B.Sc. Engg. (EE), 4th Sem.                                    Date: February 23, 2023 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

**DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING**

Mid-Semester Examination                                Summer Semester, A.Y. 2021-2022
Course No.: EEE 4407                                     Time: 90 Minutes
Course Title: Random Signals and Processes              Full Marks: 75

There are **3 (three)** questions. Answer **all** the questions. All questions carry equal marks. Marks in the margin indicate full marks. Programmable calculators are not allowed. Do not write on this question paper.

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1. a) Define sample space and event space. Explain their differences with appropriate example.

   05
   (CO1, PO1)

   b) In an experiment, $A$, $B$, $C$, and $D$ are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$. Furthermore, $A$ and $B$ are disjoint, while $C$ and $D$ are independent.

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   (CO1, PO1)

   i) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^C]$, and $P[A \cup B^C]$.
   ii) Are $A$ and $B$ independent?
   iii) Find $P[D]$, $P[C \cap D^C]$, $P[C^C \cap D^C]$, and $P[C|D]$.
   iv) Find $P[C \cup D]$ and $P[C \cup D^C]$.

   c) You have a shuffled deck of three cards: 2, 3 and 4 and you deal out the three cards. Let $E_i$ denote the event that i-th card dealt is even numbered.

   10
   (CO1, PO1)

   i.   Find $P[E_2|E_1]$, the probability the second card is even given that the first card is even.
   ii.  Find the conditional probability that the first two cards are even given that the third card is even.
   iii. Let $O_i$ represent the event that the i-th card dealt is odd numbered. Find $P[E_2|O_1]$, the conditional probability that the second card is even Given that the first card is odd.
   iv.  Find the conditional probability that the second card is odd given that the first card is odd.

2. a) Let X be a random variable with CDF

   06
   (CO2, PO2)

   $$F_X(x) = \begin{cases} 0 & x < -1, \\ x/3 + 1/3 & -1 \le x < 0, \\ x/3 + 2/3 & 0 \le x < 1, \\ 1 & 1 \le x. \end{cases}$$

   Sketch the CDF and find the following:
   i.   P $[X < -1]$ and P $[X \le -1]$,
   ii.  P $[X < 0]$ and P $[X \le 0]$,
   iii. P $[0 < X \le 1]$ and P $[0 \le X \le 1]$.

b) The probability density function of random variable Y is

$$f_Y(y) = \begin{cases} 1/10 & 0 \le y < 10, \\ 0 & Otherwise \end{cases}$$

09
(CO2, PO2)

Find the following:

i. P [Y ≤ 6]
ii. the conditional PDF $f_{Y|Y \le 6}(y)$
iii. P [Y > 8]
iv. the conditional PDF $f_{Y|Y>8}(y)$
v. E [Y |Y ≤ 6]
vi. E [Y |Y > 8]

c) The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.01. Given T > 0.02, Find E [T|T > 0.02], the conditional expected value of T.

10
(CO2, PO2)

3. a) Determine the expected value of Poisson random variable.

05
(CO2, PO2)

b) The number of buses that arrive at a bus stop in T minutes is a Poisson random variable B with expected value T/5.

10
(CO2, PO2)

i. Find the PMF of B, the number of buses that arrive in T minutes.
ii. Find the probability that in a two-minute interval, three buses will arrive.
iii. Find the probability of no buses arriving in a 10-minute interval.
iv. How much time should you allow so that with probability 0.99 at least one bus arrives?

c) Let X have the binomial PMF

$$P_X(x) = \binom{5}{x} \left(\frac{1}{2}\right)^5$$

10
(CO2, PO2)

i. Find the standard deviation of the random variable X.
ii. Find $P[\mu_X - \sigma_X \le X \le \mu_X + \sigma_X]$, the probability that X is within one standard deviation of the expected value.

# CDF chart for standard normal distribution

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|------|------|------|------|------|------|------|------|------|------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7703 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.90147 |
| 1.3 | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| 2.2 | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | $0.9^2 0097$ | $0.9^2 0358$ | $0.9^2 0613$ | $0.9^2 0863$ | $0.9^2 1106$ | $0.9^2 1344$ | $0.9^2 1576$ |
| 2.4 | $0.9^2 1802$ | $0.9^2 2024$ | $0.9^2 2240$ | $0.9^2 2451$ | $0.9^2 2656$ | $0.9^2 2857$ | $0.9^2 3053$ | $0.9^2 3244$ | $0.9^2 3431$ | $0.9^2 3613$ |
| 2.5 | $0.9^2 3790$ | $0.9^2 3963$ | $0.9^2 4132$ | $0.9^2 4297$ | $0.9^2 4457$ | $0.9^2 4614$ | $0.9^2 4766$ | $0.9^2 4915$ | $0.9^2 5060$ | $0.9^2 5201$ |
| 2.6 | $0.9^2 5339$ | $0.9^2 5473$ | $0.9^2 5604$ | $0.9^2 5731$ | $0.9^2 5855$ | $0.9^2 5975$ | $0.9^2 6093$ | $0.9^2 6207$ | $0.9^2 6319$ | $0.9^2 6427$ |
| 2.7 | $0.9^2 6533$ | $0.9^2 6636$ | $0.9^2 6736$ | $0.9^2 6833$ | $0.9^2 6928$ | $0.9^2 7020$ | $0.9^2 7110$ | $0.9^2 7197$ | $0.9^2 7282$ | $0.9^2 7365$ |
| 2.8 | $0.9^2 7445$ | $0.9^2 7523$ | $0.9^2 7599$ | $0.9^2 7673$ | $0.9^2 7744$ | $0.9^2 7814$ | $0.9^2 7882$ | $0.9^2 7948$ | $0.9^2 8012$ | $0.9^2 8074$ |
| 2.9 | $0.9^2 8134$ | $0.9^2 8193$ | $0.9^2 8250$ | $0.9^2 8305$ | $0.9^2 8359$ | $0.9^2 8411$ | $0.9^2 8462$ | $0.9^2 8511$ | $0.9^2 8559$ | $0.9^2 8605$ |
| 3.0 | $0.9^2 8650$ | $0.9^2 8694$ | $0.9^2 8736$ | $0.9^2 8777$ | $0.9^2 8817$ | $0.9^2 8856$ | $0.9^2 8893$ | $0.9^2 8930$ | $0.9^2 8965$ | $0.9^2 8999$ |
| 3.1 | $0.9^3 0324$ | $0.9^3 0646$ | $0.9^3 0957$ | $0.9^3 1260$ | $0.9^3 1553$ | $0.9^3 1836$ | $0.9^3 2112$ | $0.9^3 2378$ | $0.9^3 2636$ | $0.9^3 2886$ |
| 3.2 | $0.9^3 3129$ | $0.9^3 3363$ | $0.9^3 3590$ | $0.9^3 3810$ | $0.9^3 4024$ | $0.9^3 4230$ | $0.9^3 4429$ | $0.9^3 4623$ | $0.9^3 4810$ | $0.9^3 4991$ |
| 3.3 | $0.9^3 5166$ | $0.9^3 5335$ | $0.9^3 5499$ | $0.9^3 5658$ | $0.9^3 5811$ | $0.9^3 5959$ | $0.9^3 6103$ | $0.9^3 6242$ | $0.9^3 6376$ | $0.9^3 6505$ |
| 3.4 | $0.9^3 6631$ | $0.9^3 6752$ | $0.9^3 6869$ | $0.9^3 6982$ | $0.9^3 7091$ | $0.9^3 7197$ | $0.9^3 7299$ | $0.9^3 7398$ | $0.9^3 7493$ | $0.9^3 7585$ |
| 3.5 | $0.9^3 7674$ | $0.9^3 7759$ | $0.9^3 7842$ | $0.9^3 7922$ | $0.9^3 7999$ | $0.9^3 8074$ | $0.9^3 8146$ | $0.9^3 8215$ | $0.9^3 8282$ | $0.9^3 8347$ |
| 3.6 | $0.9^3 8409$ | $0.9^3 8469$ | $0.9^3 8527$ | $0.9^3 8583$ | $0.9^3 8637$ | $0.9^3 8689$ | $0.9^3 8739$ | $0.9^3 8787$ | $0.9^3 8834$ | $0.9^3 8879$ |
| 3.7 | $0.9^3 8922$ | $0.9^3 8964$ | $0.9^4 0039$ | $0.9^4 0426$ | $0.9^4 0799$ | $0.9^4 1158$ | $0.9^4 1504$ | $0.9^4 1838$ | $0.9^4 2159$ | $0.9^4 2568$ |
| 3.8 | $0.9^4 2765$ | $0.9^4 3052$ | $0.9^4 3327$ | $0.9^4 3593$ | $0.9^4 3848$ | $0.9^4 4094$ | $0.9^4 4331$ | $0.9^4 4558$ | $0.9^4 4777$ | $0.9^4 4988$ |
| 3.9 | $0.9^4 5190$ | $0.9^4 5385$ | $0.9^4 5573$ | $0.9^4 5753$ | $0.9^4 5926$ | $0.9^4 6092$ | $0.9^4 6253$ | $0.9^4 6406$ | $0.9^4 6554$ | $0.9^4 6696$ |
| 4.0 | $0.9^4 6833$ | $0.9^4 6964$ | $0.9^4 7090$ | $0.9^4 7211$ | $0.9^4 7327$ | $0.9^4 7439$ | $0.9^4 7546$ | $0.9^4 7649$ | $0.9^4 7748$ | $0.9^4 7843$ |
| 4.1 | $0.9^4 7934$ | $0.9^4 8022$ | $0.9^4 8106$ | $0.9^4 8186$ | $0.9^4 8263$ | $0.9^4 8338$ | $0.9^4 8409$ | $0.9^4 8477$ | $0.9^4 8542$ | $0.9^4 8605$ |
| 4.2 | $0.9^4 8665$ | $0.9^4 8723$ | $0.9^4 8778$ | $0.9^4 8832$ | $0.9^4 8882$ | $0.9^4 8931$ | $0.9^4 8978$ | $0.9^5 0226$ | $0.9^5 0655$ | $0.9^5 1066$ |
| 4.3 | $0.9^5 1460$ | $0.9^5 1837$ | $0.9^5 2199$ | $0.9^5 2545$ | $0.9^5 2876$ | $0.9^5 3193$ | $0.9^5 3497$ | $0.9^5 3788$ | $0.9^5 4066$ | $0.9^5 4332$ |
| 4.4 | $0.9^5 4587$ | $0.9^5 4831$ | $0.9^5 5065$ | $0.9^5 5288$ | $0.9^5 5502$ | $0.9^5 5706$ | $0.9^5 5902$ | $0.9^5 6089$ | $0.9^5 6268$ | $0.9^5 6439$ |
| 4.5 | $0.9^5 6602$ | $0.9^5 6759$ | $0.9^5 6908$ | $0.9^5 7051$ | $0.9^5 7187$ | $0.9^5 7318$ | $0.9^5 7442$ | $0.9^5 7561$ | $0.9^5 7675$ | $0.9^5 7784$ |
| 4.6 | $0.9^5 7888$ | $0.9^5 7987$ | $0.9^5 8081$ | $0.9^5 8172$ | $0.9^5 8258$ | $0.9^5 8340$ | $0.9^5 8419$ | $0.9^5 8494$ | $0.9^5 8566$ | $0.9^5 8634$ |
| 4.7 | $0.9^5 8699$ | $0.9^5 8761$ | $0.9^5 8821$ | $0.9^5 8877$ | $0.9^5 8931$ | $0.9^5 8983$ | $0.9^6 0320$ | $0.9^6 0789$ | $0.9^6 1235$ | $0.9^6 1661$ |
| 4.8 | $0.9^6 2067$ | $0.9^6 2453$ | $0.9^6 2822$ | $0.9^6 3173$ | $0.9^6 3508$ | $0.9^6 3827$ | $0.9^6 4131$ | $0.9^6 4420$ | $0.9^6 4696$ | $0.9^6 4958$ |
| 4.9 | $0.9^6 5208$ | $0.9^6 5446$ | $0.9^6 5673$ | $0.9^6 5889$ | $0.9^6 6094$ | $0.9^6 6289$ | $0.9^6 6475$ | $0.9^6 6652$ | $0.9^6 6821$ | $0.9^6 6981$ |

Example: $\Phi(3.39) = 0.9996505$, $\Phi(0.98) = 0.8365$.