

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)  
ORGANISATION OF ISLAMIC COOPERATION (OIC)  
DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Mid Semester Examination  
Course Number: Math 4211  
Course Title: PDE, Special Functions, Laplace and  
Fourier

Summer Semester: 2021 - 2022  
Full Marks: 75  
Time : 1.5 Hours

There are 3 (three) questions. Answer all questions. The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the brackets.

1. a) Let,  $f(x) = \sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n n}$  is a power series, determine its interval of convergence and radius of convergence. [10] CO1 PO1
- b) A differential equation is given below: [5] CO1 PO1  
$$(x^2 + 1)y'' + xy' - y = 0$$
  
(i) Determine the singular points of the above differential equation and classify whether they are regular or irregular. [5]  
(ii) Find the power series solution of the above given differential equation about an ordinary point. [10]
2. a) Define Beta and Gamma function. Using Beta and Gamma relations, evaluate  $\int_0^1 x^{5/2}(1-x)^{5/2} dx$ . Also, Prove that  $\Gamma(n+1) = n\Gamma n = n!$  [10] CO1 PO2
- b) Find a series solution of Bessel's differential equation near a regular singular point using Frobenius method where  $v^2 = \frac{1}{4}$ . [15] CO1 PO1
3. a) (i) Find the Fourier series expansion of the function: [15] CO2 PO1  
$$f(x) = \begin{cases} -x, & -\pi \leq x \leq 0 \\ x, & 0 < x \leq \pi \end{cases}$$
  
(ii) Sketch the graph of  $f(x)$  for the values of  $x$  from  $-5\pi$  to  $5\pi$ .
- b) Evaluate the followings: [5] CO2 PO1  
(i)  $L\{f(t)\}$  where  $f(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 2, & t \geq 3 \end{cases}$  [5]  
(ii)  $L^{-1}\left\{\frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)}\right\}$  [5]

The End