(26)

17 February, 2023(afternoon)

M Sc. Eng./PhD(M)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING (MPE)

Mid Semester Examination — Semester: Summer 2022
Course Code: Math 6103 — Time : 1½ hours
Course Title: Advanced Mathematics — Full Marks: 75

There are **4 (Four)** Questions. Answer any **3 (Three)** of them. All Questions carry equal Marks. Programmable calculators are not allowed. Do not write on this question paper. The Symbols have their usual meaning.

1. a) Sketch and describe the graphs of the following functions in an *xyz*-coordinate system:

(i) $f(x,y)=1-\frac{1}{2}x-y$

(*ii*) $f(x,y)=\sqrt{1-x^2-y^2}$

(*iii*) $f(x,y)=\sqrt{x^2+y^2}$

b) (i) Identify the surface and sketch the contour plot of $f(x,y)=x^2+4y^2$ using level curves of height $k = 0, 1, 2, 3, 4, 5$.

(ii) Sketch the graph of the function $f(x,y)=x^2-y^2$ in *xyz*-space and the contour plot using level curves of with different level values.

2. a) Locate all relative maxima, relative minima, and saddle points, if any of the function $f(x, y) = x^2 + xy + y^2 - 3x$.

b) Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 48 ft3, and requiring the least amount of material for its construction.

3. a) Evaluate $\iint_R (2x-y^2)dA$; over the region R enclosed between the lines $y = -x+1$, $y = x+1$, and $y = 3$, using (i) type I and (ii) type II region.

b) Use double integration to find the volume of the tetrahedron bounded by the coordinate planes and the plane $4x + 2y + z = 4$.

4. a) Find an equation for the (i) tangent plane and (ii) parametric equations for the normal line to the surface $xz - yz^3 + yz^2 = 2$ at the point P(2, –1, 1). (iii) Find the acute angle that the tangent plane at the point (2, –1, 1) makes with the xy-plane.

b) Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + xy^2 z$ at the point (1,–2, 4) in the direction of the vector $\mathbf{a} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. What is the magnitude and direction of maximum rate of change of $f(x, y, z)$ at this point?

--- --- --- --- --- --- --- --- ---