ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2021-2022**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) Determine the truth value of each of these statements: 1×3 (CO1) (PO1)
   i. $\forall x \forall y \exists z \left(z = \frac{x+y}{2}\right), where\ x, y, and\ z \in Z$
   ii. $\forall x \forall y \exists z\ (z = x + y), where\ x, y, and\ z \in Z$
   iii. $\forall x \exists y\ (y \neq 0 \rightarrow xy = 1), where\ x\ and\ y \in Z$

   b) Consider the following statements. 2×2 (CO1) (PO1)
   $P(x)$: "$x\ can\ speak\ English.$"
   $Q(x)$: "$x\ knows\ the\ computer\ language\ C++.$"

   Express following sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. Assume the domain for quantifiers consists of all students at your school.
   i. No student at your school can speak English or knows C++
   ii. There is a student at your school who can speak English but who does not know C++.

   c) Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology using the logical equivalence rules. 9 (CO1) (PO1)

   d) Consider the following system specifications: "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode." 9 (CO1) (PO1)
   Determine if the mentioned system specifications are consistent.

2. a) Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use? 6+2 (CO1) (PO1)

   b) Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1. Apply the mathematical induction principle to prove that $\forall_n P(n)$ is true. 8 (CO2) (PO1)

   c) Show that the premises "If you send me a world cup match ticket, then I will go Qatar to watch the match", "If you do not send me world cup match ticket, then I will go to a tour in Malaysia", and "If I go to a tour in Malaysia, then I will feeling refreshed" lead to the conclusion "If I do not go Qatar to watch the match, then I will feel refreshed." 9 (CO2) (PO2)

3. a) Show that the sequence $a_n = n4^n$ is a solution of the recurrence relation: 8 (CO2) (PO1)
   $$a_n = 8a_{n-1} - 16a_{n-2}.$$

   b) Show that $f\ o\ g\ (x)$ is bijection from $R\ to\ R$, where $f(x) = -3x + 4$ and $g(x) = x + 2$. Determine $f\ o\ g^{-1}\ (x)$. 9 (CO2) (PO1)

   c) Use set builder notation and logical equivalences to show that 8 (CO2) (PO1)
   $A \oplus B = (A \cup B) - (A \cap B)$, where A and B are two sets.