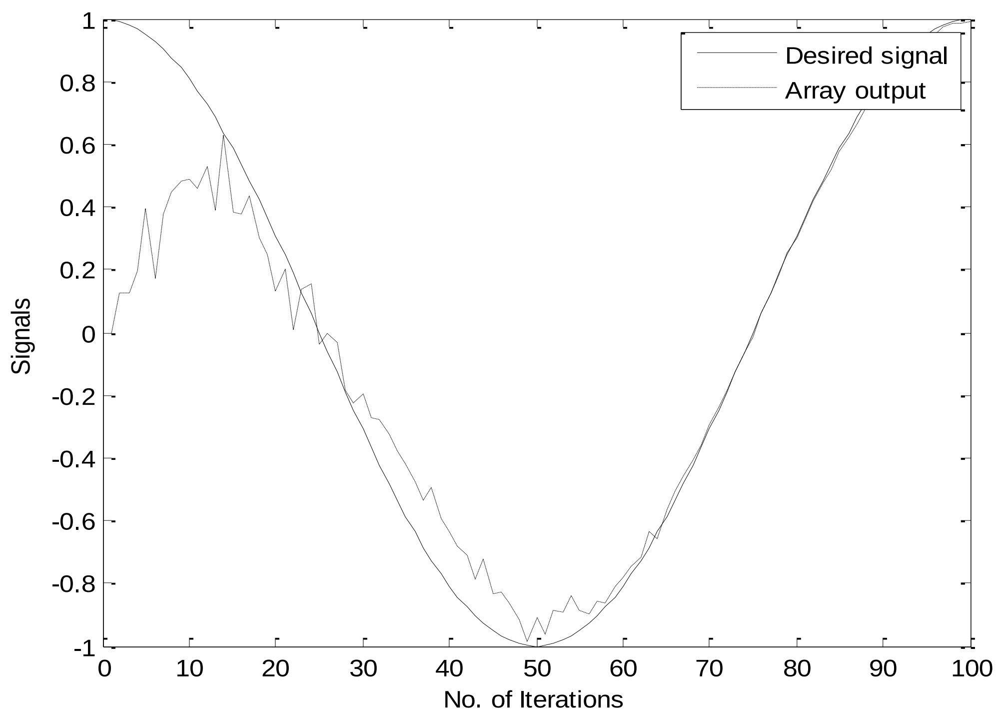


Figure 6.1.1(c): Signal vs. no of iteration

The array output is following the desired signal after 60 (approximation) iterations. LMS algorithm need fair amount of time and iteration number to follow the mother signal. That makes this algorithm a little slower and less efficient.

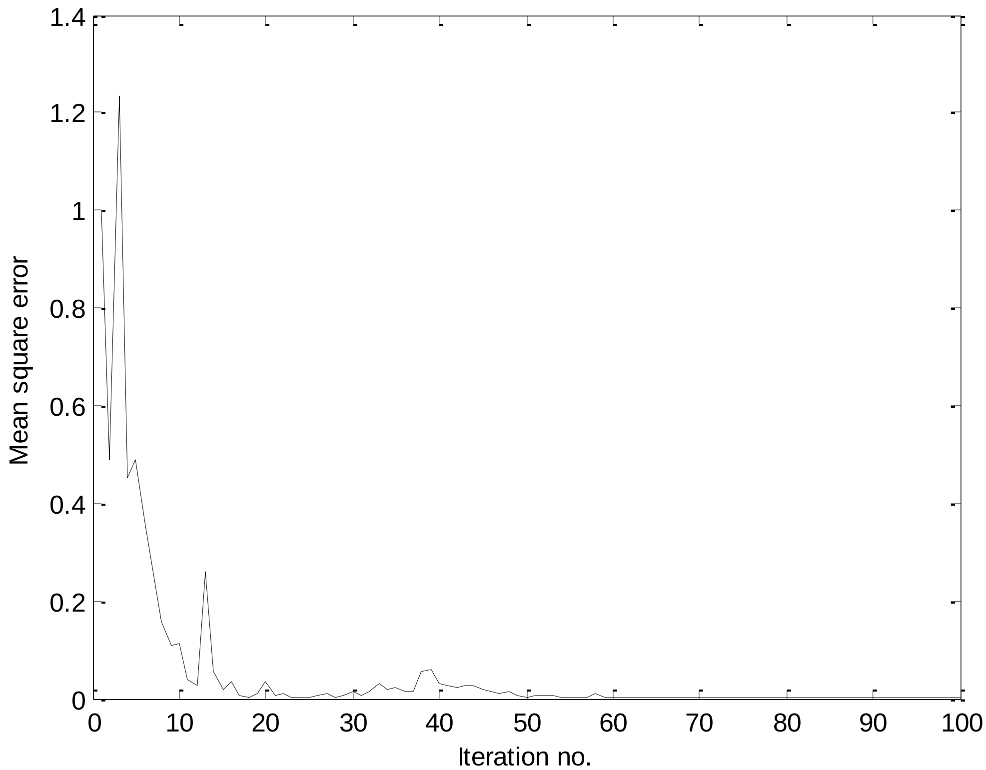


Figure 6.1.1(d): Error vs. iteration number

Another problem with the LMS algorithm is that the error is not Zero at the beginning of the iteration process. From the figure we can see that mean error is becoming zero when the iteration no is almost 60. So the receiver will have problem to identify the main signal coming from the source.

Drawbacks:

- If step size is too small, convergence is too slow (Overdamped case),so adaptive array cannot acquire SOI fast enough to track the changing signal.
- If step size is too large, algorithm will overshoot the optimum weights of interest(Underdamped case).
- The solution of this algorithm cannot support fast convergence.

6.1.2 Sample Matrix Inversion:

One of the drawbacks of the LMS adaptive scheme is that the algorithm must go through many iterations before satisfactory convergence is achieved. If the signal characteristics are rapidly changing, the LMS adaptive algorithm may not allow tracking of the desired signal in a satisfactory manner. One possible approach to circumventing the relatively slow convergence of the LMS scheme is by use of SMI method. The *sample matrix* is a time average estimate of the array correlation, matrix using *K*-time samples. If the random process is ergodic in the correlation, the time average estimate will equal the actual correlation matrix. The optimum array weights are given by the optimum Wiener solution as:

$$\bar{w}_{\text{opt}} = \overline{R}_{xx}^{-1}\bar{r}$$

Where,

$$\bar{r} = E\left[d^* \cdot \bar{x}\right]$$
$$\overline{R}_{xx} = E\left[\bar{x}\bar{x}^H\right]$$

We can estimate the correlation matrix by calculating the time average such that

$$\hat{R}_{xx} = \frac{1}{K}\sum_{k=1}^{K} \bar{x}(k)\bar{x}^H(k)$$

Where *K* is the observation interval. The correlation vector *r* can be estimated by

$$\hat{r} = \frac{1}{K}\sum_{k=1}^{K} d^*(k)\bar{x}(k)$$

Since we use a *K*-length block of data, this method is called a *block adaptive approach.*

Define the matrix X'_K (k)as the k-th block of x' vectors ranging over K-data snapshots. Thus,

$$\overline{X}_K(k) = \begin{bmatrix} x_1(1+kK) & x_1(2+kK) & \cdots & x_1(K+kK) \\ x_2(1+kK) & x_2(2+kK) & & \vdots \\ \vdots & & \ddots & \\ x_M(1+kK) & & \cdots & x_M(K+kK) \end{bmatrix}$$

Where k is the block number and K is the block length.

Thus, the estimate of the array correlation matrix is given by

$$\hat{R}_{xx}(k) = \frac{1}{K}\overline{X}_K(k)\overline{X}_K^H(k)$$

In addition, the desired signal vector can be define by,

$$\bar{d}(k) = [d(1+kK) \quad d(2+kK) \quad \cdots \quad d(K+kK)]$$

Thus the estimate of the correlation vector is given by,

$$\hat{r}(k) = \frac{1}{K}\bar{d}^*(k)\overline{X}_K(k)$$

The SMI weights can then be calculated for the k-th block of length K as

$$\begin{aligned} \bar{w}_{SMI}(k) &= \overline{R}_{xx}^{-1}(k)\bar{r}(k) \\ &= \left[\overline{X}_K(k)\overline{X}_K^H(k)\right]^{-1}\bar{d}^*(k)\overline{X}_K(k) \end{aligned}$$

Applying this algorithm in previous problem we get better array factor graph which is shown below. In this case sidelobes are much more suppresed than before.

Array factor vs AOA fig is :

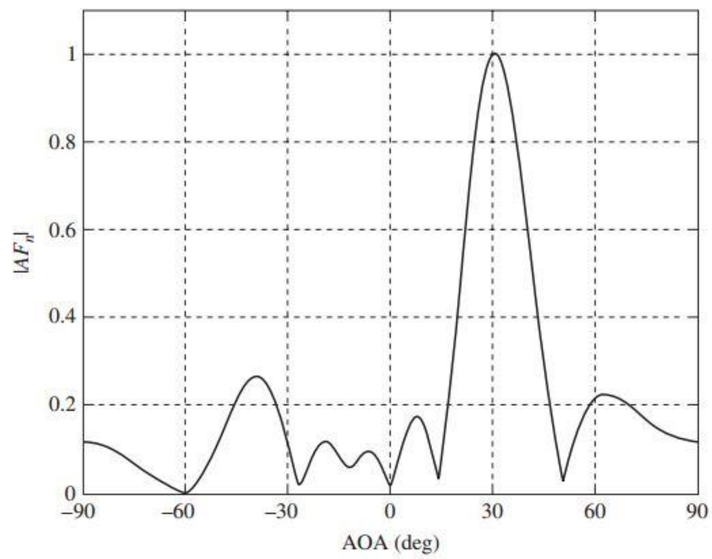


Fig 6.1.2: Weighted SMI array pattern.

6.1.3 RECURSIVE LEAST SQUARES:

As was mentioned in the previous section, the SMI technique has several drawbacks. Even though the SMI method is faster than the LMS algorithm, the computational burden and potential singularities can cause problems. However, we can recursively calculate the required correlation matrix and the required correlation vector. Thus, we can rewrite the correlation matrix and the correlation vector omitting *K* as

$$\hat{R}_{xx}(k) = \sum_{i=1}^{k} \bar{x}(i)\bar{x}^H(i)$$

$$\bar{r}(k) = \sum_{i=1}^{k} d^*(i)\bar{x}(i)$$

where k is the block length and last time sample k and $\bar{R}_{xx}(k)$, $\bar{r}(k)$ is
the correlation estimates ending at time sample k.

Since the signal sources can change or slowly move with time, we might want to deemphasize the earliest data samples and emphasize the most recent ones. This is called a *weighted estimate.* Thus

$$\hat{R}_{xx}(k) = \sum_{i=1}^{k} \alpha^{k-i} \bar{x}(i)\bar{x}^H(i)$$

$$\hat{r}(k) = \sum_{i=1}^{k} \alpha^{k-i} d^*(i)\bar{x}(i)$$

Where α is the *forgetting factor*. The forgetting factor is also sometimes referred to as the *exponential weighting* factor. α is a positive constant such that $0 \leq \alpha \leq 1$. When $\alpha = 1$, we restore the ordinary least squares algorithm. $\alpha = 1$ also indicates infinite memory. Let us break up the summation in previous Eqs. into two terms: the summation for values up to $i = k-1$ and last term for $i = k$.

$$\begin{aligned}\hat{R}_{xx}(k) &= \alpha \sum_{i=1}^{k-1} \alpha^{k-1-i} \bar{x}(i)\bar{x}^H(i) + \bar{x}(k)\bar{x}^H(k) \\ &= \alpha \hat{R}_{xx}(k-1) + \bar{x}(k)\bar{x}^H(k)\end{aligned}$$

$$\begin{aligned}\hat{r}(k) &= \alpha \sum_{i=1}^{k-1} \alpha^{k-1-i} d^*(i)\bar{x}(i) + d^*(k)\bar{x}(k) \\ &= \alpha \hat{r}(k-1) + d^*(k)\bar{x}(k)\end{aligned}$$

Thus, future values for the array correlation estimate and the vector correlation estimate can be found using previous values.

We can involve the Sherman Morrison-Woodbury to find the inverse recursion formula,

$$\hat{R}_{xx}^{-1}(k) = \alpha^{-1}\hat{R}_{xx}^{-1}(k-1) - \frac{\alpha^{-2}\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)}{1 + \alpha^{-1}\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)}$$

We can simplify the previous equation by defining the gain vector,

$$\bar{g}(k) = \frac{\alpha^{-1}\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)}{1+\alpha^{-1}\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)}$$

Thus,

$$\hat{R}_{xx}^{-1}(k) = \alpha^{-1}\hat{R}_{xx}^{-1}(k-1) - \alpha^{-1}\bar{g}(k)\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)$$

This is known as Riccati equation for the recursive least squares (RLS) method.

Now we can derive a recursion relationship to update the weight vectors. The optimum Wiener solution is repeated in terms of the iteration number *k* and get the weight equation,

$$\begin{aligned}\bar{w}(k) &= \hat{R}_{xx}^{-1}(k)\hat{r}(k)\\ &= \alpha\hat{R}_{xx}^{-1}(k)\hat{r}(k-1) + \hat{R}_{xx}^{-1}(k)\bar{x}(k)d^*(k)\end{aligned}$$

We now substitute this equation into the first correlation matrix,

$$\begin{aligned}\bar{w}(k) &= \hat{R}_{xx}^{-1}(k-1)\hat{r}(k-1) - \bar{g}(k)\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)\hat{r}(k-1)\\ &\quad + \hat{R}_{xx}^{-1}(k)\bar{x}(k)d^*(k)\\ &= \bar{w}(k-1) - \bar{g}(k)\bar{x}^H(k)\bar{w}(k-1) + \hat{R}_{xx}^{-1}(k)\bar{x}(k)d^*(k)\end{aligned}$$

Finally we get,

$$\begin{aligned}\bar{w}(k) &= \bar{w}(k-1) - \bar{g}(k)\bar{x}^H(k)\bar{w}(k-1) + \bar{g}(k)d^*(k)\\ &= \bar{w}(k-1) + \bar{g}(k)[d^*(k) - \bar{x}^H(k)\bar{w}(k-1)]\end{aligned}$$

RLS algorithm is applied for previous for previous problem and we see that the sidelobes are much more suppresed than before.

Array factor vs AOA graph of RLS is given below

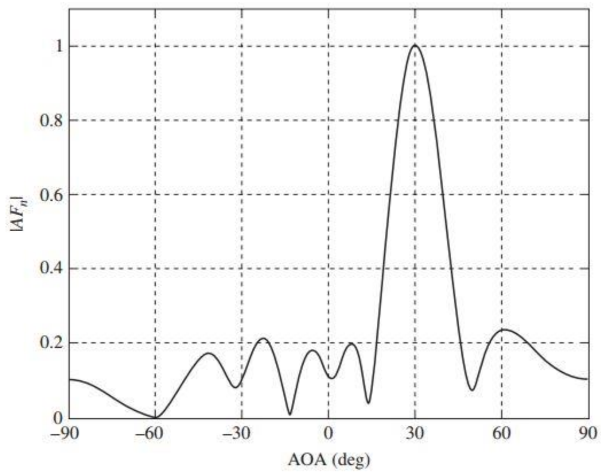


Fig 6.1.3(a): RLS array pattern

Iterarion number is also less in RLS to obtain the minimum mean square error.

Mean square error vs iteration number graph is given below:

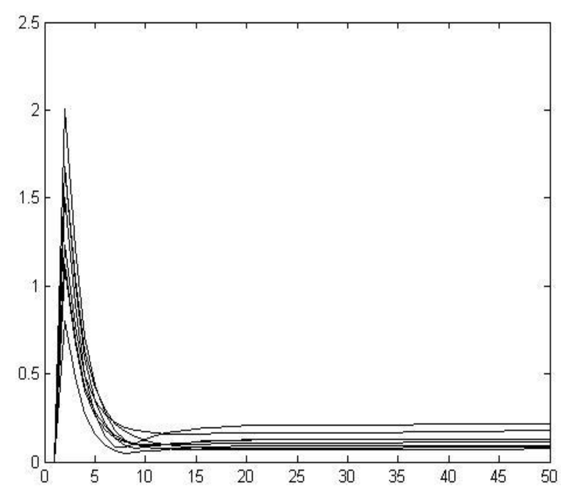
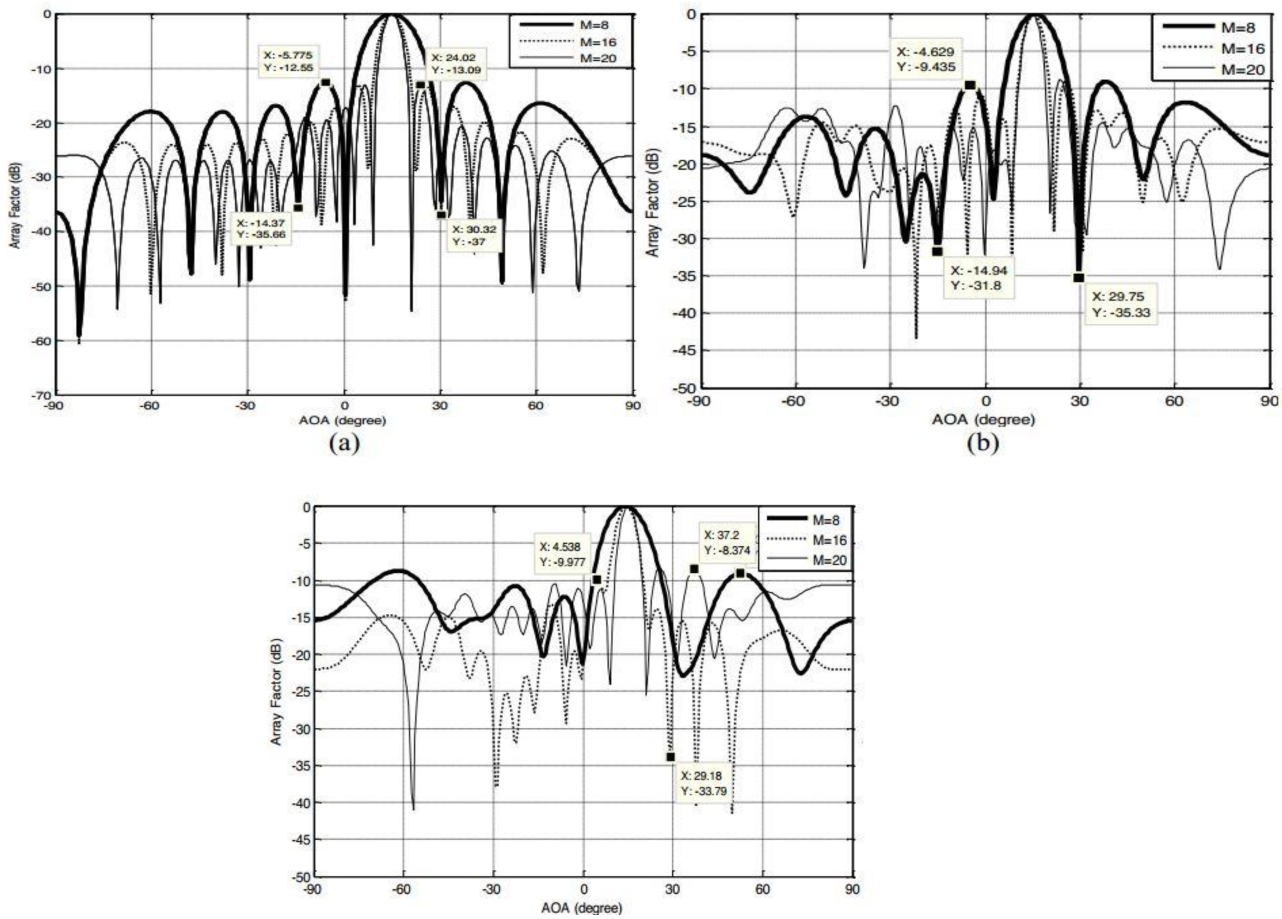


Fig 6.1.3(b): Mean square error vs iteration number

The advantage of the RLS algorithm over SMI is that it is no longer necessary to invert a large correlation matrix. The recursive equations allow for easy updates of the inverse of the correlation matrix. The RLS algorithm also converges much more quickly than the LMS algorithm.

6.1.4 Comparison among the three algorithms :



(C)

Fig 6.1.4: Comparison among the three algorithms

Fig- (a),(b) and (c) are simultaneously normalized array factor plot for LMS, SOI and RLS beamforming algorithm.From these figures , RLS beamforming algorithm is more convenient than LMS and SOI .

Rate of convergence for each algorithm:

Algorithm	No. of iterations
LMS	60
SMI	No iteration, block size 30
RLS	15

6.2 Algorithms for DOA Estimation

1) MUSIC (Multiple Signal Classification)

2) ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques)

6.2.1 Introduction to MUSIC algorithm:

Multiple Signal Classification (MUSIC) algorithm was proposed by Schmidt and his colleagues in 1979. It has created a new era for spatial spectrum estimation algorithms. The promotion of the structure algorithm characterized rise and development, and it has become a crucial algorithm for theoretical system of spatial spectrum. Before this algorithm was presented, some relevant algorithms directly processed data received from array covariance matrices. The basic idea of MUSIC algorithm is to conduct characteristic decomposition for the covariance matrix of any array output data, resulting in a signal subspace orthogonal with a noise subspace corresponding to the signal components. Then these two orthogonal subspaces are used to constitute a spectrum function, be got though by spectral peak search and detect DOA signals. It is because MUSIC algorithm has a high resolution, accuracy and stability under certain conditions that it attracts a large number of scholars to conduct in-depth research and analyses. In general, it has the following advantages when it is used to estimate a signal's DOA.

1) The ability to simultaneously measure multiple signals.

2) High precision measurement.

3) High resolution for antenna beam signals.

4) Applicable to short data circumstances.

5) It can achieve real-time processing after using high-speed processing technology

6.2.2 Eigen decomposition of array covariance:

For array output x, corresponding calculations can get its covariance matrix R_x

$Rx=E[XX^H]$

Where H is the conjugate transpose matrix.

As assumed before, signal and noise is uncorrelated, and the noise is zero mean white noise. Thus following can be obtained

$$R_x=E[(AS+N)](AS+N)^H]$$

$$=AE[SS^H]A^H+E[NN^H]$$

$$=AR_sA^H+RN,$$

Where $R_s=E[SS^H]$ is called the signal correlation matrix.

$RN=\sigma^2 I$,is the noise correlation matrix, $\sigma^2$is the power of noise, I is the unit matrix of M*M.

In practical applications, R_x usually cannot be directly obtained and only sample covariance can be used:

$$\tilde{R}_x = \frac{1}{N}\sum_{i=1}^{N} x(i)x^H(i),$$

Where $R\acute{x}$ is the maximum likelihood estimation of Rx. When the number of samples

N→∞, they are the same, but in actual situations there are some errors because of the limitation of the samples number.

According to the theory that matrix can conduct eigenvalue decomposition, conduct eigenvalue decomposition to the array covariance matrix. First consider the ideal case, where noise doesn't exist:

$R_x = AR_SA^H$.

For uniform linear array (ULA), the matrix A is a Vandermond matrix which is defined by Formula (4), as long as:

$$\theta_i \neq \theta_j, \ \mathrm{i} \neq \mathrm{j},$$

Then, each column is independent. Hence, if Rs is non-singular matrix (Rank(Rs)=D, each signal source is independent), then:

$\mathrm{Rank}(AR_SA^H)=D$,

since $Rx=E[XX^H]$, so:

$R_x^H=R_x$,

i.e. R_x is a Hermitian matrix, whose eigenvalue is real. Because R_s is positive definite, AR_sA^H is semi-positive definite, it has positive eigenvalues D and zero eigenvalues M-D.

Next, consider there is noise

$R_x=AR_SA^H+\sigma^2$, …………………………………… (1)

Since $\sigma^2>0$, R_x is a full rank matrix, R_x has M positive real eigenvalues $\lambda_1,\lambda_2,\dots,\lambda_M$, respectively corresponding to the M eigenvectors $v_1,v_2,\dots,v_M$.

For Rx is a Hermitian matrix, each eigenvector is orthogonal.

i.e. $vi^Hvj=0$ $i \neq j$,

Only D eigenvalue is relevant to signals. They are equal to the sum of matrix AR_sA^H and every eigenvalue σ^2 respectively. That is to say, σ^2 is the smallest eigenvalue of R, which is M-D dimension. For the corresponding eigenvalue v_i, i=1,2,…, M, there is still D related to the signals.

In addition, M-D is related to the noise. In the next section, the nature of characteristic decomposition will be used to determine the source of DOA.

6.2.3 The principle and implementation of MUSIC algorithm:

Characterized by an array of covariance decomposition, the following conclusions can be drawn:

The eigenvalues of the matrix Rx are sorted in accordance with size, which is

$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M > 0$,Where larger eigenvalues D are corresponding to signal while M-D smaller eigenvalues are corresponding to noise.

The eigenvalues and eigenvectors which belong to matrix R_x are corresponding to signal and noise respectively. Therefore, the eigenvalue (eigenvector) of R_x to signal eigenvalue (eigenvector) and noise eigenvalue (eigenvector) can be divided.

Let λ_i be the i-th eigenvalues of the matrix R_x, v_i is eigenvector corresponding to λ_i, then:

$R_x v_i = \lambda_i v_i$,

let $\lambda_i = \sigma^2$ be the minimum of R_x

$R_x v_i = \sigma^2 v_i \quad i=D+1, D+2, \ldots, M$, …………………….. (2)

Placing (1) into (2) the following can be acquired,

$\sigma^2 v_i = (AR_S A^H + \sigma^2 I)\, v_i$,

Expand the right side and compare to the left, the following can be obtained

$AR_S A^H v_i = 0$.

Because $A^H A$ is D*D dimensionally full rank matrix and $(A^H A)^{-1}$exists,R_s^{-1} also exists. From the above, multiply $Rs^{-1}(A^H A)^{-1} A^H$on both sides at the same time, then:

$$R_s^{-1}(A^H A)^{-1} A^H AR_S A^H v_i = 0,$$

$$A^H v_i = 0 \quad i=D+1, D+2, \ldots, M.$$

The above equation indicates that the eigenvector corresponding to the noise eigenvalue (the noise eigenvector) v_i is perpendicular with the column vector of the matrix A. Each row of A is corresponding to the direction of a signal source. That is the starting point of using noise eigenvector to get the direction of signal source.

Using noise characteristic value as each column, construct a noise matrix E_n can be constructed:

$$E_n=[V_{D+1},V_{D+2},\ldots,V_M],$$

to define spatial spectrum $P_{mu}(\theta)$ can be defined:

$$P_{mu}(\theta)=\frac{1}{a^H(\theta)E_nE_n^Ha(\theta)}=\frac{1}{\|E_n^Ha(\theta)\|^2},$$

Where the denominator of the formula is an inner product of the signal vector and the noise matrix. When (θ) is orthogonal with each column of En, the value of this denominator is zero, but because of the existence of the noise, it is actually a minimum. $P_{mu}(\theta)$ has a peak. By this formula, make θ change and estimate the arrival angle by finding the peak.

The implementation steps of MUSIC algorithm are shown below.

Obtain the following estimation of the covariance matrix based on the N received signal vector:

$$R_x=\frac{1}{N}\sum_{i=1}^{N}X(i)X^H(i),$$

to eigenvalue decompose the covariance matrix above

$R_x= AR_sA^H+ \sigma^2 I.$

According to the order of eigenvalues, take eigenvalue and eigenvector which are equal to the number of signal D as signal part of space; take the rest, M-D eigenvalues and eigenvectors, as noise part of space. Get the noise matrix En:

$$A^H v_i = 0 \quad i = D+1, D+2, \ldots, M,$$

$$E_n = [V_{D+1}, V_{D+2}, \ldots, V_M],$$

vary θ; according to the formula

$$P_{mu}(\theta) = \frac{1}{a^H(\theta) E_n E_n^H a(\theta)}.$$

Calculate the spectrum function; then obtain the estimated value of DOA by searching the peak.

6.2.4 Simulation and Result for MUSIC Algorithm

Factors affecting DOA estimation results;

1. Array element

2.Array element spacing

3. No of snapshots (time domain=sample no; frequency domain=no of time sub segment)

4.SNR

5. Incident angle difference

6.2.5 The relationship between DOA estimation and the number of array elements

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm,incident angle 20 and 60 degree

2. Signal is not co-related

3.d=half of incident signal wavelength

4. Array element =10,50,100

5. Snapshot no =200

6.SNR=20dB

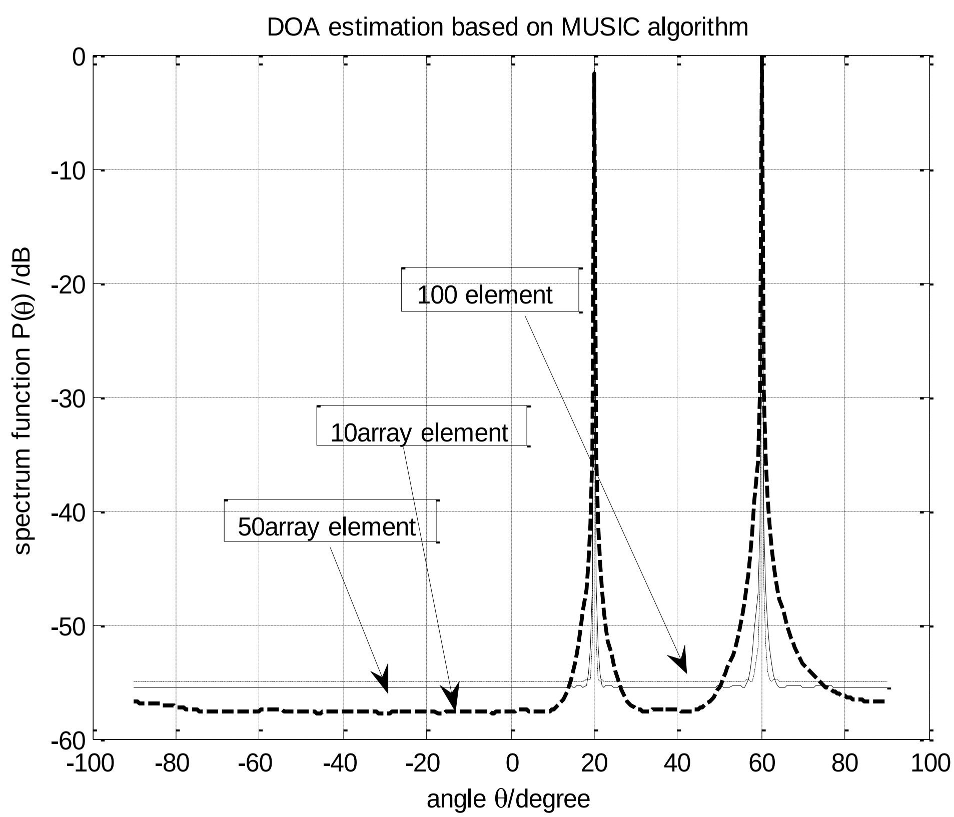


Fig 6.2.5: DOA estimation with the number of array elements

1. With the increase in the number of array elements, DOA estimation spectral beam width becomes narrow, the directivity of the array becomes good; that is, the ability to distinguish spatial signals is enhanced.

2. Got more accurate estimations of DOA, increase the number of array elements

3. But, the more the number of array elements the more the data needs processing; and the more amount of computation, the lower the speed.

4.From the above figure, when the number of the array elements amounts to 50 and 100, their beam width is very similar, so for high element is reasonable element no is used as it will not reduce the speed.

6.2.6 The relationship between DOA estimation and the array element spacing

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm, incident angle 20 and 60 degree

2. Signal is not co-related

3.d=λ/ 6 , λ/ 2

4. Array element =10

5. Snapshot no =200

6.SNR=20dB

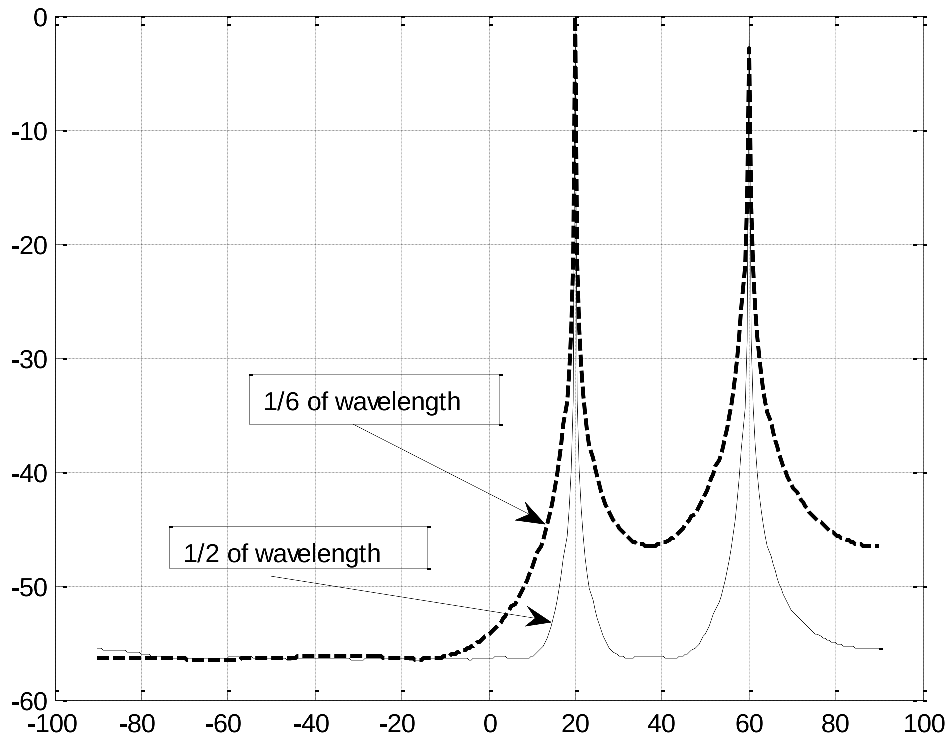


Fig 6.2.6: DOA estimation with Array element spacing

1. With increasing array element spacing, the beam width of DOA estimation spectrum becomes narrow, the direction of the array elements becomes good

2. The resolution of MUSIC algorithm improves with the increase in the spacing of array element

3. When the spacing of the array elements is larger than half the wavelength, the estimated spectrum, except for the signal source direction, shows false peaks, so it has lost the estimation accuracy.

6.2.7 The relationship between DOA estimation and the number of snapshots

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm ,incident angle 20 and 60 degree

2. Signal is not co-related

3.d=λ/ 2

4. Array element =10

5.Snapshotno =5,50,200

6.SNR=20dB

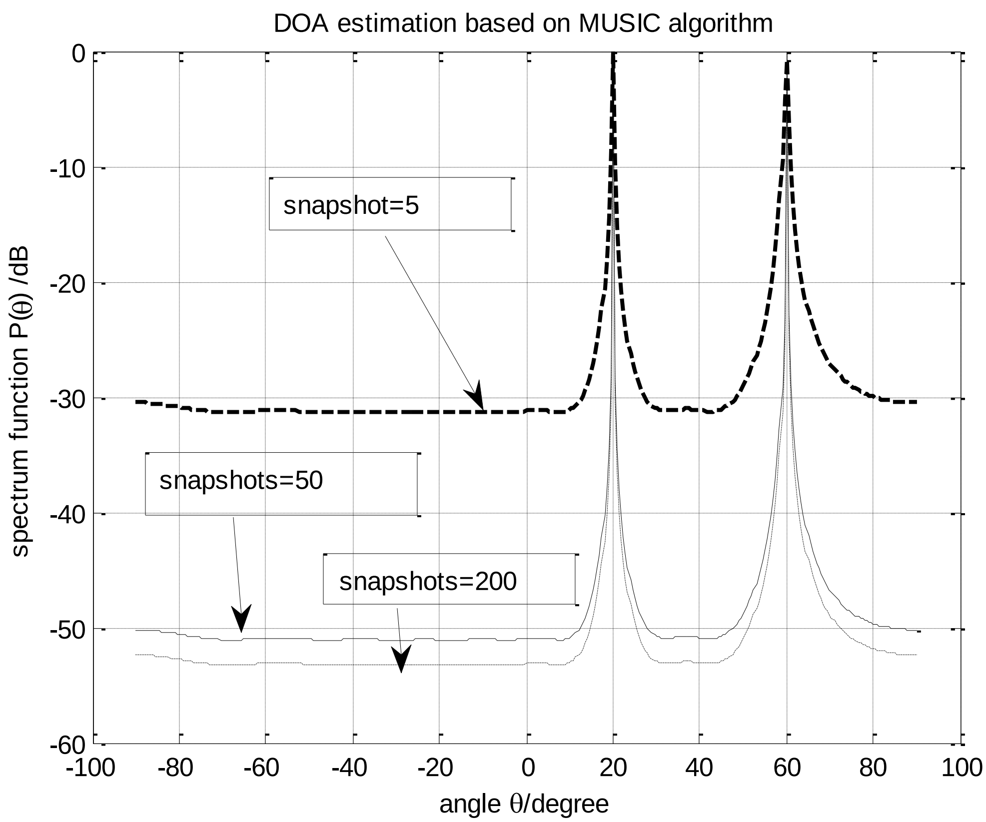


Fig 6.2.7: DOA estimation with no of snapshots

1. The increase in the number of snapshots, the beam width of DOA estimation spectrum becomes narrow, the direction of the array element becomes good and the accuracy of MUSIC algorithm is also increased

2. The number of sample snapshots can be expanded to multiply the accuracy of DOA estimation

3. But the more sample snapshots, the more the data needs to be processed; the more amount of calculation of MUSIC algorithm, the lower the speed

4. So in practical application, we select reasonable sampling snapshots which ensure the accuracy of DOA estimation, minimize the amount of computation and accelerating the speed of work and saving resources.

6.2.8 The relationship between DOA estimation and SNR

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm, incident angle 20 and 60 degree

2. Signal is not co-related

3. d=λ/ 2

4. Array element =10

5. Snapshot no =200

6. SNR=-20dB, 0dB, 20dB

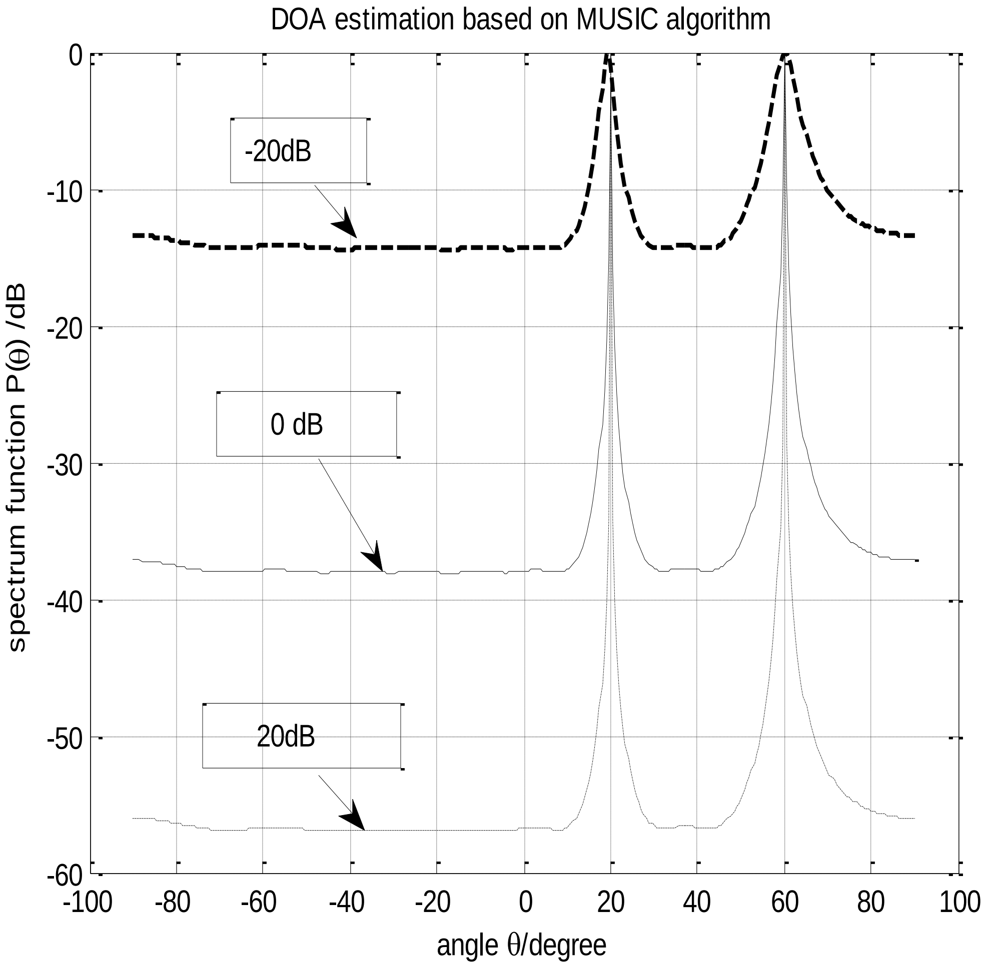


Fig 6.2.8: DOA estimation with SNR

1. With the other conditions remaining unchanged, with the increase in the number of SNR, the beam width of DOA estimation spectrum becomes narrow, the direction of the signal becomes clearer, and the accuracy of MUSIC algorithm is also increased

2. The value of SNR can affect the performance of high resolution DOA estimation algorithm directly

3. At low SNR, the performance of MUSIC algorithm will sharply decline

6.2.9The relationship between DOA estimation and the signal incident angle difference

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm, incident angle 5,10and 20 degree.

2. Signal is not co-related

3. d=λ/ 2

4. Array element =10

5. Snapshot no =200

6. SNR=20dB

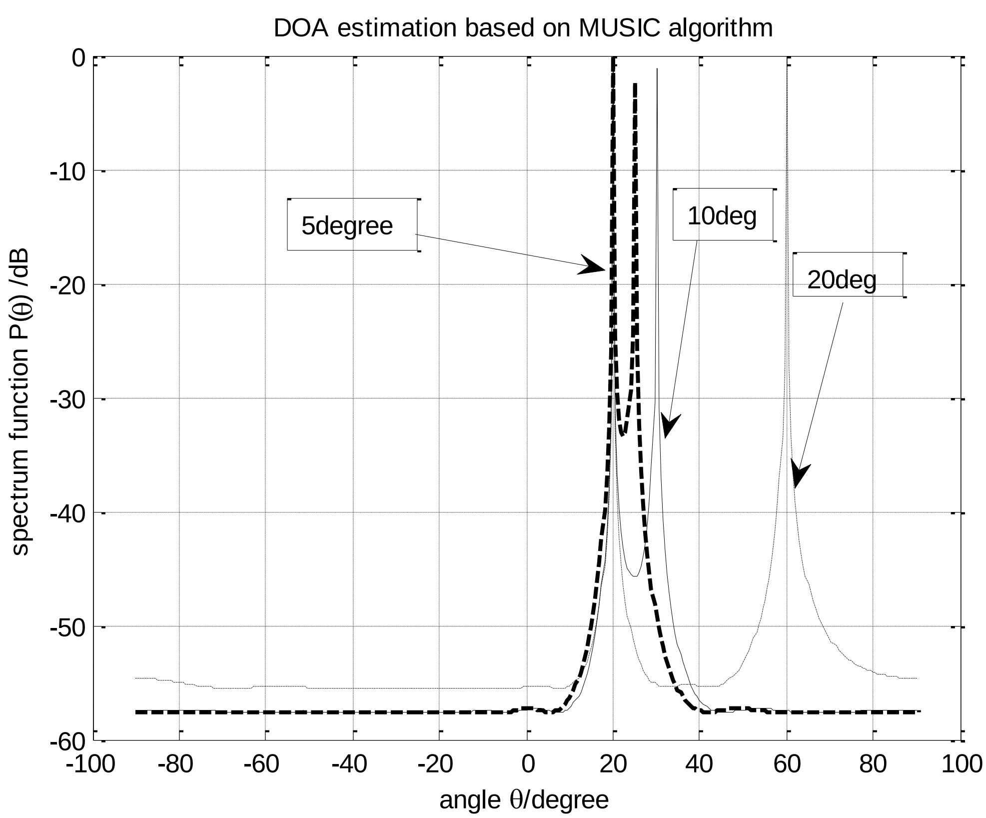


Fig 6.2.9: DOA estimation with AOA

1. With the other conditions remaining unchanged and with the increase in incidence angle difference, the beam width of the DOA estimation spectrum becomes narrow, the direction of the signal becomes clear and the resolution of MUSIC algorithm is also increased

2. When the signal wave angle space is very small, the algorithm cannot estimate the number of signal sources

3.The usual array signal source estimation method conducted under the condition of the incident angle being large, when the angle difference signal wave direction is relatively small, are estimated to be ineffective

6.2.10 The MUSIC algorithm considering best cases

PROBLEM STATEMENT

1. Two signal incoming recognized MUSIC algorithm ,incident angle 20 and 60 degree

2. Signal is coherent

3. d=λ/ 2

4. Array element =10

5. Snapshot no =200

6. SNR=20dB

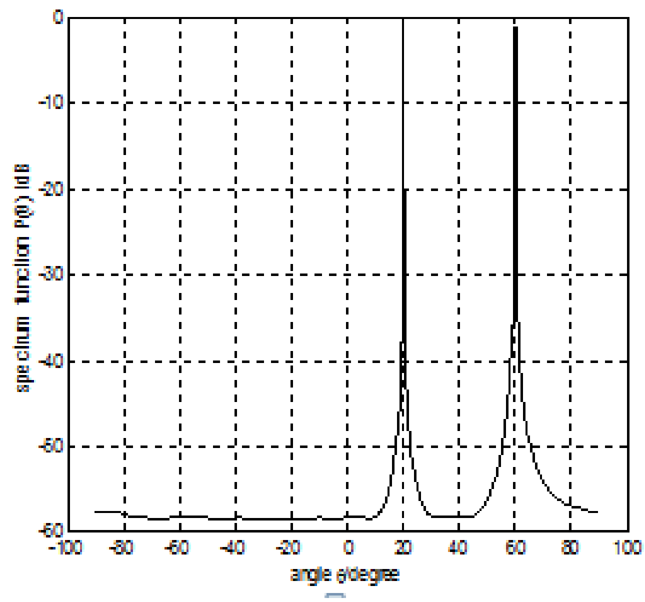


Fig 6.2.10: DOA estimation with MUSIC

1. For coherent signals, classic MUSIC algorithm has lost effectiveness, while improved MUSIC algorithm can be better applied to remove the signal correlation feature, which can distinguish the coherent signals, and estimate the angle of arrival more accurately

2. Under the right model, using MUSIC algorithm to estimate DOA can get any high resolution

3. But MUSIC algorithm only focuses on uncorrelated signals; when the signal source is correlation signal, the MUSIC algorithm estimation performance deteriorates or fails completely. This improved MUSIC algorithm can make DOA estimation more complete, and have a marked effect both on theoretical and practical study.

6.3 Esprit Algorithm

Two identical sub arrays are separated by a common displacement Δ of the signal subspace, induced by the translational invariance structure of the associated sensor array. This special structure allows the parameter estimates to be obtained without knowledge of the individual sensor responses and without computation or search of any spectral measure as in MUSIC. ESPRIT is a computationally efficient and robust method for estimating DOA.

Let us consider an array of 2M antenna elements that consists of two non-overlapping sub arrays of M elements each. The elements in each sub array have identical sensitivity pattern and are translationally separated by a known constant displacement vector Δ. The mth element of the first sub array and the corresponding element of the second sub array are displaced by the same displacement vector. Assume that there are N≤2Mnarrow band uncorrelated sources impinging on the array are planar and the sources assumed to bestationary zero-mean random processes additive noise is present in all 2Melements and is assumed to be weak stationary zero mean random process with a spatial covariance σ^2. We assume K samples are observed by the array. The signals received at the sub arrays can then be expressed as

$$\mathbf{X}_1(n) = \mathbf{A}\mathbf{S}(n) + \mathbf{W}_x(n)$$

$$\mathbf{X}_2(n) = \mathbf{A}\boldsymbol{\varphi}\mathbf{S}(n) + \mathbf{W}_y(n), \quad n = 1,2,\ldots\ldots K$$

Where $X_1(n)$ and $X_2(n)$ are 2M×K matrices of the outputs of the sub arrays, A is 2M×L array direction matrix, S(n) is L×K matrix of the source waveform and $W_x(n)$, Wy(n) is 2M×K matrices of elements noise. The matrix ϕ is a diagonal L×L matrix of the phase delays between sub array elements for the L signals.

$$\boldsymbol{\varphi} = \mathrm{diag}\left\{e^{j2\pi\Delta\sin\theta_1}, \ldots\ldots\ldots\ldots, e^{j2\pi\Delta\sin\theta_L}\right\}$$

The total array output vector is

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1(n) \\ \mathbf{X}_2(n) \end{bmatrix} = \mathbf{GS} + \mathbf{W}$$

Where $\mathbf{G} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\boldsymbol{\varphi} \end{bmatrix}$ $\mathbf{W} = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \end{bmatrix}$

The array output covariance matrix $\mathbf{R}_{XX}$ is

$$\mathbf{R}_{XX} = \mathbf{E}\left[\mathbf{XX}^H\right] = \mathbf{GPG}^H + \boldsymbol{\sigma}^2\mathbf{I}$$

$$\mathbf{P} = \mathbf{E}\left[\mathbf{S}(n)\mathbf{S}(n)^H\right]$$

is covariance matrix of the signal. The covariance matrix Rxx will have Nsignal eigen values , e_i, i=N+1,..........2M-N ordered as

The eigen vectors corresponding to the N largest eigen values are $\mathbf{E}_S = [e_1, e_2, \ldots\ldots e_N]$ and span the signal subspace and eigen vectors corresponding to the noise eigen values is $\mathbf{E}_N = [e_{N+1}, \ldots\ldots\ldots e_{2M}]$span the noise subspace. The invariance structure of the array implies $\mathbf{E}_S$ and it can be decomposed into $\mathbf{E}_{S1}$ and $\mathbf{E}_{S2}$.

$$e_1 \geq e_2 \geq e \ \geq \ldots\ldots\ldots\ldots\ldots e_N > e_{N+1} > \ldots\ldots\ldots e_M = \boldsymbol{\sigma}^2$$

$$\mathbf{E}_S = \begin{bmatrix} \mathbf{E}_{S1} \\ \mathbf{E}_{S2} \end{bmatrix}$$

By applying eigen decomposition on E_S then

$$\begin{bmatrix} \mathbf{E}_{S1}^H \\ \mathbf{E}_{S2}^H \end{bmatrix} \begin{bmatrix} \mathbf{E}_{S1} & \mathbf{E}_{S2} \end{bmatrix} = \mathbf{E} \Lambda \mathbf{E}^H$$

partition E into $N \times N$ sub matrices

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}$$

Eigen values of **E** are $\mathbf{e}_n(-\mathbf{E}_{12}\mathbf{E}_{22}^{-1})$, $n = 1,2,\ldots..N$

Then direction of arrival of the signals is

$$\theta_n = \sin^{-1}\left(\frac{-\mathbf{e}_n}{2\pi\Delta_n}\right)$$

Where $n = 1,2,\ldots..N$ is number of signals.

6.3.1 Simulation and Results

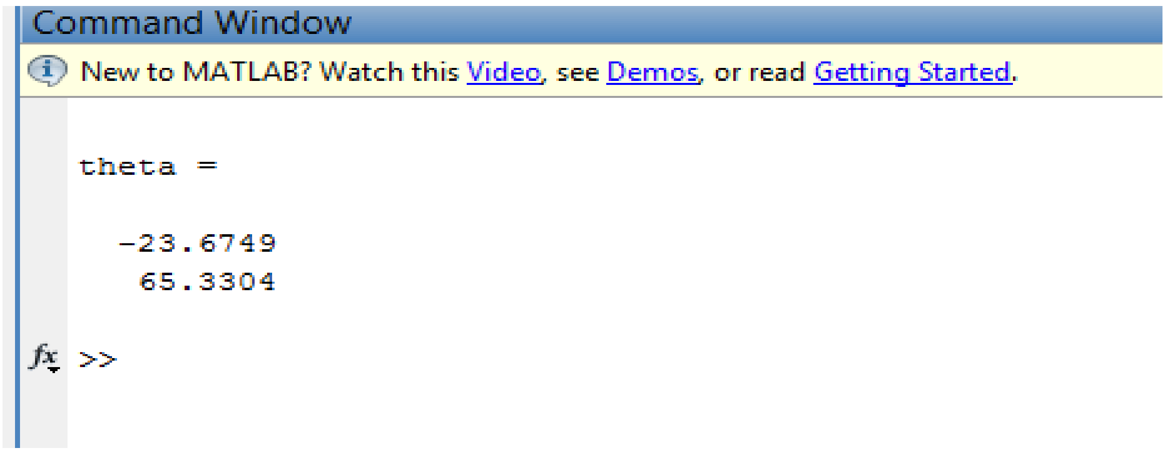
Problem Statement:

A uniform linear array (ULA) composed of 10 isotropic antennas. The array element spacing is 0.5 meters. Simulate the array output for two incident signals. Both signals are incident from 90° in azimuth. Their elevation angles are 73° and 68° respectively. In this example, we assume that these two directions are unknown and need to be estimated. Simulate the baseband received signal at the array demodulated from an operating frequency of 300 MHz.

Using Esprit algorithm, the sources are found at

Ang= 21.9988 16.9748

Generating random signal and finding AOA using this algorithm:



6.3.2 Comparison with MUSIC

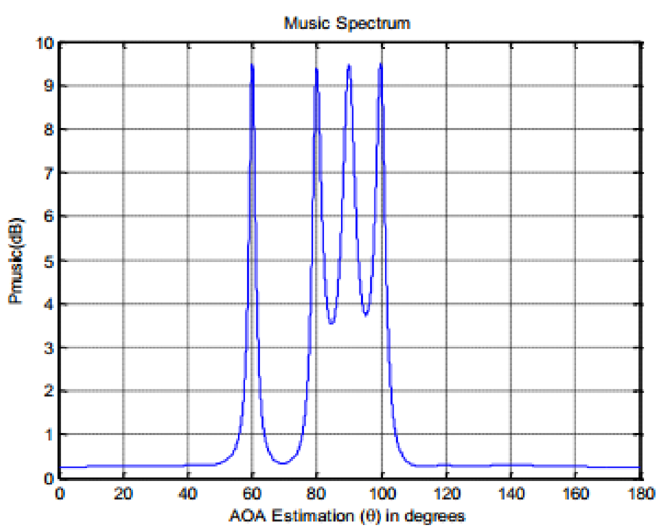


Fig 6.3.2(a): MUSIC spectrum for 64 snapshots, SNR=20dB, 10 array elements

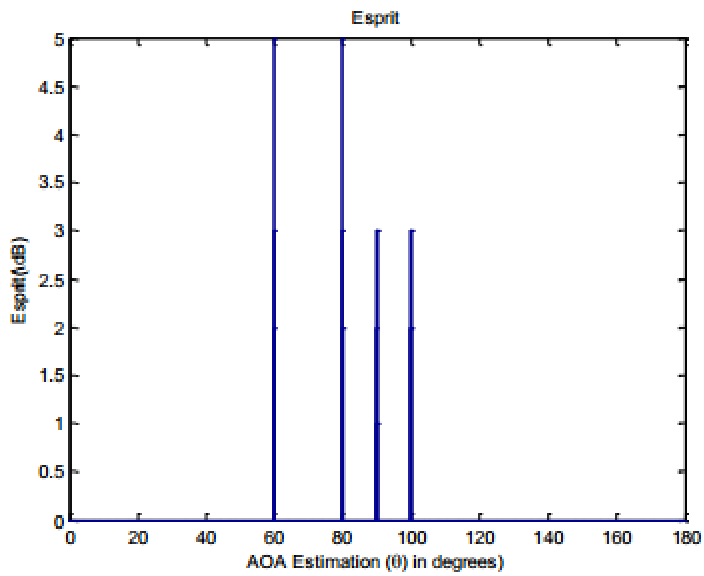


Fig 6.3.2(b): ESPRIT spectrum for 64 snapshots, SNR=20dB and 10array elements

The AOA estimation results by MUSIC and ESPRIT for snapshots = 1024, M=10, SNR=20dB (Table 1) and for snapshot =64, SNR=5dB, M=10 (Table 2) are given respectively

Table 1:

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
60	60.0000	60.0027
80	80.0000	80.0029
90	90.0000	89.9906
100	100.0000	99.9815

Table 2:

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
60	60.0002	59.0027
80	80.0000	80.0543
90	90.0000	90.0141
100	100.0000	100.0177

In MUSIC & ESPRIT more antennas results in a higher spatial resolution, less antennas will results in the reduction of the spatial resolution and using less snapshots the signals are more correlated and increased snapshots results to MUSIC spectrum more accurate detection and better resolution than low snapshots. If the signals are in low level of correlation it will result in high peaks in the spectrum and in high level of correlation will result in small peaks in the spectrum. We see that the MUSIC is more accurate and high resolution than ESPRIT.

1. ESPRIT algorithm reduces computation and storage costs presented by that of MUSIC algorithm.

2. ESPRIT algorithm has reduced sensitivity to array perturbations compared to MUSIC.

3. But ESPRIT is not so accurate compared to MUSIC algorithm.

From the above discussion we can conclude that ESPRIT algorithm has only the advantage of reduced computational complexities over MUSIC algorithm. As in communication accuracy is our main concern, Music algorithm is preferable.

References:

- Constantine A.Balanis, 'Antenna Theory Analysis and Design'
- Smart antenna using matlab by Lal Chand Godara
- Doa estimation based on music algorithm by Honghao Tang, Sven Nordebo
- Harris, F. J. "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform." Proceedings of the IEEE. Vol. 66 (January 1978)
- Adaptive Beamforming Algorithm for Smart Antenna Systems by AMARA PRAKASA RAO and N.V.S.N. SARMA
- Performance analysis of adaptive beamforming algorithms for smart antennas by Prerna Saxena & A.G. Kothari
- S. Kamboj and R. Dahiya, "Adaptive antenna array for satellite communication systems," Proceedings of the International Multiconference of Engineers and Computer Scientists, vol. 2, March 2008,Hong Kong.
- Peter Stonica etc. MUSIC Maximum Likelihood and Cramer-Rao Bond, IEEE Trans on ASSP. 1989. Vol. 37(5). P720~741.
- T J Shan, Wax M. Adaptive beamforming for Coherent Signals and Inference. IEEE Trans on ASSP. 1985. Vol. 33(4). P527~536
- Richard Roy, Thonas Kailath. ESPRIT-Estimation of Signal Parameters via Rotational Invariance Techniques. IEEE Trans on Acoustics Speech and Signal Processing. July 1989. Vol. 37. No.7 pp 984~995