C.E.E (1)

Name of the Program: B. Sc. in Civil Engineering **Date: 15 May, 2023**
Semester: 2nd semester **10:00 A.M to 1:00 P.M**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

Semester Final Examination **Summer Semester: 2021 - 2022**
Course Number: MATH 4253 **Full Marks: 150**
Course Title: Vector Algebra, Vector Calculus and ODE **Time: 3.0 Hours**

There are 6 (Six) questions. Answer all questions. Programmable calculators are not allowed. Do not write on this question paper. The figures in the right margin indicate full marks and corresponding CO and PO. The Symbols have their usual meaning.

1. Solve the following differential equations

(a) $xp^3 + (x^2 + 2xy + y)p^2 + (x^2y + xy + xy^2 + 2y^2)p + xy^2 + y^3 = 0$ (8) (CO1) (PO1)

(b) $\frac{d^5y}{dx^5} + 2\frac{d^4y}{dx^4} + 2\frac{d^3y}{dxy} - 4\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 10y = 0$ (8) (CO1) (PO1)

(c) $(D^3 - 6D^2 + 11D - 6)y = e^{2x} + x^2$ (9) (CO1) (PO1)

2.(a) Solve: $[(x^3D^3 + 3x^2D^2 + xD + 1)y = xlnx + \frac{1}{x}$ (12) (CO1) (PO1)

(b) In an L-R-C circuit an inductance of 3 henry, a resistor of 18 ohoms and a capacitor of $\frac{1}{27}$ farads have been connected through battery e.m.f. of $E = 300sint$ volts . At t=*0* the charge on the capacitor and current in a circuit are zero. Find the charge and current at any time *t >0* (13) (CO2) (PO2)

3.(a) Solve: $(D^2 - 6D + 13)y = 8e^{3x}sin2x$ (9) (CO1) (PO1)

(b) At any point of the curve $x = t^2 - 1, y = 4t - 3$ and $z = 2t^2$ - 6t, where t is any variable, find the Tangent vector, unit tangent vector, Normal vector and unit normal vector at $t = 2$ (8) (CO2) (PO2)

(c) Find the value of $\bar{r}$ satisfying the equation $\frac{d^2\bar{r}}{dt^2} = \bar{a}$, where $\bar{a}$ is a constant vector. Also is given that when $t = o, \bar{r} = o$ and $\frac{d\bar{r}}{dt} = \bar{u}$ (8) (CO2) (PO2)

4.(a) Define directional derivative of a scalar point function, find the directional derivative of $\emptyset(x,y,z) = e^{2x}\cos yz$ at the origin in the direction of the tangent to the curve $x = a\sin t, y = a\cos t$ and $z = at$, at $t = \frac{\pi}{4}$ (12) (CO3) (PO2)

(b) Obtain the equations of the tangent plane and normal to the surface $x^2 + y^2 + z^2 = 25$ *at the point* (4,0,3) (13) (CO3) (PO2)

5.(a) Find the circulation of $\overline{F}$ round the curve c, where $\overline{F} = (2x + y^2)\,\hat{\imath} + (3y - 4x)\hat{\jmath}$ and c is the curve $y = x^2$ from (0,0) to (1,1) and the curve $y^2 = x$ from (1,1) to (0,0) (8) (CO3) (PO2)

(b) Give the physical significance of the Curl of a vector point function. (9) (CO2) (PO1)

(c) Use Green's theorem to evaluate $\int_c (x^2 + xy)\,dx + (x^2 + y^2)dy$, where c is the square formed by the lines $y = \pm 1$ and $x = \pm 1$. (8) (CO3) (PO2)

6. Give the statement of Gauss' divergence theorem. Verify Gauss' theorem for the vector field $\overline{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ over the region bounded by $x^2 + y^2 = 4, z = 0$ *and* $z = 3$. (25) (CO3) (PO2)