

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Mid-Semester Examination
Course No.: Math 4221/Math 4629
Course Title: Mathematics III

Summer Semester, A. Y. 2021-2022
Time: 90 Minutes
Full Marks: 75

There are 3 (**three**) questions. Answer all 3 (**three**) questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

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1. (a) Prove that $\mathbf{a} \times \mathbf{b} \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ (9)
hence find the value of, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b})$. (CO1)
(PO1)
- (b) If the volume of the parallelepiped with edges $\mathbf{a}=2\mathbf{i}-\mathbf{j}-\mathbf{k}$, $\mathbf{b}=3\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ and $\mathbf{c}=5\mathbf{i}-m\mathbf{j}+3m\mathbf{k}$ is 28 find the value of m . (8)
(CO1)
(PO1)
- (c) Explain the physical meaning of $[\mathbf{a}+\mathbf{b} \ \mathbf{b}+\mathbf{c} \ \mathbf{c}+\mathbf{a}] = 2[\mathbf{a}\mathbf{b}\mathbf{c}]$ and prove this relation. (8)
(CO1)
(PO1)
2. (a) Prove that $\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot \mathbf{A} + \phi (\nabla \cdot \mathbf{A})$ and hence find $\nabla \cdot (r^3 \mathbf{r})$ (12)
(CO2)
(PO2)
- (b) Determine the constant a, b, c so that vector $\mathbf{v}=(-4x-3y+az)\mathbf{i}+(bx+3y+5z)\mathbf{j}+(4x+cy+3z)\mathbf{k}$ is irrotational. (13)
(CO2)
(PO2)
Find a scalar function ϕ so that $\mathbf{v}=\nabla\phi$.
3. (a) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}=-3x^2\mathbf{i}+5xy\mathbf{j}$ and C is the curve $y=4x^2$ in (12)
(CO3)
(PO2)
the xy plane from $(1, 4)$ to $(2, 16)$.
- (b) Evaluate $\iiint_V \mathbf{F} \cdot d\mathbf{V}$ where $\mathbf{F}=2x^2\mathbf{i}-xz\mathbf{j}+y^2z\mathbf{k}$ and V is the region bounded by the (13)
(CO3)
(PO2)
surfaces $x=0, x=6, y=0, z=y^2, z=4$.