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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2021-2022**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) The upward velocity of a rocket is given as a function of time according to Table 1: 7 + 5 + 6 (CO2) (PO1)

Table 1: Time vs Velocity data

t (second)	Velocity (meter/second)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

i. Determine the velocity of the rocket at $t = 11$ seconds using Second Order (quadratic) Lagrangian polynomial interpolation.

ii. Find the acceleration of the rocket at $t = 11$ seconds.

iii. Find the distance travelled by the rocket from $t = 10$ to $t = 14$ seconds.

b) Provide an analytical comparison among the solution approaches of Direct Interpolation, Newton's Divided Difference Interpolation and Lagrangian Interpolation. 7 (CO3) (PO2)

2. a) i. When we use a calculator to find the square root of a positive number R, then the calculator performs some sort of numerical method to find it. One such method is the Newton-Raphson method. Prove that the Newton-Raphson equation for finding the square root of R is: 6 (CO1) (PO1)

$$X_{i+1} = \frac{1}{2}(X_i + \frac{R}{X_i})$$

ii. Suppose you are trying to find the square root of 30. If you directly use your calculator, you will get the result as 5.477. Simulate how the calculator will find this value by using the Newton-Raphson equation you derived in 2.a)i., taking an initial guess of $X_0 = 3$. Show the absolute relative approximate error at every iteration. 7 (CO2) (PO1)

b) Compute $f(4)$, $f'(4)$ and $f''(4)$ using Taylor Series given that $f(5) = -30$, $f'(5) = 10$, $f''(5) = 16$ and $f'''(5) = 6$ and all higher derivatives of $f(x)$ at $x = 5$ are zero. 12 (CO2) (PO1)

3. a) Find the number of significant digits for each of the following figures: 2×4 (CO2) (PO1)

   i. 817000

   ii. 81.7000

   iii. 21090

   iv. 0.0314159×10^2

   b) i. Why do we need Spline Interpolation? Describe a scenario where Direct method, Newton's Divided Difference method and Lagrangian method would fail but Spline interpolation would not. 8 (CO3) (PO2)

   ii. Suppose you want to perform Spline interpolation on n data points given as (x_0, y_0), (x_1, y_1), ..., (x_{n-1}, y_{n-1}), (x_n, y_n). You want to fit the data using Cubic splines of the form $f(x) = ax^3 + bx^2 + cx + d$. How many unknowns and how many equations do you have to deal with, and how can you obtain those equations? 9 (CO1) (PO2)