B. Sc. Engg. (CEE)/ 8th Sem. 15 May 2023 (Group A)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

Final Examination **Summer Semester: 2021-2022**

Course No.: CEE 4815 **Full Marks: 150**

Course Title: Introduction to Finite Element Method **Time: 3.0 Hours**

There are 6 (Six) questions. Answer 5 questions. Questions 5 and 6 are compulsory. Answer 3 (Three) questions from questions 1 to 4. The Symbols have their usual meaning. The figures in the right margin indicate full marks.

1(a). Explain isoparametric element concept in finite element method. (6) (CO1) (PO1)

1(b). Explain the limitations of Jacobian Matrix. (6) (CO1) (PO1)

1(c). Derive the shape functions of a 3-nodal triangular element in the two-dimensional condition for the local coordinate system. (13) (CO1) (PO1)

2. Derive the general stiffness matrix for soil-water coupling condition from $\{\Delta F\} = \int_V [B]^T \{\Delta\sigma\} dV$ using effective stress theory. (25) (CO1) (PO1)

3. Derive the shape function and stiffness matrix for a 4-nodal interface element in the local coordinate system using the principle of virtual work. (25) (CO1) (PO1)

4. Answer the following questions for the finite element mesh shown in Fig.1. (25) (CO1) (PO1)
   (i) Calculate the band width of the mesh.
   (ii) Form $[K]_G \{\Delta\delta\}_{nG} = \{\Delta F\}_G$ considering 2 degrees of freedom of each node (u, v).
   (iii) Include the the boundary conditions in the global finite element equation where the nodal displacements at node 4 are (u = -0.2m, v = -0.5m).

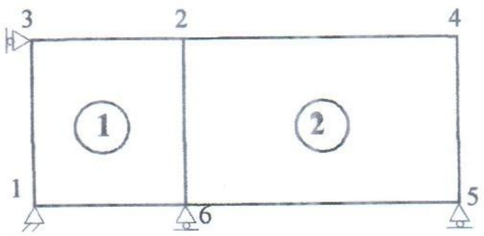


Fig.1

Stiffness matrix of each element is –

$$[K^e] = \begin{bmatrix} K^e_{11} & K^e_{12} & K^e_{13} & K^e_{14} & K^e_{15} & K^e_{16} & K^e_{17} & K^e_{18} \\ K^e_{21} & K^e_{22} & K^e_{23} & K^e_{24} & K^e_{25} & K^e_{26} & K^e_{27} & K^e_{28} \\ K^e_{31} & K^e_{32} & K^e_{33} & K^e_{34} & K^e_{35} & K^e_{36} & K^e_{37} & K^e_{38} \\ K^e_{41} & K^e_{42} & K^e_{43} & K^e_{44} & K^e_{45} & K^e_{46} & K^e_{47} & K^e_{48} \\ K^e_{51} & K^e_{52} & K^e_{53} & K^e_{54} & K^e_{55} & K^e_{56} & K^e_{57} & K^e_{58} \\ K^e_{61} & K^e_{62} & K^e_{63} & K^e_{64} & K^e_{65} & K^e_{66} & K^e_{67} & K^e_{68} \\ K^e_{71} & K^e_{72} & K^e_{73} & K^e_{74} & K^e_{75} & K^e_{76} & K^e_{77} & K^e_{78} \\ K^e_{81} & K^e_{82} & K^e_{83} & K^e_{84} & K^e_{85} & K^e_{86} & K^e_{87} & K^e_{88} \end{bmatrix}$$

5. Answer the following questions considering plane strain condition for a square element shown in Fig.2. The [B] matrix of the element is (30) (CO2) (PO2)

$$[B]=\begin{bmatrix}0.5 & 0 & -0.5 & 0 & -0.5 & 0 & 0.5 & 0\\ 0 & 0.5 & 0 & 0.5 & 0 & -0.5 & 0 & -0.5\\ 0.5 & 0.5 & 0.5 & -0.5 & -0.5 & -0.5 & -0.5 & 0.5\end{bmatrix}$$

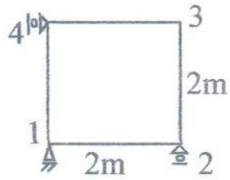


Fig. 2

(i) Form nodal displacement vector $\{d\delta\}$ considering displacements of node 3 (0, -0.20m), and node 4 (0,-0.20m).

(ii) Calculate strain increment of the element, $\{\Delta\varepsilon\}=[B]\{\Delta\delta\}$.

(iii) Calculate elastic stress-strain matrix [D] using E=200MPa and ν=1/3.

(iv) Calculate stress increment $\{\Delta\sigma\}$.

(v) Calculate stiffness matrix.

6(a). Answer the following questions for a beam element: (20) (CO1) (PO1)

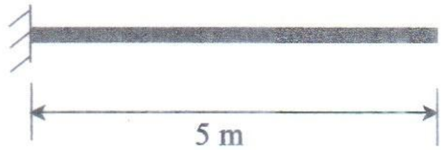
(i) Derive shape function, [N] - Matrix.

(ii) Derive strain displacement matrix, [B] – Matrix.

(iii) Derive stiffness matrix, [K] – Matrix.

6(b). Calculate axial forces, shear forces, and bending moments at both ends of a cantilever beam, shown in Fig.3, using the stiffness matrix derived in Q.6(a), where at the free end, u = -25 mm, v = -200 mm, and θ = -0.06 radian. The length of the beam is 5.0 m, axial stiffness EA = 2000 kN, and flexural stiffness EI = 500 kN-m^2. (15) (CO2) (PO2)

Fig.3



6(c). Calculate the consistent and lumped mass matrices of a truss member having a length of 5.0 m. The cross-sectional area of the truss member is 0.10 m^2, and density of the truss material is 2450 kg/m^3. Also, briefly explain which mass matrix should be used for a dynamic analysis. (10) (CO2) (PO2)