BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

EXPONENTIAL TRANSFOMATION AND MODIFIED NTH ROOT EXPONENTIAL TRANSFORMATION FOR PEAK TO AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEM

By

Raihan Uddin Ahmed (122423)

Mahir Asef Chowdhury (122413)

Mohammad Solaiman (122411)

Department of Electrical and Electronic Engineering

Islamic University of Technology (IUT)

Organization of Islamic Cooperation (OIC)

Boardbazar, Gazipur-1704, Bangladesh

November – 2016

A Dissertation
In Partial Fulfillment of the Requirement for the Degree of
Bachelor of Science in Electrical and Electronic Engineering
Academic Year: 2015-2016

EXPONENTIAL TRANSFOMATION AND MODIFIED NTH ROOT EXPONENTIAL TRANSFORMATION FOR PEAK TO AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEM

Submitted By

Raihan Uddin Ahmed (122423)
Mahir Asef Chowdhury (122413)
Mohammad Solaiman (122411)

Approved By

Dr. Mohammad Rakibul Islam
Professor
Department of Electrical and Electronic Engineering
IUT, Gazipur, Bangladesh

DECLARATION OF CANDIDATE

We hereby declare that the work reported in the BSc. Thesis entitled "**EXPONENTIAL TRANSFOMATION AND MODIFIED NTH ROOT EXPONENTIAL TRANSFORMATION FOR PEAK TO AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEM"** submitted at **Islamic University of Technology (IUT)**, Boardbazar, Gazipur, Bangladesh, is an authentic record of our work carried out under the supervision of **Prof. Dr. Mohammad Rakibul Islam**. We have not submitted this work elsewhere for any degree or diploma. We are fully responsible for the contents of our BSc thesis

____

Raihan Uddin Ahmed
Student Number: 122423
Academic Year: 2015-2016

____

Mahir Asef Chowdhury
Student Number: 122413
Academic Year: 2015-2016

____

Mohammad Solaiman
Student Number: 122411
Academic Year: 2015-2016

CERTIFICATE OF APPROVAL

The thesis titled "**EXPONENTIAL TRANSFOMATION AND MODIFIED NTH ROOT EXPONENTIAL TRANSFORMATION FOR PEAK TO AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEM",** submitted by **Raihan Uddin Ahmed (122423), Mahir Asef Chowdhury (122413), Mohammad Solaiman (122411)** of Academic Year: 2015-2016 has been found as satisfactory and accepted as partial fulfillment of the requirement for the degree of Bachelor of Science in Electrical and Electronic Engineering on 10 November, 2016.

Dr. Mohammad Rakibul Islam
(Supervisor)
Professor
Department of Electrical and Electronic Engineering
IUT, Gazipur-1704, Bangladesh

Dr. Md. Ashraful Hoque
Professor & Head
Department of Electrical and Electronic Engineering
IUT, Gazipur-1704, Bangladesh

TO OUR PARENTS & TEACHERS

Table of Contents

DECLARATION OF CANDIDATE 3
CERTIFICATE OF APPROVAL 4
LIST OF ACCRONYMS AND ABBREVIATION 8
LIST OF FIGURES 12
LIST OF TABLES 14
ACKNOWLEDGEMENTS 15
ABSTRACT 16
CHAPTER 1 17
INTRODUCTION 17
1.1 Third Generation Wireless Systems 17
1.2 Fourth generation Wireless Systems 18
1.3 Background to OFDM 19
1.4 Problem Definition 20
1.5 Goal and Scope of the Thesis 21
1.6 Thesis Layout 22
CHAPTER 2 23
FUNDAMENTALS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING 23
2.1 Orthogonality 24
2.2 Frequency Domain Orthogonality 25
2.3 OFDM Generation and Reception 26
2.4 Guard Interval for Eliminating Inter Symbol Interference 27
2.5 Cyclic Prefix 28
2.6 Idealized System Models 30
Chapter 3 32
PEAK TO AVERAGE POWER RATIO 32
3.1 Distribution of PAPR 33
3.2 Complementary Cumulative Distribution Function (CCDF) of PAPR 34
3.3 Bit Error Rate 35
3.4 Current State of The Art PAPR reduction technique 37
3.4.1 Clipping and Filtering 37
3.4.2 Partial Transmit Sequence 39
3.4.3 Selective Mapping 40

3.4.4 Active Constellation Extension 41
CHAPTER 4 43
EXPONENTIAL TRANSFORMATION TECHNIQUE 43
4.1 Mathematical Representation 43
4.2 Block Diagram 44
4.3 Modification at the Receiver 45
4.4 Simulation Results 45
CHAPTER 5 48
Nth ROOT EXPONENTIAL TRANSFORM 48
5.1 Mathematical Representation 48
5.2 Block Diagram 49
5.3 Simulation Results 50
CHAPTER 6 53
CONCLUSION 53
Future Works 54
REFERENCES 55

LIST OF ACCRONYMS AND ABBREVIATION

Abbreviation	Definition
3GPP	3rd Generation Partnership Product
4G	4th Generation
A/D	Analog to Digital Conversion
ACE	Active Constellation Extension
ACI	Adjacent Channel Interference
ADC	Analog to Digital Converter
ADSL	Asymmetric Digital Subscriber Line
AFC	Adaptive Frequency Correction
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CCDF	Complementary Cumulative Distribution Function
CCM	Concentric Circle Constellation Mapping
CDF	Cumulative Distribution Function
CF	Crest Factor
CFO	Carrier Frequency Offset
CIR	Carrier to Interference Ratio
CP	Cyclic Prefix
D/A	Digital to Analog Conversion
DAB	Digital Audio Broadcast

DAC	Digital to Analog Converter
DFT	Discrete Fourier Transform
DQPSK	Differential Quadrature Phase Shift Keying
DVB	Digital Video Broadcast
EC	Exponential Companding
ETSI	European Telecommunications Standard Institute
FCC	Federal Communications Commissions
FDM	Frequency Division Multiplexing
G.P.	Geometric Progression
HDTV	High Definition Television
HiperLAN	High Performance Radio LAN
HPA	High Power Amplifier
IBI	Inter Block Interference
ICI	Inter Carrier Interference
IDFT	Inverse Discrete Fourier Transform
IEEE	Institute of Electrical and Electronic Engineers
LST	Linear Symmetrical Transform
MATLAB	Matrix Laboratory
MBWA	Mobile Broad Band Wireless Access
MCM	Multicarrier Modulation
MIMO	Multi Input Multi Output
NLST	Non-linear Symmetrical Transform

OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiplexing Access
OOB	Out of Band
P/S	Parallel to Serial
PAPRR	Peak to Average Power Ratio
PDF	Probability Distribution Function
PHY	Physical
POCS	Projection onto Convex Sets
PRT	PAPRR Reduction Tones
PSD	Power Spectral Density
PSK	Phase Shift Keying
PTS	Partial Transmit Sequence
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
S/P	Serial to Parallel
SER	Symbol Error Rate
SGP	Smart Gradient Project
SI	Side Information
SLM	Selective Mapping
SNR	Signal to Noise Ratio
SUI	Stanford University Interim

TC	Trapezoidal Comapanding
TDBC	Trapezium Distribution based Companding
TI	Tone Injection
TR	Tone Rejection
VHDSL	Very High Speed Digital Subscriber Line
VLSI	Very Large Scale Integration
WiMAX	Worldwide Mobility for Wireless Access
WLAN	Wireless Local Area Network

LIST OF FIGURES

Figure Number	Caption	Page Number
1-1	Mobility vs. Data Rate for different Generations of Telecommunication Technology	11
2-1	Frequency response of the subcarriers in a 5 tone OFDM signal.	18
2-2	Basic Block Diagram of OFDM Transceiver	19
2-3	Cyclic Prefix	22
2-4	OFDM Idealized Transmitter	23
2-5	OFDM Idealized Receiver	24
3-1	Square root of PAPR ratio for a 16-channel OFDM signal, modulated with the same initial phase for all sub carriers.	25
3-2	BER for BPSK, QPSK, 8-QAM, 16-QAM, 32-QAM, 64-QAM	28
3-3	BER for 128-QAM, 256-QAM, 512-QAM, 1024-QAM, 2048-QAM, 4096-QAM	29
3-4	BER for 8-PSK, 16-PSK, 32-PSK, 64-PSK, 128-PSK, 256-PSK	29
3-5	Spectrum of OFDM signal with clipping distortion.	31
3-6	Block Diagram of PTS	33
3-7	Block Diagram of SLM	34
3-8	ACE Constellation Diagram	35
4-1	Block Diagram Representation of Exponential Transform	38

4-2	CCDF vs PAPR Plot for 64, 128 and 256 subcarriers using Exponential Transform.	40
4-3	Comparison of PAPR Reduction between Existing and Proposed Exponential Transform Method	41
5-1	Block Diagram of Nth root Exponential Transform	43
5-2	CCDF vs. PAPR Plot for Nth Root Exponential Transform	45
5-3	Comparison between the two proposed methods	45
5-4	Comparison between the Modified exponential and existing methods	46
5-5	Representation of the Output Using Nth Root Exponential	46

LIST OF TABLES

Table No.	Description
4-1	Simulation Parameters for Exponential Transform
4-2	Numeric Reduction Result for Exponential Transform
5-1	Simulation Parameters for Nth Root Exponential Transform
5-2	Numeric Reduction Result for Nth Root Exponential Transform

ACKNOWLEDGEMENTS

First and foremost, we would like to express our cordial gratitude to our parents who have supported us and always believed in us.

We would like to convey our sincerest appreciation to our supervisor Dr. Mohammad Rakibul Islam, Professor in Electrical and Electronic Engineering Department of Islamic University of Technology (IUT), Dhaka, Bangladesh. His guidance, support and encouragement paved the way towards our achievement. We learnt from the knowledgeable discussions and regular sessions we had with him. Through constructive criticism and advice he guided us. It was fruitful to learn from the suggested theoretical background and other research works he proposed. Forever we shall remain grateful to him.

We would also like to express our gratitude towards Prof. Dr. Md. Ashraful Hoque, Head of the Department of Electrical and Electronic Engineering, Islamic University of Technology, Dhaka, Bangladesh. His welcoming and favorable approach has enabled us to develop our potential.

Last but not the least, we would like to thank and congratulate our fellow classmates for the cherished and precious moments that we shared together.

ABSTRACT

Due to the High data rate requirement of modern cellular network, multicarrier modulation (MCM) is now becoming more and more mandatory for Wireless Networks. Orthogonal Frequency Division Multiplexing (OFDM) has proved to a promising MCM technology for application requiring High Data Rate. For instance, in the case of 3GPP and 4G cellular networks, OFDM is used for its huge spectral efficiency and zero Inter Symbol Interference (ISI). However OFDM has a major drawback in terms of Peak to Average Power Ratio (PAPRR) and Inter Carrier Interference (ICI). The resulting peaks from superposition sum, creates high peaks in the signal, which drives the Amplifier saturation region. The saturation region has non-linear characteristics which creates Out of Band (OOB) radiations that are filtered resulting in a data loss. To restrict the amplifier from driving in to the saturation region, high Power Amplifiers (HPAs) must be used to keep the signal In Bound. Thus the power dissipation increases and the battery drains out quickly. Unless the peak power problem of OFDM is not addressed, then the power inefficiency of the system surpasses the advantage of Higher Data Rate. Researches, over the years have proposed versatile methods for reducing the PAPRR of OFDM signals. The methods are categorized using numerous parameters such as computational complexity, necessity of Side Information (SI), noise performance, implementation process and so on. Performance of each method is affected by the parameters considered for the transmission. Because of this, the generality of PAPRR Reduction implementation cannot be defined. Some of the methods involve the scrambling of signal like Partial Transmit Sequence (PTS), Selective Mapping (SLM) and Active Constellation Extension (ACE), while others employ signal distortion technique as Iterative Clipping and Filtering, Tone Injection (TI) and Tone Rejection (TR). In this Thesis PAPRer, we have explored some non-conventional methods for the purpose of PAPRR reduction. Our first proposal involves an exponential conversion the time domain signal in order to achieve lower PAPRR values. The method introduces no further need of SD and any change of phase or frequency whatsoever. The computation of the exponential equivalent is simple and requires less numbers of complex addition and multiplication resulting in low complexity. Next proposed method is a modification of the exponential conversion using the signal mean square average to scale the transformed signal. Combining this modified proposal with n times root, we get even higher PAPRR reduction. The reduction performance is illustrated as Complementary Cumulative Distribution Function (CCDF) vs PAPRR plot using MATLAB. The advantages of the proposed models over the current state of the art technologies are demonstrated by tabulating the reduction results achieved for various methods. Though the better reduction result is achieved, there exists a tradeoff between the PAPRR and Bit Error Rate (BER). The optimization of BER is excluded from the capacity of this Paper, though a representation is drawn. Mathematical expression and Simulation results of the proposals outperform the result of both PTS and SLM.

CHAPTER 1

INTRODUCTION

Wireless communications is an emerging field, which has seen enormous growth in the last several years. The huge uptake rate of mobile phone technology, Wireless Local Area Networks (WLAN) and the exponential growth of the Internet have resulted in an increased demand for new methods of obtaining high capacity wireless networks. The huge growth of cellular industry, exponential growth of internet connectivity and expansion of WLANs have necessitated the development of high capacity wireless network. Traditional single carrier modulation scheme is no longer capable of delivering such high performance. As a result Multi Carrier Modulation (MCM) technologies are being developed. Higher data rate requirement through a single carrier is achievable though there would be a cost of lower symbol rate. Therefore the communication system using single carrier suffers severely from Inter Symbol Interference (ISI) caused by dispersive-channel impulse response and thereby need a complex equalization scheme. Orthogonal Frequency Division Multiplexing (OFDM) is an MCM technology which has the potential of being the replacement of single carrier systems that require higher data rate.

1.1 Third Generation Wireless Systems

Third generation mobile systems such as the Universal Mobile Telecommunications System (UMTS) [1], [2], [3], [4] and CDMA2000 has been introduced over the years (2002 onwards) [5]. These systems are striving to provide higher data rates than current 2G systems such as the Global System for Mobile communications (GSM) [6], [7] and IS-95. Second generation systems are mainly targeted at providing voice services, while 3rd generation systems has shifted to more data oriented services such as Internet access.

Most of the third generation systems used Wideband Code Multiplexing Access (WCDMA). This modulation scheme has a high multipath tolerance, flexible data rate, and allows a greater cellular spectral efficiency than 2G systems. Third generation systems provide a significantly higher data rate compared to the 2nd Generation Systems. The higher data rate 3G system supports a wide variety of Internet based application and as such gained popularity quickly. The demand for use of the radio spectrum is very high, with terrestrial mobile phone systems being just one of many applications vying for suitable bandwidth. These applications require the system to operate reliably in non-line of-sight environments with a propagation distance of 0.5 - 30 km, and at velocities up to 100 kmph or higher. This operating environment limits the maximum RF frequency to 5 GHz, as operating above this frequency results in excessive channel path loss, and excessive Doppler Shift at high velocity making the radio bandwidth more costly.

In order to develop more cost efficient design along with higher data rate availability fourth generation systems has been introduced. The perspective for developing such systems was fueled by the need of the ever growing communication technology and the huge economic benefits that comes along.

1.2 Fourth generation Wireless Systems

Incorporating the benefits of a 3G system, 4^{th} generation (4G) systems were designed to provide maximum realizable data rete along with minimal path loss and restrictions. Single carrier based modulation schemes were no longer capable of providing such a system. That is why researches adapted to MCMs. To develop an effective and efficient MCM technology many different aspects of signal was tested for modification. A special form of such MCM technology is OFDM, where a single carrier of frequency selective fading channel is divided into many orthogonal narrow band flat fading sub channel, capable of transmitting parallel streams of higher data rates. This is turn reduces the ISI appreciably.

A significant improvement in the spectral efficiency is compulsory for high data rate applications. This will only be achieved by significant advances in multiple aspects of cellular network systems, such as network structure, network management, smart antennas, RF modulation, user allocation, and general resource allocation.

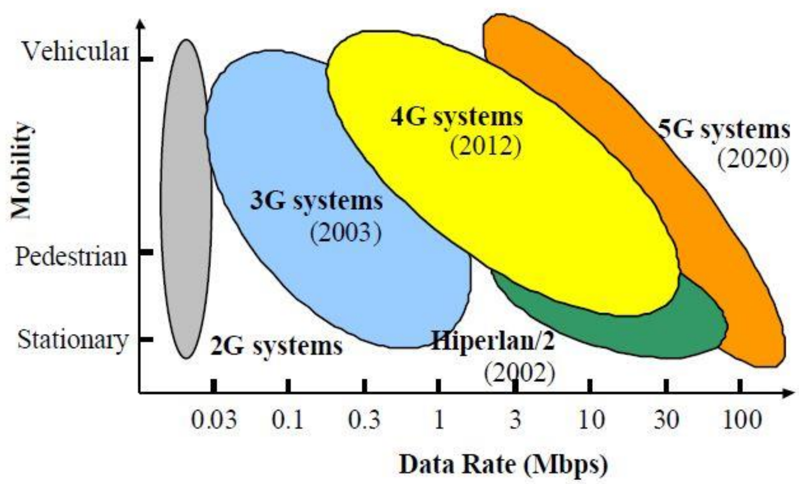


Figure 1-1 Mobility vs. Data Rate for different Generations of Telecommunication Technology

OFDM is capable of supporting all these requirements of 4G and higher generation communication technologies. Thus rigorous studies and numerous experiments are conducted to improve this frequency division multiplexing method.

1.3 Background to OFDM

The fundamental principal of OFDM originated from the PAPRer of Chang [7], and over the years a number of researchers have evolved this technique. Initially, its use was restricted to Military systems such as KINEPLEX, KATHRYN and ANEFT [8]. Wienstein and Ebert's proposal to use Discrete Time Fourier Transform (DFT) to perform subcarrier modulation using a single oscilloscope was a pioneering effort. Ebert, Salz and Schwartz demonstrated the efficacy of Cooley-Tukey Fast Fourier Transform (FFT) algorithm to further simplify the computational complexity of DFT, thereby making it possible to utilize OFDM in commercial Communication Systems. The use of OFDM in commercial purpose originated a series of wire-line standards such as, High bit rate Digital Subscriber Line (HDSL) [9], Asymmetric Digital Subscriber Line (ADSL) [10] and so on. The use of OFDM enabled these systems to perform at a bit rate of 100Mbps. Afterwards, it was used in DAB [11], WLAN [12], DVB [13, 14] and WMAN.

OFDM uses cyclic prefix insertion to eliminate the effect of ISI and require a simple one-tap equalizer at the receiving end. OFDM brings about an unparalleled bandwidth saving, resulting into higher spectral efficiency. Properties like these makes OFDM highly attractive for Higher data Rate applications. It is possible to use different modulation scheme for the subcarrier channels to improve the efficiency of symbol generation. This is suitable for fading channels of highly dispersive nature.

Despite the many advantages introduced by OFDM it has some drawbacks too.

- OFDM signals with their high peak to average power ratio (PAPRR) requires highly linear amplifiers.
- In absence of amplifier linearity Out of Band (OBO) power requirement will occur.
- OFDM systems are more sensitive to Doppler Spread than single carrier system.
- Phase noise, originated by the imperfect transmitter and receiver synchronization degrades the performance. Thus an accurate frequency and time sync in mandatory.
- Also addition of Cyclic Prefix (CP) results in loss of spectral efficiency.

Scarcity and higher price of the available bandwidth have driven the engineers to develop more and more efficient methods for communications maintaining the ever increasing need of high data rate created by increased number of users. OFDM with its spectral efficient versions like MIMO-OFDM or multiple access versions like OFDMA has proved to be a promising candidate for fulfilling the requirements of present and future.

Large envelope fluctuation in OFDM signal is one of the major drawbacks of OFDM technique. Such fluctuations create difficulties, because practical communication systems are peak-power limited. Envelope peaks require the system to accommodate an instantaneous signal power that is much larger than the signal average power, necessitating either low operating power efficiencies or Power Amplifier (PA) saturation.

In addition to this, OFDM system requires tight system synchronization, in comparison to single carrier systems, due to narrowband subcarriers. Therefore, it is sensitive to a small frequency offset between the transmitted and the received signal. The frequency offset may arise due to Doppler Effect or due to mismatch between transmitter and the receiver local oscillator frequencies. The Carrier Frequency Offset (CFO) disturbs the orthogonality between the subcarriers, and therefore, the signal on any particular subcarrier will not remain independent of the remaining subcarriers. This phenomenon is known as Inter-Carrier Interference (ICI), which is a big challenge for error-free demodulation and detection of OFDM symbols.

The current implementation of OFDM does not fully exploit its capabilities. There are still several avenues which can be explored to reduce the Peak-to-Average Power Ratio (PAPRR) and Inter-Carrier Interference (ICI) of OFDM signal. Therefore, the necessity to reduce the PAPRR of standard OFDM signal is a prime motivating factor for this work.
Clipping and filtering [15] is the simplest solution to reduce the PAPRR but it achieves the PAPRR reduction capability at the cost of BER performance degradation. Existing companding transforms [16]-[20] achieve acceptable PAPRR reduction capability but they introduce high non-linear distortion, due to which their BER performances degrade. Moreover, both the schemes result in out-of-band radiation and in-band distortion. Improving BER performance using companding based PAPRR reduction schemes is a prime motivating future for our work.

The distortion-less schemes proposed in literature [21-25] provide PAPRR reduction capability at the cost of increased complexity. In most of these techniques like Selected Mapping (SLM) [26] and Partial Transmit Sequence (PTS) [27], Side Information (SI) is required at the receiver to recover the original data signal, which not only causes data-rate loss but also results in severe BER performance degradation, if SI gets corrupted during transmission. In order to avoid spectral deficiency, various SI embedding schemes have been proposed [28-31], but in most of them the SI detection capability depends on the available signal-to-noise ratio. Elimination of SI requirement at the receiving end motivates a major part of this work.

ICI cancellation schemes effectively mitigate the effect of ICI but PAPRR performance of existing ICI cancellation schemes is either same or worse than that of standard OFDM signal. Therefore, there is a necessity to reduce the PAPRR of OFDM signal obtained from ICI cancellation schemes. Hence the need for a joint ICI cancellation and PAPRR reduction scheme is another motivating factor for this week.

1.4 Problem Definition

High PAPRR and Inter-Carrie-Interference (ICI) are the two major issues in the implementation of an OFDM system. The thesis aims at exploring and arriving at efficient, low complexity schemes for PAPRR reduction in OFDM based of practical use. The first problem addressed in this thesis is the high PAPRR of an OFDM signal. We begin by exploring the existing PAPRR reduction techniques and to find out their advantages and major limitations for implementing a practical OFDM system. Investigation of efficient PAPRR reduction schemes for an OFDM system is thus considered as one of the problem areas explored in this thesis.

The OFDM based multiple access schemes have been adopted for downlink communication in 3GPP LTE standards, but not chosen in uplink communication due to high PAPRR. In uplink communication, when a mobile terminal has to transmit an OFDM signal, it requires an expensive and bulky high power amplifier, which is difficult to accommodate, due to its cost and size limitation. The mobile terminal is also having a limited computational power and battery life. Therefore, the proposed quadrilateral companding transform can be utilized in uplink communication because the proposed quadrilateral companding schemes provide good PAPRR reduction capability with low computational complexity. The quadrilateral companding transform requires only one additional block at transmitter and receiver to perform companding and expanding operations respectively.

Being a multicarrier modulation scheme, OFDM brings all major benefits of a multicarrier scheme but unlike single carrier modulation schemes, it suffers from the problem of ICI. In this thesis, we explore the existing ICI cancellation schemes and perform a comparison of CIR and BER performance. As discussed above, the PAPRR is an important parameter that must be taken into consideration while designing an ICI cancellation scheme for the OFDM system of practical use. Therefore, investigation of PAPRR performance of OFDM systems utilizing ICI cancellation schemes is also considered as another area to be explores in this thesis.

1.5 Goal and Scope of the Thesis

The goals met by the thesis are listed as follows:

- Investigation of companding transforms for PAPRR reduction scheme and development of a generalized companding transform with good PAPRR
- Providing a simple computation based robust technique to reduce the PAPRR and investigate the BER
- Exploring new method of exponential conversion of the original signal to ensure the ruction of PAPRR
- Exploiting the exponential companding method of PAPRR reduction to restrain the signal average from abrupt modification
- Investigation combined effect of exponential and Nth rooting method in order to achieve the goal of PAPRR reduction.

1.6 Thesis Layout

The thesis is organized into six chapters:

- **Chapter 1** includes the introduction, evolution of communication technology, historical background of OFDM, definition of the problem, goals and scope of this PAPRer.
- **Chapter 2** this chapter includes a detailed discussion of OFDM systems, its construction, process of signal generation, advantages and disadvantages of it. The chapter also contains theoretical analysis of the OFDM system and illustrates the block diagram of the system.
- **Chapter 3** This chapter elaborates the problem of high PAPRR and includes state of the art reduction technique such as PTS, SLM, ACE, Exponential Companding etc.
- **Chapter 4** covers the first method proposed by the thesis, its mathematical expression and block diagram. Also the modification in the demodulation process is discussed in details with pictorial representation.
- **Chapter 5** introduces a modified exponential transformation method where the transformed values are scaled to their mean square average values in order to maintain the Power Spectral Density (PSD) of the actual signal.
- **Chapter 6** includes the proposal for future works and avenues of developments.

CHAPTER 2

FUNDAMENTALS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

Orthogonal Frequency Division Multiplexing (OFDM) is very similar to the well-known and used technique of Frequency Division Multiplexing (FDM). OFDM uses the principles of FDM to allow multiple messages to be sent over a single radio channel. It is however in a much more controlled manner, allowing an improved spectral efficiency.

OFDM is different from FDM in several ways. In conventional broadcasting each radio station transmits on a different frequency, effectively using FDM to maintain a separation between the stations. There is however no coordination or synchronization between each of these stations. With an OFDM transmission such as DAB, the information signals from multiple stations is combined into a single multiplexed stream of data. This data is then transmitted using an OFDM ensemble that is made up from a dense packing of many subcarriers. All the subcarriers within the OFDM signal are time and frequency synchronized to each other, allowing the interference between subcarriers to be carefully controlled. These multiple subcarriers overlap in the frequency domain, but do not cause Inter-Carrier Interference (ICI) due to the orthogonal nature of the modulation. Typically with FDM the transmission signals need to have a large frequency guard-band between channels to prevent interference. This lowers the overall spectral efficiency. However with OFDM the orthogonal packing of the subcarriers greatly reduces this guard band, improving the spectral efficiency.

All wireless communication systems use a modulation scheme to map the information signal to a form that can be effectively transmitted over the communications channel. A wide range of modulation schemes has been developed, with the most suitable one, depending on whether the information signal is an analogue waveform or a digital signal. Some of the common analogue modulation schemes include Frequency Modulation (FM), Amplitude Modulation (AM), Phase Modulation (PM), Single Side Band (SSB), Vestigial Side Band (VSB), Double Side Band Suppressed Carrier (DSBSC) [16], [17]. Common single carrier modulation schemes for digital communications include Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK), Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) [16] - [18].

Each of the carriers in a FDM transmission can use an analogue or digital modulation scheme. There is no synchronization between the transmission and so one station could transmit using FM and another in digital using FSK. In a single OFDM transmission all the subcarriers are synchronized to each other, restricting the transmission to digital modulation schemes. OFDM is symbol based, and can be thought of as a large number of low bit rate carriers transmitting in parallel. All these carriers transmit in unison using synchronized time and frequency, forming a single block of spectrum. This is to ensure that the orthogonal nature of the structure is maintained. Since these multiple carriers form a single OFDM transmission, they are commonly referred to as 'subcarriers', with the term of 'carrier' reserved for describing the RF carrier mixing the signal from base band. There are several ways of looking at what make the subcarriers in an OFDM signal orthogonal and why this prevents interference between them.

2.1 Orthogonality

Signals are orthogonal if they are mutually independent of each other. Orthogonality is a property that allows multiple information signals to be transmitted perfectly over a common channel and detected, without interference. Loss of orthogonality results in blurring between these information signals and degradation in communications. Many common multiplexing schemes are inherently orthogonal. Time Division Multiplexing (TDM) allows transmission of multiple information signals over a single channel by assigning unique time slots to each separate information signal. During each time slot only the signal from a single source is transmitted preventing any interference between the multiple information sources. Because of this TDM is orthogonal in nature. In the frequency domain most FDM systems are orthogonal as each of the separate transmission signals are well spaced out in frequency preventing interference. Although these methods are orthogonal the term OFDM has been reserved for a special form of FDM. The subcarriers in an OFDM signal are spaced as close as is theoretically possible while maintain orthogonality between them.

Sets of functions are orthogonal to each other if they match the conditions in equation (2-1). If any two different functions within the set are multiplied, and integrated over a symbol period, the result is zero, for orthogonal functions. Another way of thinking of this is that if we look at a matched receiver for one of the orthogonal functions, a subcarrier in the case of OFDM, then the receiver will only see the result for that function. The results from all other functions in the set integrate to zero, and thus have no effect.

$$\int_0^T s_i(t)s_j(t)dt = \begin{cases} C\,; i = j \\ 0\,; i \neq j \end{cases} \qquad (2\text{-}1)$$

Equation (2-2) shows a set of orthogonal sinusoids, which represent the subcarriers for an un-modulated real OFDM signal.

$$s_k(t) = \begin{cases} \sin(2\pi f_0 kt)\,; 0 < t < T \; k = 1,2,3, \dots \dots, M \\ 0\,; otherwise \end{cases} \qquad (2\text{-}2)$$

where f_0 is the carrier spacing, M is the number of carriers, T is the symbol period. Since the highest frequency component is Mf_0 the transmission bandwidth is also Mf_0 .

These subcarriers are orthogonal to each other because when we multiply the waveforms of any two subcarriers and integrate over the symbol period the result is zero. Multiplying the two sine waves together is the same as mixing these subcarriers. This results in sum and difference frequency components, which will always be integer subcarrier frequencies, as the frequency of the two mixing subcarriers has integer number of cycles. Since the system is linear we can integrate the result by taking the integral of each frequency component separately then combining the results by adding the two sub-integrals. The two frequency components after the mixing have an integer number of cycles over the period and so the sub-integral of each component will be zero, as the integral of a sinusoid over an entire period is zero. Both the sub-integrals are zeros and so the resulting addition of the two will also be zero, thus we have established that the frequency components are orthogonal to each other.

2.2 Frequency Domain Orthogonality

Another way to view the orthogonality property of OFDM signals is to look at its spectrum. In the frequency domain each OFDM subcarrier has a sinc, *sin(x)/x,* frequency response, as shown in Figure 2-2. This is a result of the symbol time corresponding to the inverse of the carrier spacing. As far as the receiver is concerned each OFDM symbol transmitted for a fixed time (T_{FFT}) with no tapering at the ends of the symbol. This symbol time corresponds to the inverse of the subcarrier spacing of $1/T_{FFT}$ Hz 1. This rectangular, boxcar, waveform in the time domain results in a *sinc* frequency response in the frequency domain. The *sinc* shape has a narrow main lobe, with many side-lobes that decay slowly with the magnitude of the frequency difference away from the center. Each carrier has a peak at the center frequency and nulls evenly spaced with a frequency gap equal to the carrier spacing.

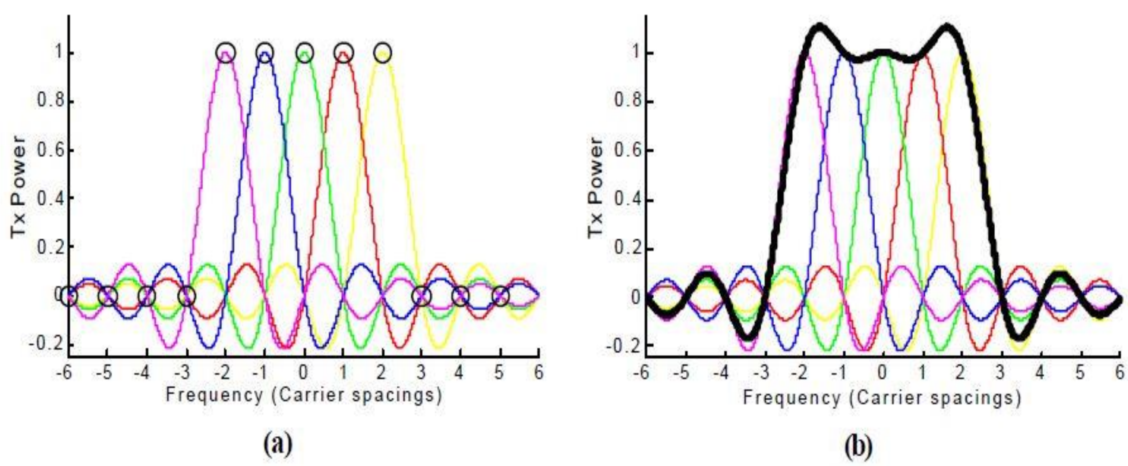


Figure 2-1 Frequency response of the subcarriers in a 5 tone OFDM signal.

2.3 OFDM Generation and Reception

OFDM signals are typically generated digitally due to the difficulty in creating large banks of phase lock oscillators and receivers in the analog domain. Figure 2-3 shows the block diagram of a typical OFDM transceiver. The transmitter section converts digital data to be transmitted, into a mapping of subcarrier amplitude and phase. It then transforms this spectral representation of the data into the time domain using an Inverse Discrete Fourier Transform (IDFT). The Inverse Fast Fourier Transform (IFFT) performs the same operations as an IDFT, except that it is much more computationally efficiency, and so is used in all practical systems. In order to transmit the OFDM signal the calculated time domain signal is then mixed up to the required frequency.

The receiver performs the reverse operation of the transmitter, mixing the RF signal to base band for processing, then using a Fast Fourier Transform (FFT) to analyse the signal in the frequency domain. The amplitude and phase of the subcarriers is then picked out and converted back to digital data.

The IFFT and the FFT are complementary function and the most appropriate term depends on whether the signal is being received or generated. In cases where the signal is independent of this distinction then the term FFT and IFFT is used interchangeably.

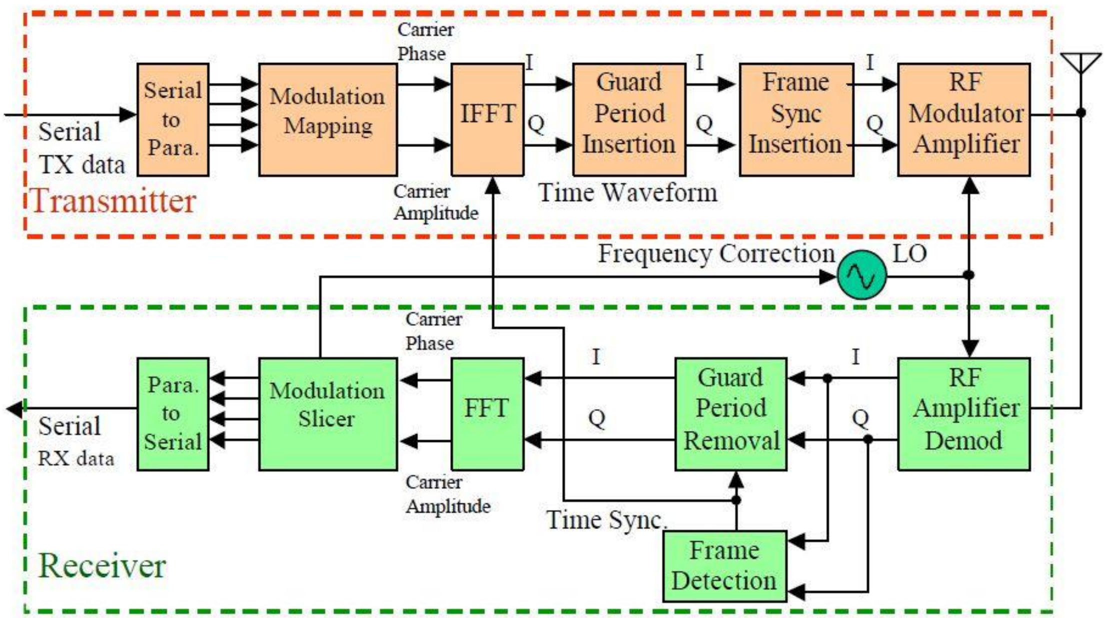


Figure 2-2 Basic Block Diagram of OFDM Transceiver

A simple example: A useful symbol duration T_U = 1 ms would require a sub-carrier spacing of $\Delta f = \frac{1}{1ms}$ =1 KHz (or an integer multiple of that) for orthogonality. N = 1,000 sub-carriers would result in a total pass-band bandwidth of $N\Delta f$ = 1 MHz. For this symbol time, the required bandwidth in theory according to Nyquist is N=1/2T_U = 0.5 MHz (i.e., half of the achieved bandwidth required by our scheme). If a guard interval is applied (see below), Nyquist bandwidth requirement would be even lower. The FFT would result in N = 1,000 samples per symbol. If no guard interval was applied, this would result in a base band complex valued signal with a sample rate of 1 MHz, which would require a baseband bandwidth of 0.5 MHz according to Nyquist. However, the pass-band RF signal is produced by multiplying the baseband signal with a carrier waveform (i.e., double-sideband quadrature amplitude-modulation) resulting in a pass-band bandwidth of 1 MHz. A single-side band (SSB) or vestigial sideband (VSB) modulation scheme would achieve almost half that bandwidth for the same symbol rate (i.e., twice as high spectral efficiency for the same symbol alphabet length). It is however more sensitive to multipath interference.

OFDM requires very accurate frequency synchronization between the receiver and the transmitter; with frequency deviation the sub-carriers will no longer be orthogonal, causing *inter-carrier interference* (ICI) (i.e., cross-talk between the sub-carriers). Frequency offsets are typically caused by mismatched transmitter and receiver oscillators, or by Doppler shift due to movement. While Doppler shift alone may be compensated for by the receiver, the situation is worsened when combined with multipath, as reflections will appear at various frequency offsets, which is much harder to correct. This effect typically worsens as speed increases, and is an important factor limiting the use of OFDM in high-speed vehicles. In order to mitigate ICI in such scenarios, one can shape each sub-carrier in order to minimize the interference resulting in a non-orthogonal subcarriers overlapping. For example, a low-complexity scheme referred to as WCP-OFDM (*Weighted Cyclic Prefix Orthogonal Frequency-Division Multiplexing*) consists of using short filters at the transmitter output in order to perform a potentially non-rectangular pulse shaping and a near perfect reconstruction using a single-tap per subcarrier equalization. Other ICI suppression techniques usually increase drastically the receiver complexity.

2.4 Guard Interval for Eliminating Inter Symbol Interference

One key principle of OFDM is that since low symbol rate modulation schemes (i.e., where the symbols are relatively long compared to the channel time characteristics) suffer less from inter symbol interference caused by multipath propagation, it is advantageous to transmit a number of low-rate streams in parallel instead of a single high-rate stream. Since the duration of each symbol is long, it is feasible to insert guard interval between the OFDM symbols, thus eliminating the inter symbol interference.

The guard interval also eliminates the need for a pulse-shaping filter, and it reduces the sensitivity to time synchronization problems.

A simple example: If one sends a million symbols per second using conventional single-carrier modulation over a wireless channel, then the duration of each symbol would be one microsecond or less. This imposes severe constraints on synchronization and necessitates the removal of multipath interference. If the same million symbols per second are spread among one thousand sub-channels, the duration of each symbol can be longer by a factor of thousand (i.e., one millisecond) for orthogonality with approximately the same bandwidth. Assume that a guard interval of 1/8 of the symbol length is inserted between each symbol. Inter symbol interference can be avoided if the multipath time-spreading (the time between the reception of the first and the last echo) is shorter than the guard interval (i.e., 125 microseconds). This corresponds to a maximum difference of 37.5 kilometers between the lengths of the paths.

The cyclic prefix, which is transmitted during the guard interval, consists of the end of the OFDM symbol copied into the guard interval, and the guard interval is transmitted followed by the OFDM symbol. The reason that the guard interval consists of a copy of the end of the OFDM symbol is so that the receiver will integrate over an integer number of sinusoid cycles for each of the multi paths when it performs OFDM demodulation with the FFT. In some standards such as Ultra wide-band, in the interest of transmitted power, cyclic prefix is skipped and nothing is sent during the guard interval. The receiver will then have to mimic the cyclic prefix functionality by copying the end part of the OFDM symbol and adding it to the beginning portion.

2.5 Cyclic Prefix

In the OFDM symbol transmission, ISI occurs at the receiver due to delay spread of the channel. ISI stands for Inter Symbol Interference. In order to avoid ISI, guard interval is inserted between two OFDM symbols. This guard internal is referred as cyclic Prefix (CP). This is kept greater than delay spread of the channel to avoid ISI.
Following are basics of cyclic prefix (CP):

- Zeros used in the guard time can alleviate interference between OFDM symbols.

- Orthogonality of carriers is lost when multipath channels are involved, which can be restored by cyclic prefix.

- It converts a linear convolution channel into a circular convolution channel. This restores the orthogonality at the receiver.

- The disadvantage is wastage of energy in the cyclic prefix samples.

The cyclic prefix is created so that each OFDM symbol is preceded by a copy of the end part of that same symbol.

Different OFDM cyclic prefix lengths are available in various systems. For example within LTE a normal length and an extended length are available and after Release 8 a third extended length is also included, although not normally used.

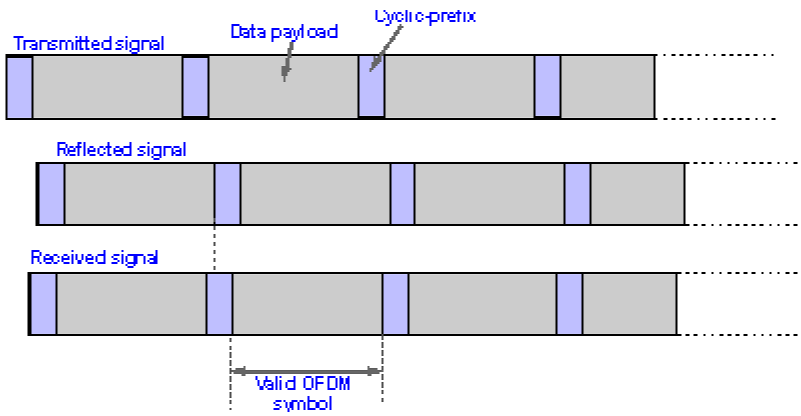


Figure 2-3 OFDM signal with cyclic prefix

Cyclic prefix advantages and disadvantages

There are several advantages and disadvantages attached to the use for the cyclic prefix within OFDM.
Advantages

Provides robustness:
The addition of the cyclic prefix adds robustness to the OFDM signal. The data that is retransmitted can be used if required.

Reduces inter-symbol interference:
The guard interval introduced by the cyclic prefix enables the effects of inter-symbol interference to be reduced.

Disadvantages
Reduces data capacity:
As the cyclic prefix re-transmits data that is already being transmitted, it takes up system capacity and reduces the overall data rate.

2.6 Idealized System Models

Transmitter

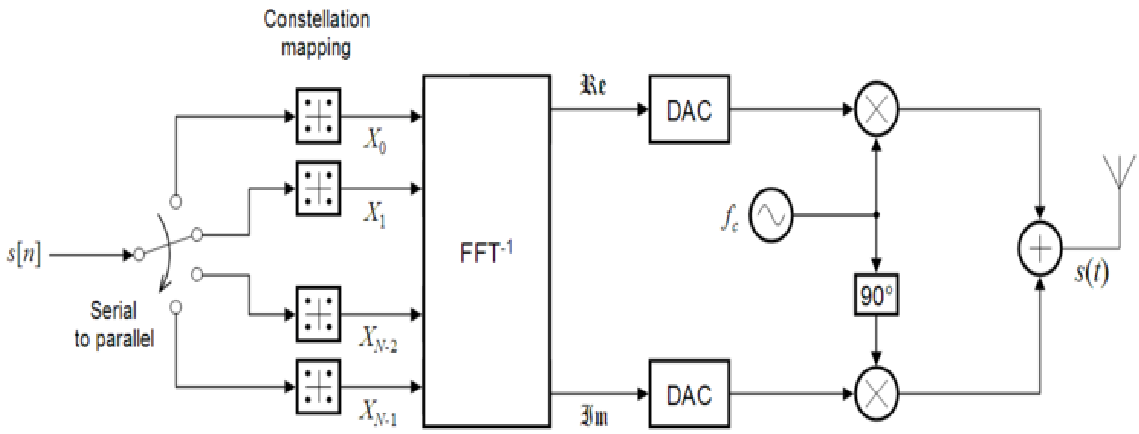


Figure 2-4 OFDM Transmitters

An OFDM carrier signal is the sum of a number of orthogonal sub-carriers, with baseband data on each sub-carrier being independently modulated commonly using some type of quadrature amplitude modulation (QAM) or phase-shift keying (PSK). This composite baseband signal is typically used to modulate a main RF carrier. δ[n] is a serial stream of binary digits. By inverse multiplication, these are first demultiplexed into parallel streams, and each one mapped to a (possibly complex) symbol stream using some modulation constellation (QAM, PSK, etc.). Note that the constellations may be different, so some streams may carry a higher bit-rate than others.

An inverse FFT is computed on each set of symbols, giving a set of complex time-domain samples. These samples are then quadrature-mixed to pass-band in the standard way. The real and imaginary components are first converted to the analogue domain using digital-to-analogue converters (DACs); the analogue signals are then used to modulate cosine and sine waves at the carrier frequency, f_c, respectively. These signals are then summed to give the transmission signal, δ(t).

Receiver

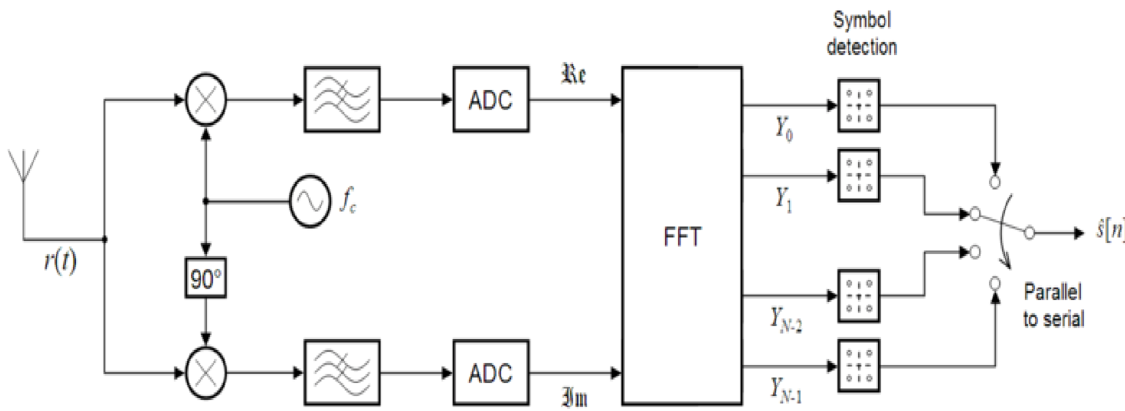


Figure 2-5 OFDM Receivers

The receiver picks up the signal r(t), which is then quadrature-mixed down to baseband using cosine and sine waves at the carrier frequency. This also creates signals centered on $2f_c$, so low-pass filters are used to reject these. The baseband signals are then sampled and digitised using analog-to-digital converters (ADCs), and a forward FFT is used to convert back to the frequency domain.

This returns N parallel streams, each of which is converted to a binary stream using an appropriate symbol detector. These streams are then re-combined into a serial stream, δ[n], which is an estimate of the original binary stream at the transmitter.

Chapter 3

PEAK TO AVERAGE POWER RATIO

An OFDM signal consists of a number of independently modulated SCs, which can give a large peak-to-average power (PAPR) ratio when added up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. This effect is illustrated in Figure 6.1. For this example, the peak power is 16 times the average value. The peak power is defined as the power of a sine wave with amplitude equal to the maximum envelope value. Hence, an un-modulated carrier has a PAPR ratio of 0 dB. An alternative measure of the envelope variation of a signal is the Crest factor, which is defined as the maximum signal value divided by the RMS signal value. For an un-modulated carrier, the Crest factor is 3 dB. This 3-dB difference between the PAPR ratio and Crest factor also holds for other signals, provided that the center frequency is large in comparison with the signal bandwidth.

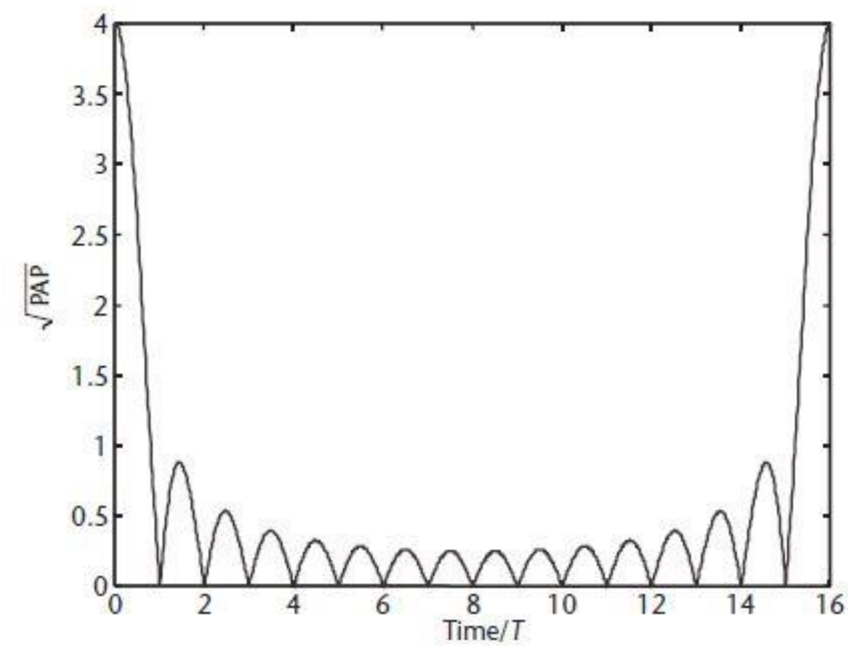


Figure 3-1 Square root of PAPR ratio for a 16-channel OFDM signal, modulated with the same initial phase for all sub carriers.

A large PAPR ratio brings disadvantages like an increased complexity of the analog-to-digital (A/D) and digital-to-analog (D/A) converters and a reduced efficiency of the RF power amplifier. To reduce the PAPR ratio, several techniques have been proposed which basically can be divided into three categories. First, there are signal distortion techniques, which reduce the peak amplitudes simply by nonlinearly distorting the OFDM signal at or around the peaks. Examples of distortion techniques are clipping, peak windowing, and peak cancellation. Second, there are coding techniques that use a special FEC code set that excludes OFDM symbols with a large PAPR ratio. The third technique scrambles each OFDM symbol with different scrambling sequences and selecting the sequence that gives the smallest PAPR ratio. This chapter discusses all of these techniques, but first analyzes the PAPR ratio distribution function. This will give a better insight in the PAPR problem and will explain why PAPR reduction techniques can be quite effective.

3.1 Distribution of PAPR

The PAPR of the transmitted signal can be expressed as,

$$PAPR = \frac{\max\limits_{0 \le t < NT} |x(t)|^2}{\frac{1}{NT}\int_0^{NT} |x(t)|^2 \, dt} \tag{3-1}$$

An approximation of NL equidistant points of $x(t)$, is made where L is an integer having the value equal to larger than 1. This L is called the "Oversampling Factor" and "L-times oversampled" time domain samples are represented as a vector $\mathbf{x} = [x_0, x_1, \ldots x_{NL-1}]^T$ and obtained as,

$$x_k = x(kT/L) = \frac{1}{\sqrt{N}} \sum_{n=0,k=0,1,\ldots,\mathrm{NL}\text{-}1}^{N-1} X_n e^{j2\pi kn\Delta f \, T/L} \tag{3-2}$$

The sequence $\{x_k\}$ can be deduced as the IFFT of data block $\mathbf{X}$ with $(L-1)N$ zero padding. The accuracy of PAPR is greatly increased with the choice of $L = 4$ [21]. The PAPR computed from the L-times oversampled time domain signal samples is calculated as,

$$PAPR = \frac{\max\limits_{0 \le k \le NL-1} |x_k|^2}{E[|x_k|^2]} \tag{3-3}$$

where $E[.]$ implicates the expectance.

For a discretized time domain samples of signal, the PAPR is expressed in the unit of decibel [22, 23]. So the equation (6) will be rewritten as,

$$PAPR = 10\log \frac{\max |x_k|^2}{E[|x_k|^2]} \text{ [dB]} \tag{3-4}$$

An OFDM system having N subcarriers, the sum of all the values of the signal, divided by the total number of subcarriers, is the mean power of the signal and both being N, gives the maximum PAPR as,

$$PAPR_{\max} = \frac{N^2}{N} = N \tag{3-5}$$

3.2 Complementary Cumulative Distribution Function (CCDF) of PAPR

The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. In the literature, the complementary CDF (CCDF) is commonly used instead of the CDF itself. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. In [19] a simple approximate expression is derived for the CCDF of the PAPR of a multicarrier signal with Nyquist rate sampling. From the central limit theorem, the real and imaginary parts of the time domain signal samples follow Gaussian distributions, each with a mean of zero and a variance of 0.5 for a multicarrier signal with a large number of subcarriers. Hence, the amplitude of a multicarrier signal has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom. The CDF of the amplitude of a signal sample is given by,

$$F(z) = 1 - e^{-z} \qquad (3\text{-}6)$$

The CDF of the PAPR of a data block with Nyquist rate sampling is derived as,

$$P(PAPR \le z) = F(z)^N = (1 - e^{-z})^N \qquad (3\text{-}7)$$

from which the complementary CDF may be expressed as,

$$\begin{aligned} P(PAPR > z) &= 1 - P(PAPR \le z) \qquad (3\text{-}8) \\ &= 1 - F(z)^N \\ &= 1 - (1 - e^{-z})^N \end{aligned}$$

The assumption on which equation (11) is based on is, the N time domain samples are mutually independent and uncorrelated. One limitation of this expression is that, the accuracy of determining CCDF decreases if the number of subcarrier is reduced, since a Gaussian assumption will no longer be satisfied. Therefore, a number of methods have been proposed to derive more accurate distribution of PAPR [20-23].

The CCDFs are usually compared in a graph such as Fig. 1, which shows the CCDFs of the PAPR of an OFDM signal with 256 and 1024 subcarriers (N = 256,1024) for quaternary phase shift keying (QPSK) modulation and oversampling factor 4 (L = 4). The CCDFs of the PAPR after applying one of the PAPR reduction techniques (i.e., the selected mapping, SLM, technique with 16 candidates) are also shown in Fig. 1. For details of the SLM technique, see the next section. The horizontal and vertical axes represent the threshold for the PAPR and the probability that the PAPR of a data block exceeds the threshold, respectively. It is shown that the unmodified OFDM signal has a PAPR that exceeds 11.3 dB for less than 0.1 percent of the data blocks for N = 256. In this case, we say that the 0.1 percent PAPR of the unmodified signal is 11.3 dB. The 0.1 percent PAPR of the unmodified signal is 11.7 dB for N = 1024. When SLM is used as a PAPR reduction technique, the 0.1 percent PAPR for N = 256 and that for N = 1024 reduce to 8.1 dB and 8.9 dB, resulting in 3.2 dB and 2.8 dB reductions, respectively. Speaking roughly, the closer the CCDF curve is to the vertical axis, the better its PAPR characteristic.

3.3 Bit Error Rate

The symbol rate of single carrier systems has to be high if they are to obtain a high bit rate, and as a result, systems such as GSM require complex equalization (up to 4 symbol periods) to cope with multipath propagation. GSM systems are designed to cope with a maximum delay spread of 15 ms, which corresponds to the typical delay spread experienced at a transmission distance of 30 - 35 km. The symbol rate for GSM is 270 kHz corresponding to a symbol period of 3.7 ms, thus ISI caused by the multipath spans over 4 symbol periods. This would normally completely destroy the transmitted information, but is recovered in practice by using complex adaptive equalization. Although this works for robust modulation schemes such as Gaussian Minimum Shift Keying (GMSK) [22], [23] as used in the GSM system, it is difficult to successfully apply to higher modulation schemes, as the residual errors in the equalization will cause a high error rate.

The BER greatly is dependent on the scheme used for symbol generation. QPSK and BPSK have the same BER. In Figure 3-2 the BER for common symbol generation methods are depicted,

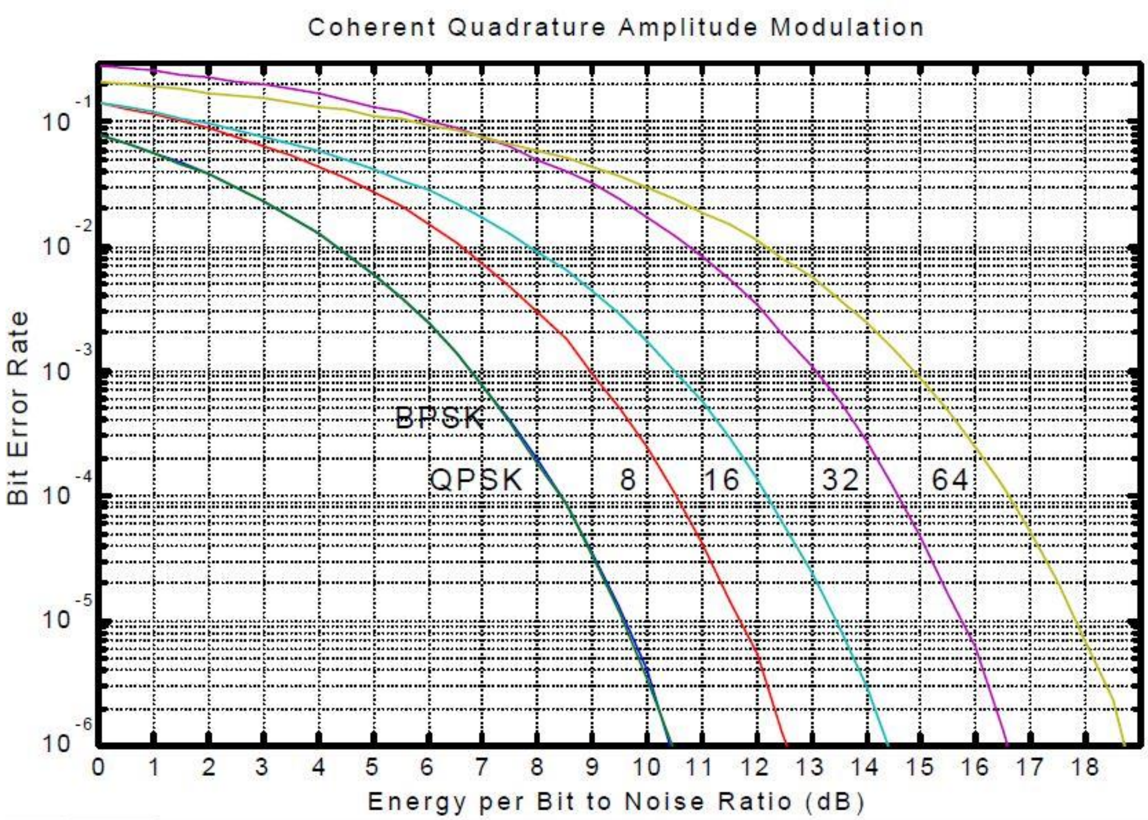


Figure 3-2 BER for BPSK, QPSK, 8-QAM, 16-QAM, 32-QAM, 64-QAM

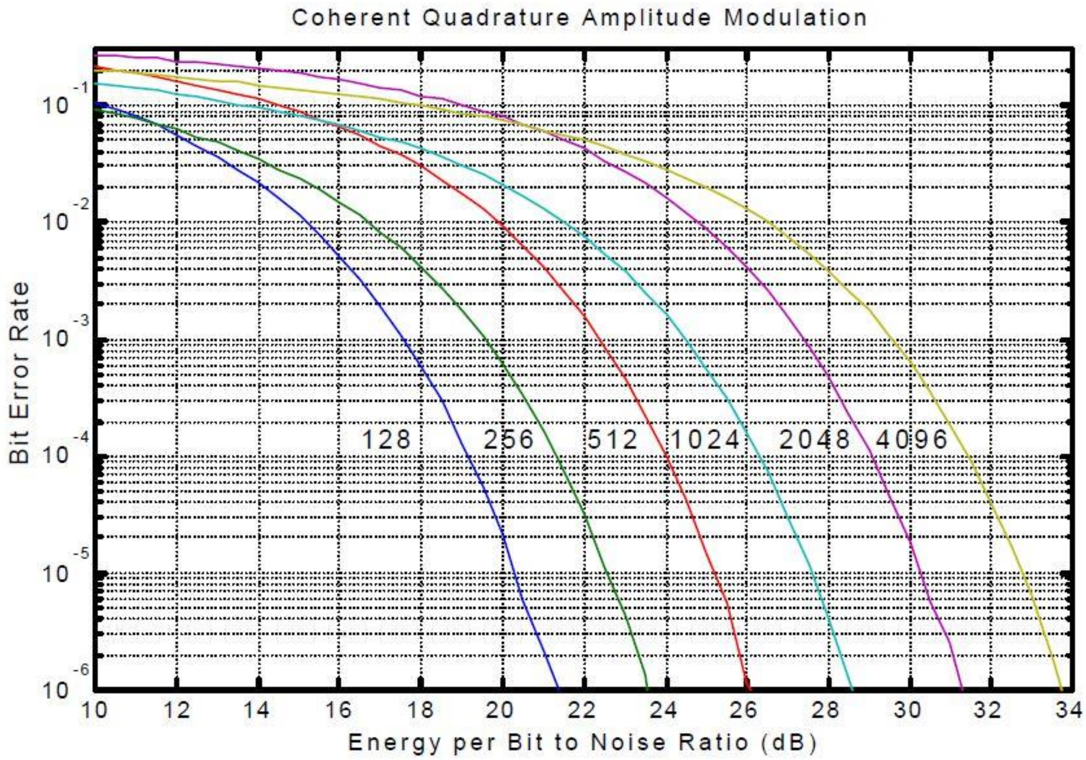


Figure 3-3 BER for 128-QAM, 256-QAM, 512-QAM, 1024-QAM, 2048-QAM, 4096-QAM

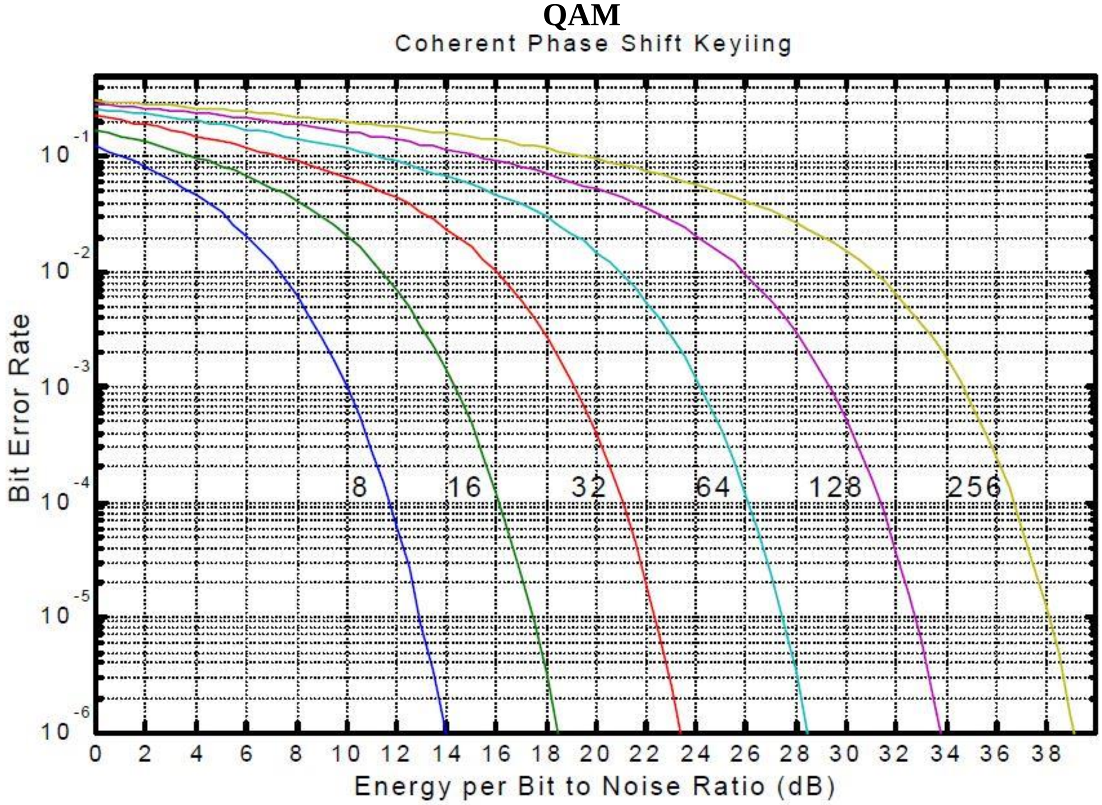


Figure 3-4 BER for 8-PSK, 16-PSK, 32-PSK, 64-PSK, 128-PSK, 256-PSK

The BER for each symbol generating method can be calculated from,

$$SNR = 10\log_{10}(N_b) + EBNR_{dB} \quad (3\text{-}9)$$

3.4 Current State of The Art PAPR reduction technique

Recently deployed OFDM systems use various state of the art PAPR reduction techniques. Most common of them are:

- Iterative Clipping and Filtering
- Partial Transmit Scheme
- Selective Mapping
- Companding

Researchers over the years have exploited many other reduction methods that are quite promising. They are evaluated in terms of reduction achieved, BER, SER and implementation complexity and efficiency. Some of these methods are:

- Hybrid PTS and SLM
- Active Constellation Extension
- Tone Rejection and Injection
- Distortion based methods as Exponential Companding

The existing methods are presented here with the reduction result in brief in the following

3.4.1 Clipping and Filtering

The simplest technique for PAPR reduction might be amplitude clipping [10]. Amplitude clipping limits the peak envelope of the input signal to a predetermined value or otherwise passes the input signal through unperturbed [42], that is,

$$B(x) = \begin{cases} x\,; |x| < A \\ Ae^{j\emptyset(x)}; |x| > A \end{cases} \quad (3\text{-}10)$$

where f(x) is the phase of x. The distortion caused by amplitude clipping can be viewed as another source of noise. The noise caused by amplitude clipping falls both in-band and out-of- band. In-band distortion cannot be reduced by filtering and results in an error performance degradation, while out-of-band radiation reduces spectral efficiency. Filtering after clipping can reduce out-of-band radiation but may also cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce overall peak regrowth, a repeated clipping-and-filtering operation can be used [11, 12]. Generally, repeated clipping-and-filtering takes many iterations to reach a desired amplitude level. When repeated clipping-and-filtering is used in conjunction with other PAPR reduction techniques described below, the deleterious effects may be significantly reduced. There are a few techniques proposed to mitigate the harmful effects of the amplitude clipping. In [43] a method to iteratively reconstruct the signal before clipping is proposed. This method is based on the fact that the effect of clipping noise is mitigated when decisions are made in the frequency domain. When the decisions are converted back to the time domain, the signal is recovered somewhat from the harmful effects of clipping, although this may not be perfect. An improvement can be made by repeating the above procedures. Another way to compensate for the performance degradation from clipping is to reconstruct the clipped samples based on the other samples in the oversampled signals. In [44] oversampled signal reconstruction is used to compensate for signal-to-noise ratio (SNR) degradation due to clipping for low values of clipping threshold. In [45] iterative estimation and cancellation of clipping noise is proposed. This technique exploits the fact that clipping noise is generated by a known process that can be recreated at the receiver and subsequently removed. In the later figures it is illustrated that how much distortion is introduced through clippilg,

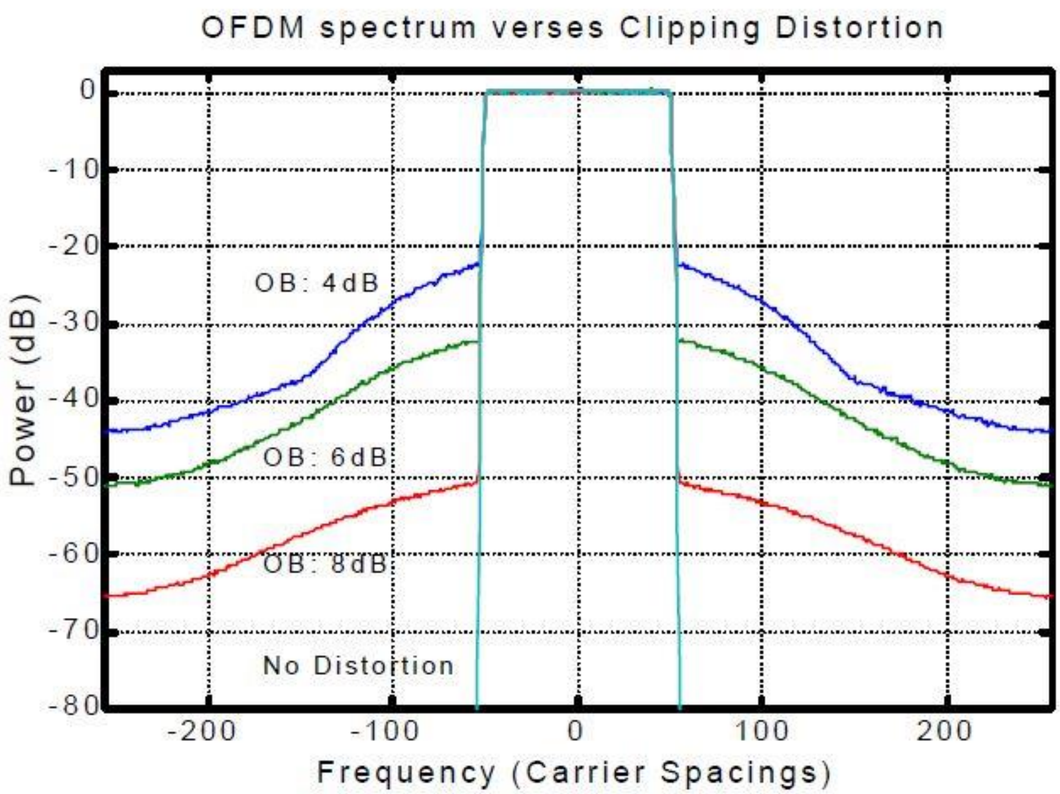


Figure 3-5 Spectrum of OFDM signal with clipping distortion.

The distortion is caused by clipping of the signal. The spectrum of the OFDM signal goes up to 50% of the signal bandwidth (BW) from center and so the 55% BW from the center result corresponds to the out of band energy just outside the pass band.

3.4.2 Partial Transmit Sequence

In the PTS technique, an input data block of *N* symbols is partitioned into disjoint subframes. The subcarriers in each subframe are weighted by a phase factor for that subframe. The phase factors are selected such that the PAPR of the combined signal is minimized. Figure 2 shows the block diagram of the PTS technique. In the ordinary PTS technique [24, 25] input data block **X** is partitioned into *M* disjoint subframes **X***m* = [*Xm*,0, *Xm*,1, …, *Xm*,*N*–1]*T*, *m* = 1, 2, …, *M*, such that S*m*=1 *M* **X***m* = **X** and the subframes are combined to minimize the PAPR in the time domain. The *L*-times oversampled time domain signal of **X***m*, *m* = 1, 2, …, *M*, is denoted **x***m* = [*xm*,0, *xm*,1, …, *xm*,*NL*–1]*T*. **x***m*, *m* = 1, 2, …, *M*, is obtained by taking an IDFT of length *NL* on **X***m* concatenated with (*L* – 1)*N* zeros. These are called the partial transmit sequences. Complex phase factors, *bm* = *ej*f*m*, *m* = 1, 2, …, *M*, are introduced to combine the PTSs. The set of phase factors is denoted as a vector **b** = [*b*1, *b*2, …, *bM*]*T*. The time domain signal after combining is given by,

$$x'(b) = \sum_{m=1}^{M} b_m x_m \qquad \text{(3-11)}$$

where **x**¢(**b**) = [x0¢(**b**), x1¢(**b**). … xNL–1¢(**b**)]T. The objective is to find the set of phase factors that minimizes the PAPR. Minimization of PAPR is related to the minimization of

$$\max_{0<k<NL-1} |x'_k(b)| \qquad \text{(3-12)}$$

In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is written as *P* = {*ej*2p*l*/*W* ç*l* = 0, 1, …, *W* – 1}, where *W* is the number of allowed phase factors. In addition, we can set *b*1 = 1 without any loss of performance. So, we should perform an exhaustive search for (*M* – 1)phase factors. Hence, *WM*–1 sets of phase factors are searched to find the optimum set of phase factors. The search complexity increases exponentially with the number of subframes *M*. PTS needs *M* IDFT operations for each data block, and the number of required side information bits is log2*WM*–1, where denotes the smallest integer that does not exceed *y*. The amount of PAPR reduction depends on the number of subframes *M* and the number of allowed phase factors *W*. Another factor that may affect the PAPR reduction performance in PTS is the subframe partitioning, which is the method of division of the subcarriers into multiple disjoint subframes. There are three kinds of subframe partitioning schemes: adjacent, interleaved, and pseudo-random partitioning [25]. Among them, pseudo-random partitioning has been found to be the best choice. The PTS technique works with an arbitrary number of subcarriers and any modulation scheme.

As mentioned above, the ordinary PTS technique has exponentially increasing search complexity. To reduce the search complexity, various techniques have been suggested. In [26] iterations for updating the set of phase factors are stopped once the PAPR drops below a preset threshold. In [27–29] various methods to reduce the number of iterations are presented. These methods achieve significant reduction in search complexity with marginal PAPR performance degradation.

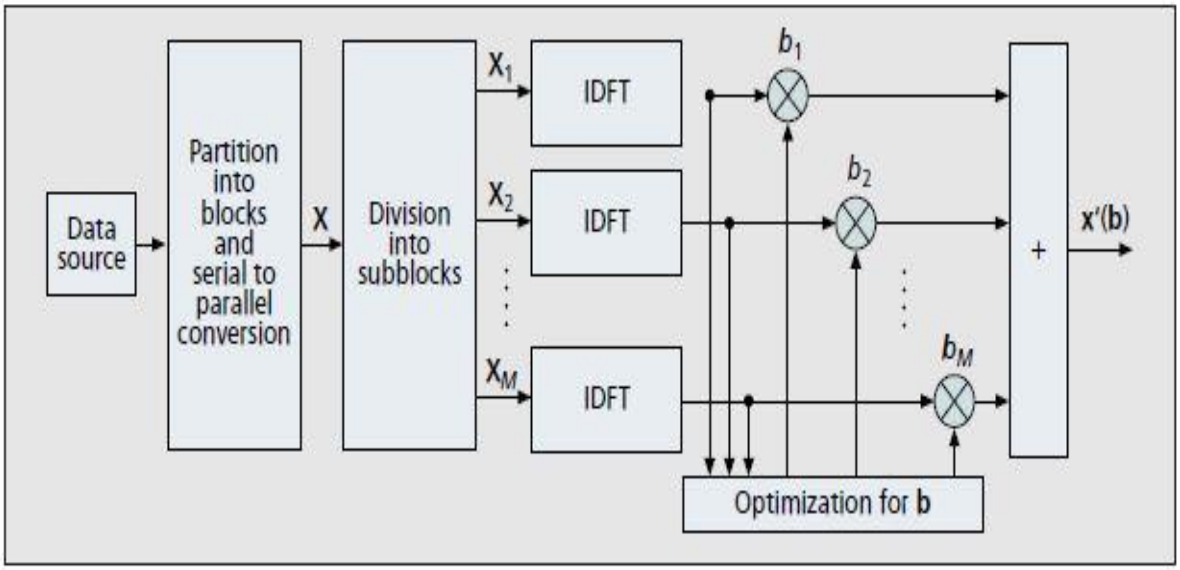
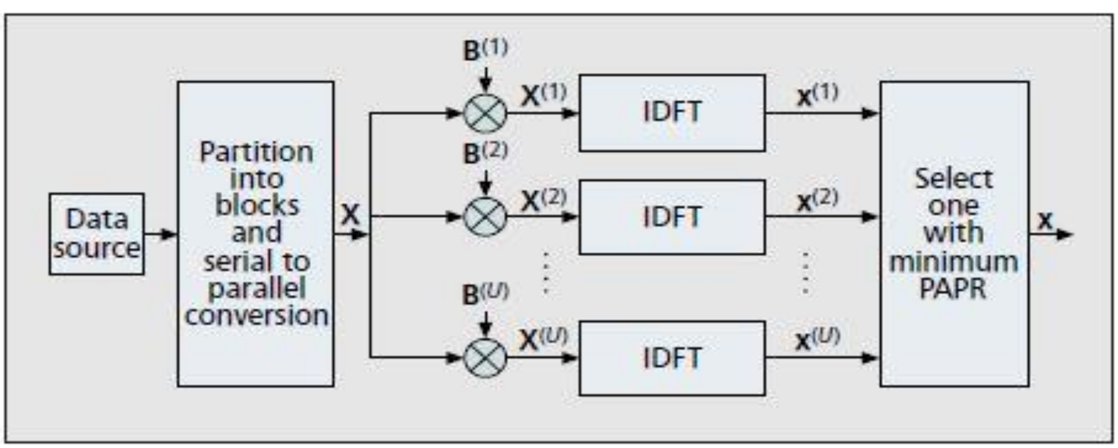


Figure 3-6 Block diagram for PTS

3.4.3 Selective Mapping

In the SLM technique, the transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and selects the most favorable for transmission [30, 31]. A block diagram of the SLM technique is shown in Fig. 4. Each data block is multiplied by U different phase sequences, each of length N, B(u) = [bu,0, bu,1, …, bu,N–1]T, u = 1, 2, …, U, resulting in U modified data blocks. To include the unmodified data block in the set of modified data blocks, we set B(1) as the all-one vector of length N. Let us denote the modified data block for the uth phase sequence X(u) = [X0bu,0, X1bu,1, …, XN–1bu,N–1]T, u = 1, 2, …, U. After applying SLM to X, the multicarrier signal becomes,

$$x^{(u)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n, b_{u,n} e^{j2\pi\Delta fnt}; 0 < t < NT, u = 1,2, \dots, U \quad (3\text{-}12)$$



Among the modified data blocks $\mathbf{X}(u)$, $u = 1, 2, \ldots, U$, the one with the lowest PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information. At the receiver, the reverse operation is performed to recover the original data block. For implementation, the SLM technique needs U IDFT operations, and the number of required side information bits is $\lceil \log_2 U \rceil$ for each data block. This approach is applicable with all types of modulation and any number of subcarriers. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the design of the phase sequences. In [32] an SLM technique without explicit side information is proposed.

Example: Here, we show a simple example of the SLM technique for an OFDM system with eight subcarriers. We set the number of phase sequences to $U = 4$. The data block to be transmitted is denoted $\mathbf{X} = [1, -1, 1, 1, 1, -1, 1, -1]^T$ whose PAPR before applying SLM is 6.5 dB. We set the four phase factors as $\mathbf{B}(1) = [1, 1, 1, 1, 1, 1, 1, 1]^T$, $\mathbf{B}(2) = [-1, -1, 1, 1, 1, 1, 1, -1]^T$, $\mathbf{B}(3) = [-1, 1, -1, 1, -1, 1, 1, 1]^T$, and $\mathbf{B}(4) = [1, 1, -1, 1, 1, -1, 1, 1]^T$. Among the four modified data blocks $\mathbf{X}(u)$, $u = 1, 2, 3, 4$, $\mathbf{X}(2)$, has the lowest PAPR of 3.0 dB. Hence, $\mathbf{X}(2)$ is selected and transmitted to the receiver. For this data block, the PAPR is reduced from 6.5 to 3.0 dB, resulting in a 3.5 dB reduction. In this case, the number of IDFT operations is 4 and the amount of side information is 2 bits. The amount of PAPR reduction may vary from data block to data block, but PAPR reduction is possible for all data blocks.

3.4.4 Active Constellation Extension

Active constellation extension (ACE) is a PAPR reduction technique similar to TI [23]. In this technique, some of the outer signal constellation points in the data block are dynamically extended toward the outside of the original constellation such that the PAPR of the data block is reduced. The main idea of this scheme is easily explained in the case of a multicarrier signal with QPSK modulation in each subcarrier. In each subcarrier there are four possible constellation points that lie in each quadrant in the complex plane and are equidistant from the real and imaginary axes. Assuming white Gaussian noise, the maximum likelihood decision regions are the four quadrants bounded by the axes; thus, a received data symbol is decided according to the quadrant in which the symbol is observed. Any point that is farther from the decision boundaries than the nominal constellation point (in the proper quadrant) will offer increased margin, which guarantees a lower BER. We can therefore allow modification of constellation points within the quarter-plane outside of the nominal constellation point with no degradation in performance. This principle is illustrated in Fig. 3-7, where the shaded region represents the region of increased margin for the data symbol in the first quadrant. If adjusted intelligently, a combination of these additional signals can be used to partially cancel time domain peaks in the transmit signal. The ACE idea can be applied to other constellations as well, such as QAM and MPSK constellations, because data points that lie on the outer boundaries of the constellations have room for increased margin without degrading the error probability for other data symbols. This scheme simultaneously decreases the BER slightly while substantially reducing the peak magnitude of a data block. Furthermore, there is no loss in data rate and no side information is required. However, these modifications increase the transmit signal power for the data block, and the usefulness of this scheme is rather restricted for a modulation with a large constellation size. It is possible to combine the TR and ACE techniques to make the convergence of TR much faster [46].

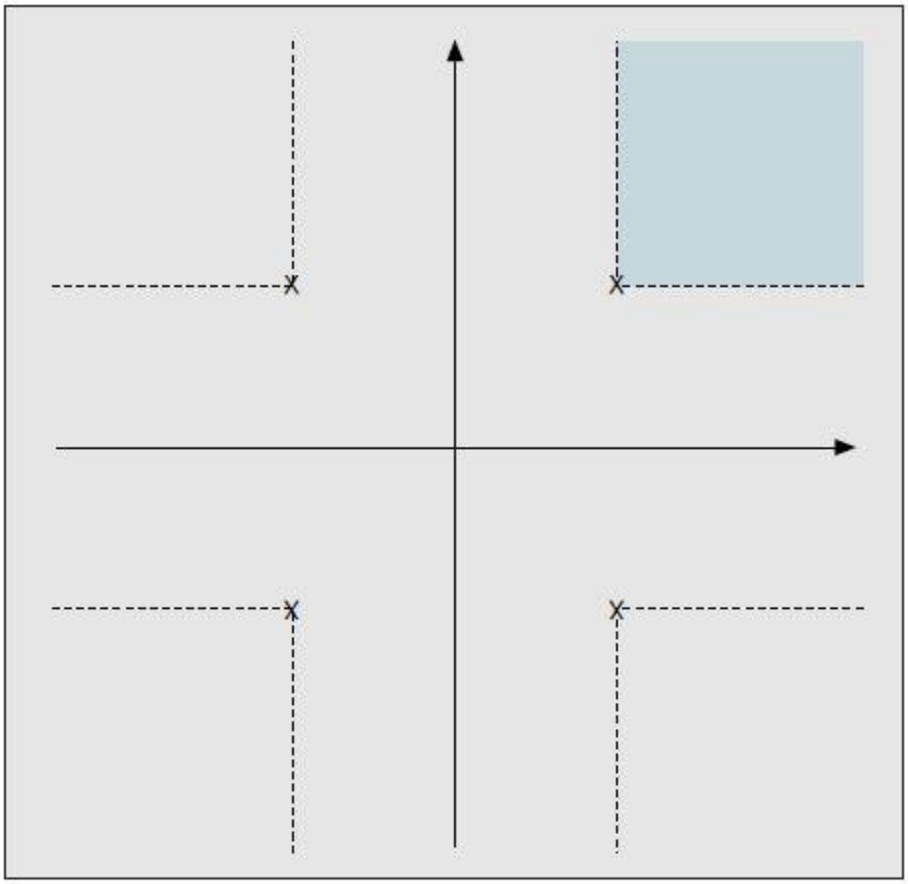


Figure 3-8 ACE Constellation Diagram

CHAPTER 4

EXPONENTIAL TRANSFORMATION TECHNIQUE

In the proposed method, the sequential steps of generating the time domain signal prevail, with some modification to achieve the purpose of PAPR reduction.

4.1 Mathematical Representation

To portray a thorough depiction of the modification, a brief procedure of OFDM signal generation is presented. First the data to be transmitted is converted from one serial stream to several parallel bit streams. Then to generate data symbols from parallel bit streams, QPSK-Modulation scheme is applied on the data bits. After the application of the aforementioned mapping technique, each symbol is comprised of 2 data bits. Next depending on the value of oversampling factor, the parallel data symbol streams are padded with zeroes. Mathematically, these parallel streams of QPSK-Modulated data symbols can be denoted as a vector, **X** = [X_1, X_2,…...,, X_{N-1}] where N is the number of parallel data streams. Afterwards, these parallel data undergoes the IFFT operation and generates the time domain subcarrier signals. For simplicity of mathematical interpretation, let us consider one such time domain subcarrier signal represented as,

$$x_c(t) = |X_c| e^{j\phi(t)} \qquad (4\text{-}1)$$

where $|X_c|$ stands for the magnitude and $e^{j\phi(t)}$ represents the phase.

Now in the case of conventional OFDM signal generation, these parallel subcarrier signals are converted into one serial representation which is transmitted via the channel.

The modification according to the proposed method is introduced right after the IFFT conversion of parallel frequency signals and immediately before the parallel to serial conversion of the time domain subcarrier signals. Each time domain data stream is transformed to its equivalent negative exponential value, represented by the following equation,

$$y(t) = e^{-|X_c| e^{j\phi(t)}} ; 0 \le t \le NT \quad (4\text{-}2)$$

where $y(t)$ refers to the negative exponential value of the signal $x_c(t)$.

After the transformation of the parallel signals, derived parallel streams are converted into a single stream and are ready to be transmitted now.

4.2 Block Diagram

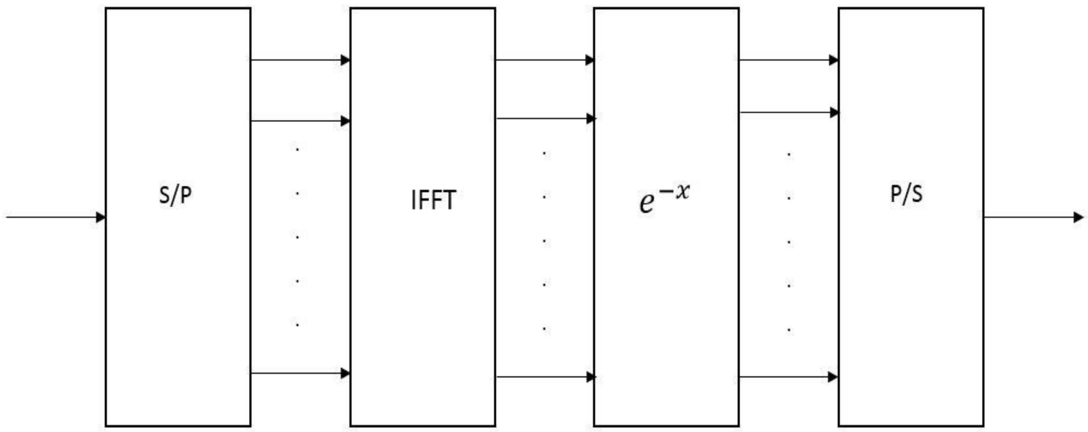


Figure 4-1 A Block Diagram Representation of Exponential Transform

Above figure illustrates the sequence of operations to generate the OFDM transmitter signal, with reduced PAPR according to the proposed method.

Using the negative exponential, instead of the actual time domain signal achieved after the IFFT, changes the statistical distribution of the signal. Dissimilar to the method of iterative clipping and filtering, where the amplitudes of the resultant peaks from the superposition sum are bounded and clipped off beyond a threshold [28] or the modification of phase introduced by PTS implementation [29], the proposed method alters both magnitude and phase, regardless of the manner of symbol generation from the digital bit stream.

In most of the techniques of PAPR reduction, the peaks, along with the adjacent values to the peaks are reformed to achieve the anticipated reduction result. Inferred from the equation (7) which computes the PAPR of a signal, there is a fact that the reduction of peak values would also decrease the average power of the signal. Since the average power of a signal holds an inversely proportional relation to the peak to average power ratio, a deduction of average power would result in an increase in PAPR consequently. But in the proposed routine, instead of reducing only the peak values, the other values smaller compared to the peak are raised in magnitude and corresponding phases are modified resulting in an incremented average power of the signal, while at the same instant, diminishing the peak values.

4.3 Modification at the Receiver

Resultant modification in the demodulation of the received signal is the complementary operation, performed in the transmitter. Right after receiving, the whole signal is required to go under a logarithmic operation to complement the exponential transformation performed at the transmitter and each data symbol is multiplied by $e^{j\pi}$ to get back the actual time domain signal, intended to be transmitted without introducing the adaptation imposed by the suggested process. The equation referring to the logarithmic transformation can be defined as in the following

$$x_c(t) = \log_e\{y(t)\} \times e^{j\pi} \tag{4-3}$$

where $y(t)$ corresponds to the received signal and $x_c(t)$ represents the time domain signal obtained after the logarithmic conversion.

Mathematically using Euler's equation multiplication with $e^{j\pi}$ can be interpreted as,

$$\begin{aligned} e^{j\pi} &= \cos(\pi) + j \times \sin(\pi) \\ &= -1 \end{aligned}$$

After the logarithmic transformation, the signal undergoes the Fast Fourier Transform (FFT) and the rest of the processing follows the successive procedures as particularized in the rudimentary perception of demodulation and detection of OFDM signal.

4.4 Simulation Results

In this section, we present the computer based simulation of our proposed technique. The simulation results are obtained using MATLAB 2016a software. The simulation parameters are listed in the following table,

Simulation Parameters	
Number of Bits	10000
Number of Subcarrier, N	64, 128, 256
Oversampling Factor, L	4
Phase Factor for PTS	1,-1, 1j, -1j
Candidates for SLM	16, 32, 64
Companding Factor, μ	115

Table 4-1 Listing of Simulation parameters

In order to generate the data symbols from the number of test bits, we used QPSK modulation technique. So each data symbol carries equivalent information of two bits of data. Since we utilized an oversampling factor of four, a Gaussian distribution assumption of the data block prevails. Accordingly, the simple expression for derivation of PAPR can be used to determine the accurate distribution of PAPR for a data block. The oversampling factor of 4 together with the number of subcarriers would determine the amount of zero padding. For the case of 64 subcarriers, there would be 192 number of zeroes padded to each data block. The amount of zero padding would be calculated from $(L-1)N$. The IFFT and FFT implementation, necessary for the OFDM modeling, are implemented using the built in functions of MATLAB since it provides results similar to the theoretical derivation. For clear distinction, we have displayed the simulation results of different methods in different subsections. To illustrate the superiority of the proposed technique, we have also portrayed a comparison between our method and conventional methods such as PTS and SLM. Also we have reproduced the output of some recently developed methods such as Hybrid method [30] and combined implementation of both Companding and Precoding [31] in order to reduce PAPR. To summarize the reduction results, finally a table is presented, where a comparison of existing and proposed method will be drawn.

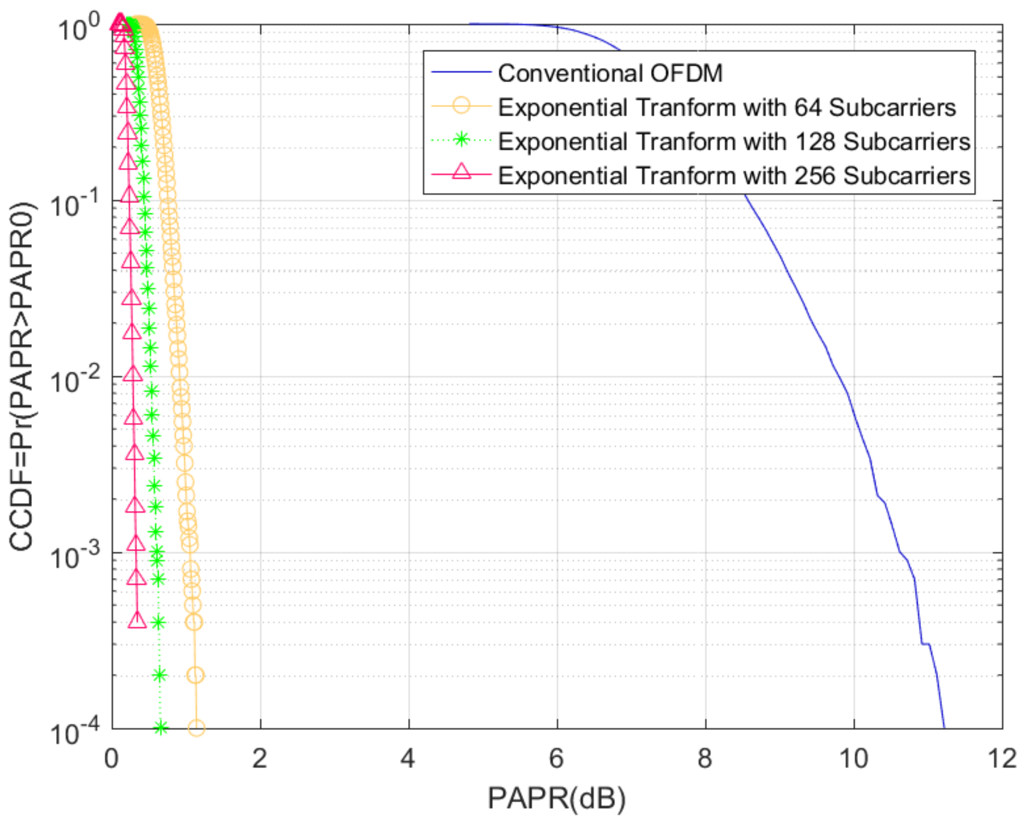


Figure 4-2 CCDF vs PAPR Plot for 64, 128 and 256 subcarriers using Exponential Transform.

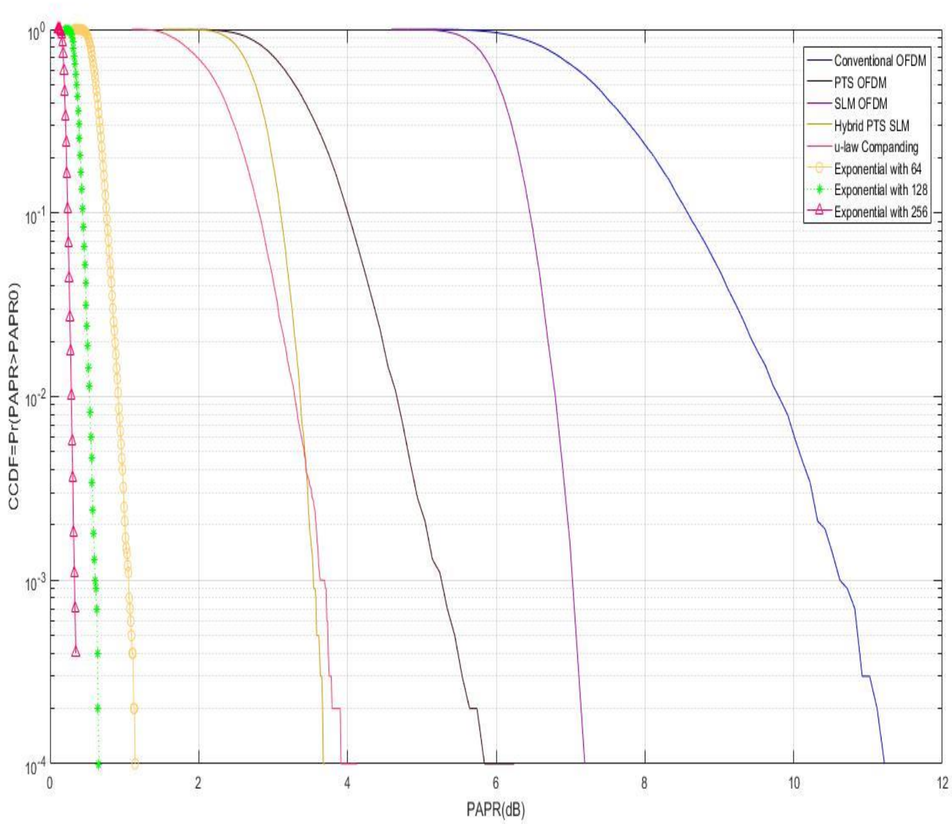


Figure 4-3 Comparison of PAPR Reduction between Existing and Proposed Exponential Transform Method

For evaluation of the numerical reduction achieved we considered a CCDF level of 10^{-3} and noted the values of the corresponding PAPR. The result is presented in the form of a table below

Reduction Compared To	Exponential Transform with N subcarriers		
	N = 64	N = 128	N = 256
PTS	2.6934dB	2.9611dB	3.1287dB
SLM	5.3946dB	5.7127dB	5.8955dB
PTS and SLM Hybrid	2.1239dB	2.3910dB	2.5576dB
Companding	1.6137dB	1.8923dB	2.0649dB

Table 4-2 Numerical Reduction Results

CHAPTER 5

Nth ROOT EXPONENTIAL TRANSFORM

With exponential transform, it is possible to achieve satisfactory PAPR reduction. However, since exponential transformation is distortion based technique, it alters the statistical distribution of the signal spectra and causes the min values to rise significantly. Thus the signal becomes more susceptible to noise and other channel induced interference.

If we introduce an AWGN channel of SNR say 50 dB the resultant BER for exponential method is quite high. In order to mitigate this loss of BER, we need to utilize coded channel. A convolution coding with code constraint length of k = 8, will give a satisfactory outcome at the cost of redundant bits.

A solution, to this limitation of exponential transform would be to introduce some scaling factor that maintains the signal average value close to a constant level and at the same time fulfills the goal of achieving reduced PAPR. With a simple modification of the proposed method of Exponential Transform and Combining it with N times root we can attain desired results. This method involves the computation of mean square average power of the signal and is named as Nth Root Exponential Transform.

5.1 Mathematical Representation

To portray a thorough depiction of the modification, a brief procedure of OFDM signal generation is presented. First the data to be transmitted is converted from one serial stream to several parallel bit streams. Then to generate data symbols from parallel bit streams, QPSK-Modulation scheme is applied on the data bits. After the application of the aforementioned mapping technique, each symbol is comprised of 2 data bits. Next depending on the value of oversampling factor, the parallel data symbol streams are padded with zeroes. The representation of data blocks given in Chapter 4 persists here too. The only modification is in the block of exponential transform. First we compute the mean square average of the sub frame under consideration,

$$\mu = \frac{1}{NT}\int_0^{NT} |x(t)|^2\, dt \qquad (5\text{-}1)$$

where μ denotes the mean square average of the signal.

Afterward a modified form of equation (4-2) is used to calculate the Nth root exponential equivalent of the time domain parallel stream of data.

If we represent the computed exponential values as y(t), then it can be written as,

$$y(t) = \sqrt[n]{\frac{1+e^{-x_c(t)}}{1+\frac{1}{\mu}}} \qquad (5\text{-}2)$$

where $x_c(t)$ is the time domain signal.

This modified method maintains the signal average power level after the alteration. Also this transforms the signal into a Rayleigh Distribution which has is more efficient in a noisy channel.

5.2 Block Diagram

Below is a block diagram representation of the proposed method

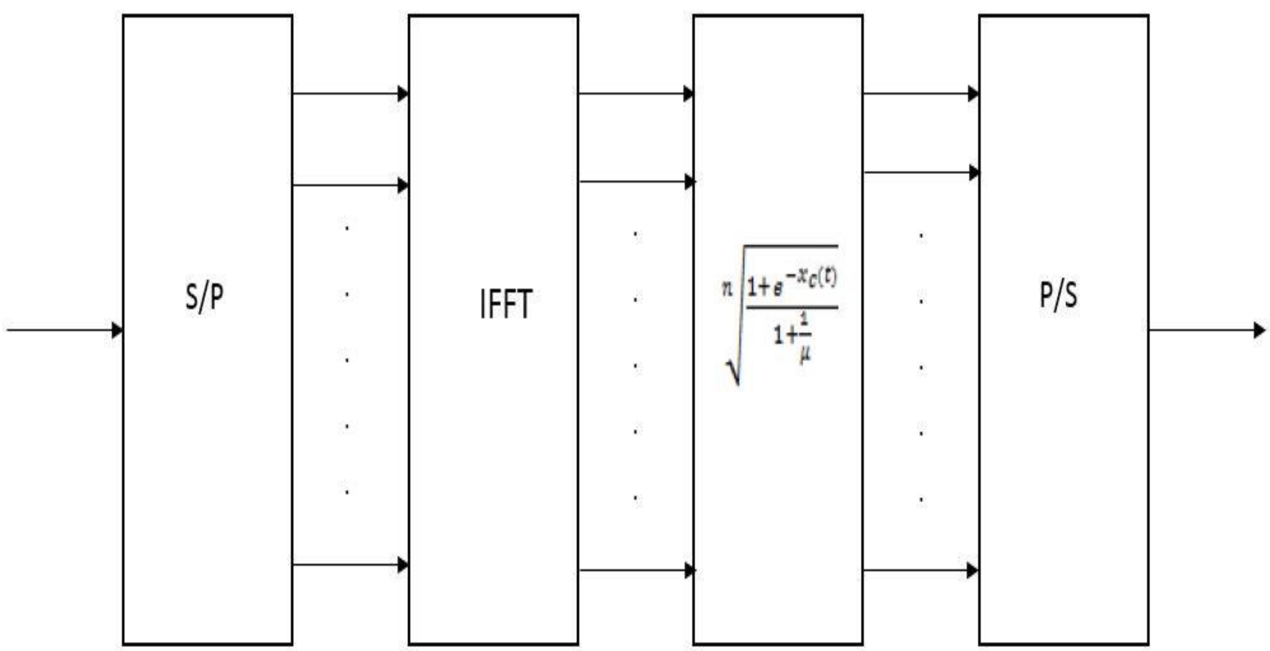


Figure 5-1 Block Diagram of Nth root Exponential Transform

The modification can be achieved by only replacing the exponential block with the required modified expression. Using the negative exponential, instead of the actual time domain signal achieved after the IFFT, changes the statistical distribution of the signal. Dissimilar to the method of iterative clipping and filtering, where the amplitudes of the resultant peaks from the superposition sum are bounded and clipped off beyond a threshold [28] or the modification of phase introduced by PTS implementation [29], the proposed method alters both magnitude and phase, regardless of the manner of symbol generation from the digital bit stream.

5.3 Simulation Results

The oversampling factor of 4 together with the number of subcarriers would determine the amount of zero padding. For the case of 64 subcarriers, there would be 192 number of zeroes padded to each data block. The amount of zero padding would be calculated from $(L-1)N$. The IFFT and FFT implementation, necessary for the OFDM modeling, are implemented using the built in functions of MATLAB since it provides results similar to the theoretical derivation. For clear distinction, we have displayed the simulation results of different methods in different subsections. To illustrate the superiority of the proposed technique, we have also portrayed a comparison between our method and conventional methods such as PTS and SLM. Also we have reproduced the output of some recently developed methods such as Hybrid method [30] and combined implementation of both Companding and Precoding [31] in order to reduce PAPR. To summarize the reduction results, finally a table is presented, where a comparison of existing and proposed method will be drawn.

In implementing the Companding along with precoding method for PAPR reduction, we used μ-law as proposed in [31]. The value of μ, the companding factor, has the range from 1 to 255. For our case we chose μ to be 115. The table contains the summery of reduction analysis from a numerical point of view. Rather than using a definitive comparison scale, we have investigated the average reduction realizable using the proposed method for different numbers of subcarriers.

Simulation Parameters	
Number of Bits	10000
Root Power, N	1, 2, 3, 10
Oversampling Factor, L	4
Phase Factor for PTS	1,-1, 1j, -1j
Candidates for SLM	16, 32, 64
Companding Factor, μ	115

Table 5-1 List of Simulation Parameters

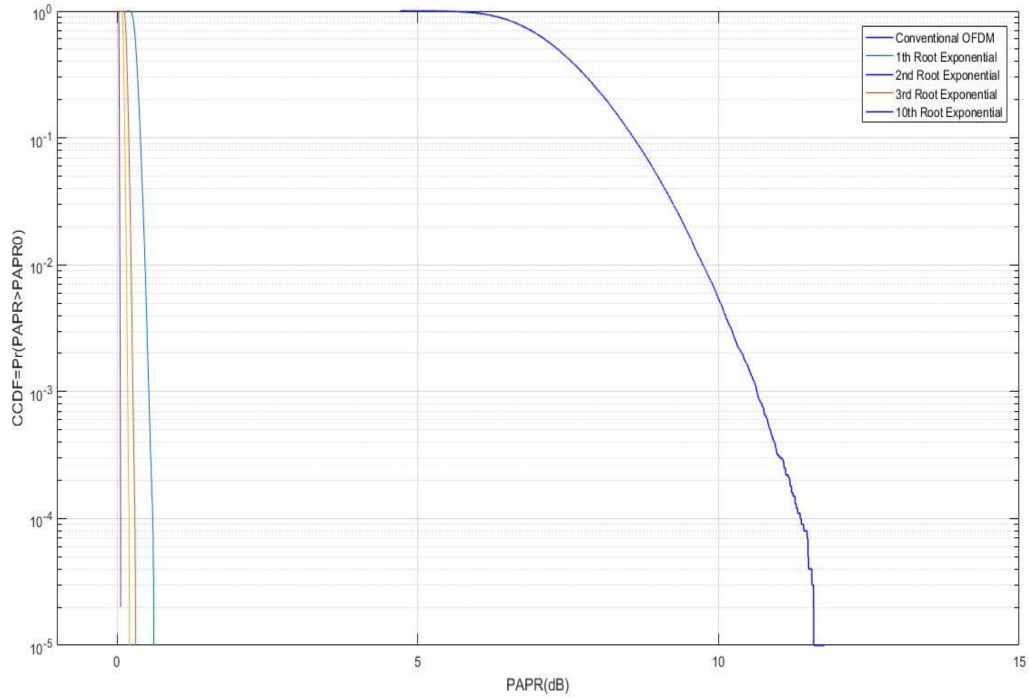


Figure 5-2 CCDF vs. PAPR Plot for Nth Root Exponential Transform

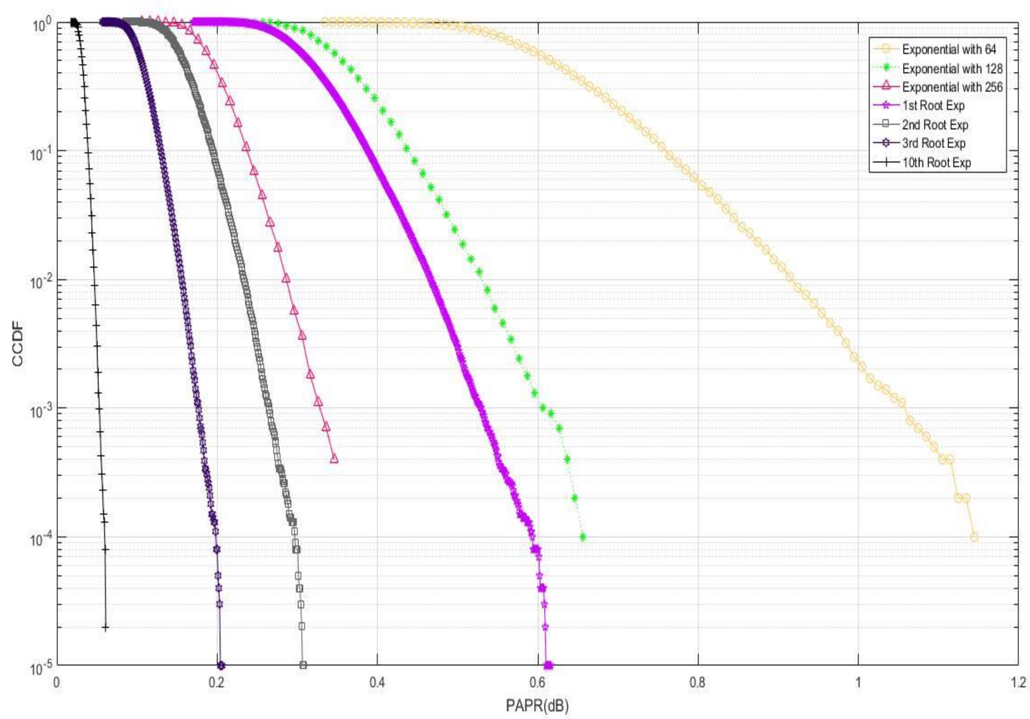


Figure 5-3 Comparison between the two proposed methods

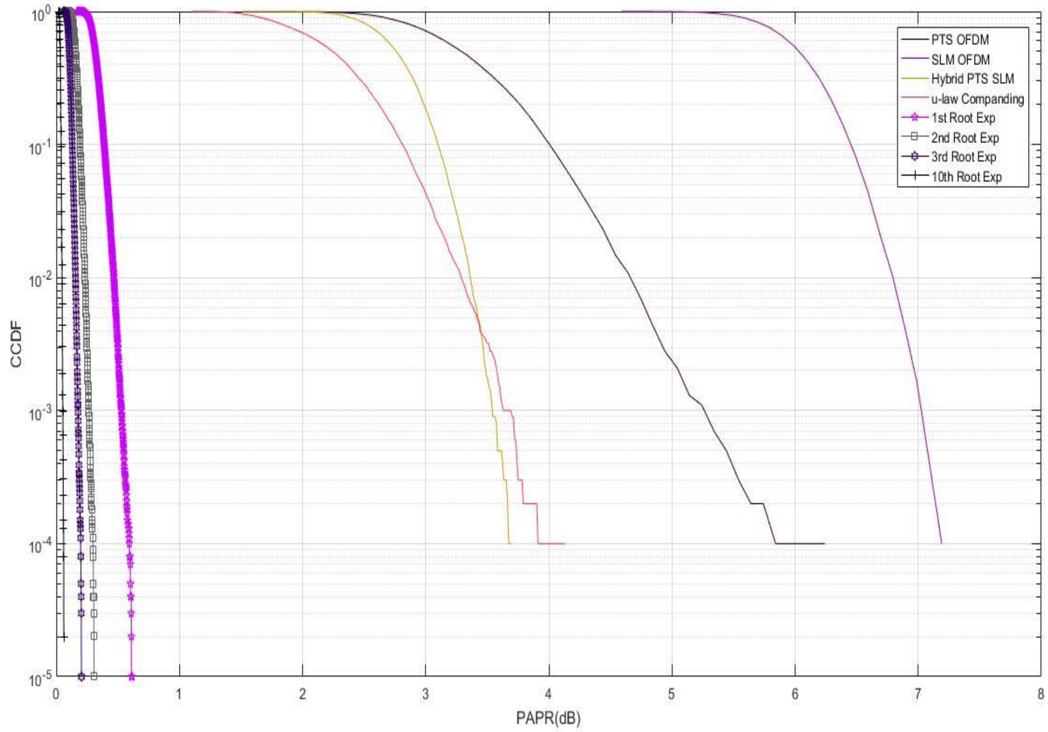


Figure 5-4 Comparison between the Modified exponential and existing methods

Now in order to estimate the reliability of the proposed method we applied it to transmit a picture. In the transmitter side equation (5-2) was implemented and in the receiving side we performed complementary operations. The before and after transmission result is shown below

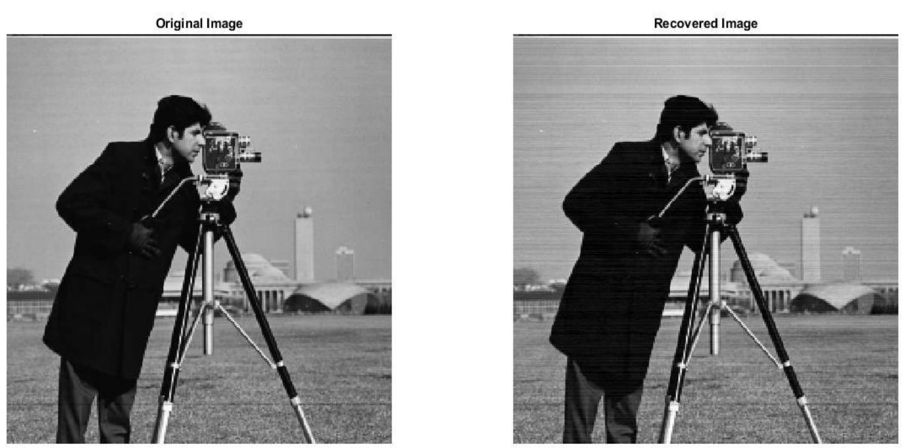


Figure 5-5 Representation of the Output Using Nth Root Exponential

CHAPTER 6

CONCLUSION

The two proposed methods are reasonably simple to implement. Both provide excellent PAPR reduction than the current state of the art technologies used. Since OFDM is a candidate for being the adequate system for 5^{th} Generation communication technology, it needs rigorous improvement. The shortcomings of OFDM restrict its deployment only to the uplink of 4G LTE systems. If it is possible to eliminate these problems than OFDM can be used with much ease and flexibility in both uplink and downlink.

Multicarrier transmission is a very attractive technique for high-speed transmission over a dispersive communication channel. The PAPR problem is one of the important issues to be addressed in developing multicarrier transmission systems. In this article we describe some PAPR reduction techniques for multicarrier transmission. Many promising techniques to reduce PAPR have been proposed, all of which have the potential to provide substantial reduction in PAPR at the cost of loss in data rate, transmit signal power increase, BER increase, computational complexity increase, and so on. No specific PAPR reduction technique is the best solution for all multicarrier transmission systems. Rather, the PAPR reduction technique should be carefully chosen according to various system requirements. In practice, the effect of the transmit filter, D/A converter, and transmit power amplifier must be taken into consideration to choose an appropriate PAPR reduction technique.

In this thesis, a simple probabilistic approach for reducing PAPR is exploited. The transformation into exponential equivalent alters the signal average and makes it close to the peak values. And as such implementation of such a system might make it impractical. If the peak powers are not scaled down then, it would result in more power consumption and the battery will drain out quickly. So it will not be favorable with a view to energy efficiency. But the current techniques are not out of the flaw as well. PTS and SLM require the transmission of sufficient data as SI. This creates less bit rate efficiency and spectral redundancy. Exponential Transform is free from SI.

In this paper, the problem of high PAPR of OFDM systems has been addressed. Simulation Results depict the improved performance of the proposed method over the various conventional techniques currently employed. It is to be noted that general applicability of a reduction method for a multi-carrier system is based on its utilization. Better performance can be achieved with the increased number of subcarriers. One of the major drawbacks of the proposed scheme is that, it does not address the concern of bit error probability. Rather with the increase of subcarriers, the bit error rate would increase exponentially. Future enhancement of the exponential transformation technique would comprise of error correction coding for improved bit error rate along with reduced PAPR at the transmitter.

Future Works

We have evaluated our proposals in terms of PAPR reduction only. We did not took the BER or SER into consideration

Future elaboration of these proposals would be achieving good BER along with reduced PAPR.

Also in order to decrease noise vulnerability, we would try to implement convenient channel coding techniques of higher order than that was introduced in this thesis.

- Analytical expression for BER with joint ICI would be developed
- Analytical expression of BER for exponential transform would be investigated
- Using more advanced RGB image the reliability of the system is going to be tested
- Higher order Nth rooting would be implemented considering average signal level
- BER for Nth root Exponential Transform will be established

REFERENCES

[1] K. W. Richard, “UMTS overview”, IEE Electronics and Communication Engineering Journal, Vol. 12, No. 3, June 2000, pp. 93 – 100

[2] Ermanno Berruto, Giovanni Colombo, Pantelis Monogioudis, Antonella Napolitano, Kyriacos Sabatakakis, “Architectural Aspects for the Evolution of Mobile Communications Toward UMTS”, IEEE Journal on Selected Areas in Communications, Vol. 15, No. 8, October 1997,pp. 1477 – 1487

[3] Erik Dahlman, Björn Gudmundson, Mats Nilsson, Johan Sköld, “UMTS/IMT-2000 Based on Wideband CDMA”, *IEEE Communications Magazine*, September 1998, pp. 70 – 80

[4] Fumiyuki Adachi, Mamoru Sawahasi, Hirohito Suda, “Wideband DS-CDMA for Next-Generation Mobile Communications Systems”, IEEE Communications Magazine, September 1998, pp. 56 – 69

[5] Josef Huber, “3G Forum Leveraging Spectum Capacity In Accordance with 3G Deployment”, *3G Forum* 17th-18th April 2001, Washington D. C., Online: http://www.umts-forum.org/iPAPRers/Leveraging_Spectrum_Capacity.pdf

[6] Douglas N. Knisely, Sarath Kumar, Subhasis Laha, Sanjiv Nanda, “Evolution of Wireless Data Services: IS-95 to cdma2000”, *IEEE Communications Magazine*, October 1998, pp. 140 - 149

[7] Powers E. and Zimmeramann, “A Digital Implementation of Multicarrier Data Modem”, Proc. Of IEEE international conference of communication technology, Philadelphia, USA, Aug., 1968

[8] Chang R. and Gibby R., “A theoretical study of performance of an orthogonal multiplexing data transmission scheme”, IEEE Transaction on Communications Technology, vol. 16, pp 529-540, Aug. 1968.

[9] BAHAI, AHMAD, RS., SALTZBERG, R., et al. Multi-carrier digital communications: theory and applications of OFDM. Springer Science & Business Media, October 2004. ISBN: 0-387-22575-7

[10] S. Dehghan, D. Lister, R. Owen, P. Jones, “W-CDMA capacity and planning issues”, *IEE Electronics and Communications Engineering Journal*, Vol. 12, No. 3, June 2000, pp. 101 – 118

[11] Louis Thibault, Minh Thien Le, “Performance Evaluation of COFDM for Digital Audio Broadcasting Part I: Parametric Study”, IEEE Transactions on Broadcasting, Vol. 43, No.1, March 1997, pp. 64 – 75

[12] Terje Tjelta, Agne Nordbotten, Marco Annoni, Enrico Scarrone, Simone Bizzarri, Laurissa Tokarchuk, John Bigham, Chris Adams, Ken Craig, Manuel Dinis, “Future Broadband Radio Access Systems for Integrated Services with Flexible Resource Management”, IEEE Communications Magazine, August 2001, pp. 56 - 63

[13] “A Guideline for the use of DVB specifications and standards”, DVB Blue Book A20, pp. 9, Online: www.dvb.org/news/pdf/dvb_cook.pdf

[14] ETSI ETS 300 744, “Framing Structure, Channel Coding and Modulation for Digital Terrestrial Television”, March 1997, Online: http://www.etsi.org

[15] ARMSTRONG, J. Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering. IEE Electronics Letters,

February 2002, vol. 38, no. 5, pp. 246–247, ISSN: 0013-5194, DOI: 10.1049/el:20020175
[16] Martin S. Roden, “Analog and Digital Communication Systems”, Prentice-Hall International, Inc, 4th Edition, 1996, ISBN: 0-13-399965-3
[17] B. P. Lathi, “Modern Digital and Analog Communication Systems”, CBS College Publishing, 1983, ISBN: 0-03-058969-X
[18] Yoshihiko Akaiwa, “Introduction to Digital Mobile Communication”, John Wiley & Sons, Inc, 1997, ISBN: 0-471-17545-5
[19] HOEHER, P. A statistical discrete-time model for the WSSUS multipath channel. IEEE Transactions on Vehicular Technology, November 1992, vol. 41, no. 4, pp.461-468. ISSN: 1939-9359, DOI: 10.1109/25.182598
[20] RUSSELL, M. and STUBER, G.L. Interchannel interference analysis of OFDM in a mobile environment. In Vehicular Technology Conference, 1995 IEEE 45th, vol. 2, pp. 820-824, ISSN: 1090-3038, DOI: 10.1109/VETEC.1995.504982
[21] TELLAMBURA, C. Computation of the Continuous–Time PAR of an OFDM Signal with BPSK Subcarriers. IEEE Communications Letter. May 2001, vol. 5, no. 5, pp. 185–87, ISSN : 1089-7798, DOI: 10.1109/4234.922754
[22] ZHU, X., ZHU, G. and JIANG, T. Reducing the peak-to-average power ratio using unitary matrix transformation. IET communications, February 2009, vol. 3, no. 2, pp.161-171, ISSN: 1751-8636, DOI: 10.1049/iet-com:20080194
[23] VAN NEE, R. and DE WILD, A. Reducing the peak-to-average power ratio of OFDM. In *Vehicular technology conference, 1998. VTC 98. 48th IEEE*, May 1998, vol. 3, pp. 2072-2076, IEEE. ISSN: 1090-3038, DOI: 10.1109/VETEC.1998.686121
[24] WUNDER, G., BOCHE, H., RAIDA, Z. Peak Value Estimation of Band Limited Signals from their Samples, noise enhancement, and a local characterization in the neighborhood of an extremum. IEEE Transactions on Signal Processing, vol. 51, No. 2003, pp. 771-780, ISSN: 1053-587, DOI: 10.1109/TSP.2002.808086
[25] SHARIF, M., GHARAVI-ALKHANSARI, M., KHALAJ, B.H. On the peak-to-average power of OFDM signals based on oversampling. IEEE Transactions on Communications, January 2003, vol. 51(1), pp.72-78, ISSN: 0090-6778, DOI: 10.1109/TCOMM.2002.807619
[26] J. Tellado, Peak to Average Power Reduction for Multicarrier Modulation, Ph.D. dissertation, Stanford Univ., 2000.
[27] JIANG, T., GUIZANI, M., CHEN, H.H., et al. Derivation of PAPR distribution for OFDM wireless systems based on extreme value theory. IEEE Transactions on Wireless Communications, April 2008, vol. 7(4), pp.1298-1305, ISSN: 1536-1276, DOI: 10.1109/TWC.2008.060862
[28] S. Boyd, “Multitone signals with low crest factor”, IEEE Transactions on Circuits and Systems, vol. CAS-33, pp. 1018 - 1022, Oct. 1986
[29] D. Wulich, “Reduction of peak to mean ratio of multicarrier modulation using cyclic coding”, Electronic Letters, 1996, Vol. 32, pp. 432 – 433

[30] S. H. Müller and J. B. Huber, "OFDM with Reduced Peak–to–Average Power Ratio by Optimum Combination of Partial Transmit Sequences," Elect. Letter, vol. 33, no. 5, Feb. 1997, pp. 368–69.

[31] H. Breiling, S. H. Müller–Weinfurtner, and J. B. Huber, "SLM Peak-Power Reduction without Explicit Side Information," IEEE Commun. Lett., vol. 5, no. 6, June 2001, pp. 239–41.

[32] G. R. Hill, M. Faulkner, and J. Singh, "Reducing the Peak-to-Average Power Ratio in OFDM by Cyclically Shifting Partial Transmit Sequences," Elect. Lett., vol. 36, no. 6, Mar. 2000, pp. 560–61.

[33] G. Wunder and H. Boche, "Upper Bounds on the Statistical Distribution of the Crest-Factor in OFDM Transmission," IEEE Trans. Info. Theory, vol. 49, no. 2, Feb. 2003, pp. 488–94.

[34] J. Heiskala and J. Terry, OFDM Wireless LANs: A Theoretical and Practical Guide, Sams Publishing, 2002.

[35] D. Kim and G. L. Stüber, "Clipping Noise Mitigation for OFDM by Decision–Aided Reconstruction," IEEE Commun. Lett., vol. 3, no. 1, Jan. 1999, pp. 4–6.

[36] H. Saeedi, M. Sharif, and F. Marvasti, "Clipping Noise Cancellation in OFDM Systems Using Oversampled Signal Reconstruction," IEEE Commun. Lett., vol. 6, no. 2, Feb. 2002, pp. 73–75.

[37] H. Chen and M. Haimovish, "Iterative Estimation and Cancellation of Clipping Noise for OFDM Signals," IEEE Commun. Lett., vol. 7, no. 7, July 2003, pp. 305–07.

[38] RYU, H.G., LEE, J.E. and PARK, J.S. Dummy sequence insertion (DSI) for PAPR reduction in the OFDM communication system. IEEE Transactions on Consumer Electronics, June 2008, vol. 50, no. 1, pp.89-94, ISSN: 0098-3063, DOI: 10.1109/TCE.2004.1277845

[39] EDFORS, O., SANDELL, M., VAN DE BEEK, J.J., et al. An introduction to orthogonal frequency-division multiplexing. Div. of Signal Processing, Research Report, September 1996.

[40] POLAK, L. and KRATOCHVIL, T. Exploring of the DVB-T/T2 performance in advanced mobile TV fading channels. In 36th International Conference on Telecommunications and Signal Processing (TSP), IEEE, 2013, pp. 768-772. ISBN: 978-1-4799-0404-4, DOI: 10.1109/TSP.2013.6614042

[41] WILSON, S.K. Digital audio broadcasting in a fading and dispersive channel, PhD Thesis, Stanford University, CA, August 1994.

[42] HOEHER, P. A statistical discrete-time model for the WSSUS multipath channel. IEEE Transactions on Vehicular Technology, November 1992, vol. 41, no. 4, pp.461-468. ISSN: 1939-9359, DOI: 10.1109/25.182598

[43] RUSSELL, M. and STUBER, G.L. Interchannel interference analysis of OFDM in a mobile environment. In Vehicular Technology Conference, 1995 IEEE 45th, vol. 2, pp. 820-824, ISSN: 1090-3038, DOI: 10.1109/VETEC.1995.504982

[44] TELLAMBURA, C. Computation of the Continuous–Time PAR of an OFDM Signal with BPSK Subcarriers. IEEE Communications Letter. May 2001, vol. 5, no. 5, pp. 185–87, ISSN : 1089-7798, DOI: 10.1109/4234.922754

[45] ZHU, X., ZHU, G. and JIANG, T. Reducing the peak-to-average power ratio using unitary matrix transformation. IET communications, February 2009, vol. 3, no. 2, pp.161-171, ISSN: 1751-8636, DOI: 10.1049/iet-com:20080194

[46] VAN NEE, R. and DE WILD, A. Reducing the peak-to-average power ratio of OFDM. In Vehicular technology conference, 1998. VTC 98. 48th IEEE, May 1998, vol. 3, pp. 2072-2076, IEEE. ISSN: 1090-3038, DOI: 10.1109/VETEC.1998.686121

[47] OCHIAI, H. and IMAI, H. On the distribution of the peak-to-average power ratio in OFDM signals. IEEE transactions on communications, February 2001, vol. 49, no. 2, pp.282-289. ISSN: 0090-6778, DOI: 10.1109/26.905885

[48] WEI, S., GOECKEL, D.L. and KELLY, P.E. A modern extreme value theory approach to calculating the distribution of the peak-to-average power ratio in OFDM systems. In Communications, 2002. ICC 2002. IEEE International Conference on, vol. 3, pp. 1686-1690. IEEE. ISBN: 0-7803-7400-2, DOI: 10.1109/ICC.2002.997136

[49] SHARIF, M., GHARAVI-ALKHANSARI, M. and KHALAJ, B.H. On the peak-to-average power of OFDM signals based on oversampling. IEEE Transactions on Communications, January 2013, vol. 51, no. 1, pp.72-78. ISSN: 0090-6778, DOI: 10.1109/TCOMM.2002.807619

[50] WUNDER, G. and BOCHE, H. Upper bounds on the statistical distribution of the crest-factor in OFDM transmission. IEEE Transactions on Information Theory, February 2003, vol. 49, no. 2, pp.488-494. ISSN: 1557-9654, DOI: 10.1109/TIT.2002.807311

[51] ZHU, X., PAN, W., LI, H., et al. Simplified approach to optimized iterative clipping and filtering for PAPR reduction of OFDM signals. IEEE Transactions on Communications, May 2013, vol. 61, no. 5, pp.1891-1901. ISSN: 0090-6778, DOI: 10.1109/TCOMM.2013.021913.110867

[52] YE, C., LI, Z., JIANG, T., et al. PAPR reduction of OQAM-OFDM signals using segmental PTS scheme with low complexity. IEEE Transactions on Broadcasting, March 2014, vol. 60, no. 1, pp.141-147. ISSN: 1557-9611, DOI: 10.1109/TBC.2013.2282732

[53] SINGH, A. and SINGH, H., 2016. Peak to average power ratio reduction in OFDM system using hybrid technique. Optik-International Journal for Light and Electron Optics, March 2016, vol. 127, no. 6, pp.3368-3371. DOI: 10.1016/j.ijleo.2015.12.105

[54] AGRAWAL, D., SHARAN, N., RAJA, M.P., et al. PAPR reduction using precoding and companding techniques for OFDM systems. 2015 International Conference on Advances in Computer Engineering and Applications (ICACEA), IEEE, March 2015, pp. 18-23, ISBN: 978-1-4673-6911-4, DOI: 10.1109/ICACEA.2015.7164696

[55] M. J. Fernández–Getino García, J. M. Páez–Borrallo, and O. Edfors, “Orthogonal Pilot Sequences for Peak- to- Average Power Reduction in OFDM,” Proc. IEEE VTC 2001-Fall, Atlantic City, NJ, Oct. 2001, pp. 650–54.

[56] Z. Yunjun, A. Yongacoglu, and J.-Y. Chouinard, “Orthogonal Frequency Division Multiple Access Peak–to–Average Power Ratio Reduction Using Optimized Pilot Symbols,” Proc. ICCT 2000, Beijing, China, Aug. 2000, pp. 574–77.
[57] G. J. Foschini, “Layered Space–Time Architecture for Wireless Communication in a Fading Environment When Using Multi-Element Antennas,” Bell Labs Tech. J., vol. 1, no. 2, Autumn 1996, pp. 41–59.
[58] G. J. Foschini and M. J. Gans, “On Limits of Wireless Communications in Fading when Using Multiple Antennas,” Wireless Pers. Commun., vol. 6, no. 3, Mar. 1998, pp. 311–35.
[59] Y. Li, J. H. Winters, and N. R. Sollenberger, “MIMO–OFDM for Wireless Communications: Signal Detection with Enhanced Channel Estimation,” IEEE Trans. Commun., vol.50, no. 9, Sept. 2002, pp. 1471–77.
[60] A. J. Paulraj et al., “An Overview of MIMO Communications — a Key to Gigabit Wireless,” Proc. IEEE, vol. 92, no. 2, Feb. 2004, pp. 198–218.
[61] G. L. Stüber et al., “Broadband MIMO-OFDM Wireless Communications,” Proc. IEEE, vol. 92, no. 2, Feb. 2004, pp. 271–94.