ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION   **SUMMER SEMESTER, 2021-2022**
DURATION: 1 HOUR 30 MINUTES   **FULL MARKS: 75**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. There is a pair of identical twins named Edward and Edwin, who are indistinguishable in appearance. One day, shortly after they were grown, a strange disease struck them both and changed their lives forever.

   20 + 5
   (CO2)
   (PO2)

   Henceforth, each twin was in one of three psychological states – State 1, State 2, or State 3, which alternated in a constant cyclical pattern: 1, 2, 3, 1, 2, 3, 1, ... and so on. Curiously enough, at any given time, both brothers were in the same state — both were in either State 1, or State 2, or State 3. There was, however, a crucial difference –

   i. Edward always lied when he was in State 1, but told the truth in the other two states.

   ii. Edwin, on the other hand, lied when in State 2, but told the truth when in State 1 or State 3.

   One day, one of the brothers was asked:

   "Are you either Edwin in State 2 or Edward not in State 1?"

   Based on your examination of his reply, answer the following questions -

   a) Can one deduce whether he is Edward or Edwin?

   b) Is it possible to deduce what state he is in?

2. a) Three friends, p, q, and r of the same university, intend to go out on a short tour. However, everything was going fine until p and q fell apart due to differences in opinion regarding the tour-destination. r on the other hand, has no clash with either of them. However, if he is free, he prefers the destination that q proposed over p. Given the clash between p and q, consider the following two propositions.

   3 + 6
   (CO1)
   (PO1)

   Proposition 1: It appears that, r will accompany whenever q wants to go on a tour. If that is the case, it may be concluded that r will go and p will avoid the tour. In short, the tour will take place.

   Proposition 2: Alternatively speaking, the tour will take place when all the following conditions are satisfied –

   i. q goes on a tour alone, or he may take r along with him and not p.

   ii. Either r or p do not go on a tour.

   Generate the logical connectives of these propositions. From your understanding of propositional equivalence, verify whether they are equivalent or not.

b) Consider the following two propositions - 8 + 8

Proposition 1: Every student in this class has taken a course in computer programming or (CO1)
participated in programming contests. (PO1)
Proposition 2: There is a student in this class who has secured A+ in at least two courses
of the previous semester. Now, answer the following questions -

    i. Express each of these propositions in terms of predicates and quantifiers considering
both local and universal domain.

    ii. Considering the logical connectives of these propositions, correlate and validate De Morgan's law for predicates and quantifiers with proper justification.

3. a) Generate the logical expression of the following statements and find their converse and con- 3 × 2
trapositive - (CO1)

    i. If it snows tonight, then I will stay at home. (PO1)

    ii. I go to the beach whenever it is a sunny summer day.

    iii. When I stay up late, it is necessary that I sleep until noon.

b) Prove that if x is a real number, then $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$. 7
(CO2)
(PO2)

c) The English alphabet consists of 26 letters. There are 5 vowels and the rest are consonants. 4 × 3
Among these 26 letters, 13 of them are odd and 13 are even. For instance, (a, c, e, ...) are (CO1)
odd letters and (b, d, f, ...) are even letters. (PO1)

Consider the set of lowercase English alphabets as the universal set U, where elements are
arranged in alphabetical order from a to z. Consider the five sets A, B, C, D, and E, all of
which are subsets of U. The sets are defined as follows -

$A = \{x_i \mid x_i \in U$ and x_i is a vowel$\}$
$B = \{x_i \mid x_i \in U$ and x_i is an odd letter$\}$
$C = \{x_i \mid x_i \in U$ and i is a multiple of 3$\}$
$D = \{x_i \mid x_i \in U$ and i is a perfect square$\}$
$E = \{x_i \mid x_i \in U$ and i is a multiple of 7$\}$

Based on the above information, answer the following questions -

    i. What are the elements of the sets A, B, C, D, and E?

    ii. Find P(E) and show that, $|P(E)| = 2^{|E|}$.

    iii. Find out the bit-string representation of all the sets.

    iv. Find out the sets resulting from the operations - $(A \cap B)$, $(C \cap D)$, $(B - A)$, and $(A - B)$.