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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2021-2022

DURATION: 3 HOURS FULL MARKS: 150

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Suppose you are given the data shown in Table 1:

Table 1: t vs $f(t)$ data for Question 1.a)

t	$f(t)$
0	0
0.5	0.19
1.0	0.26
1.5	0.29
2.0	0.31

i. Using Taylor series, derive the Central Difference Approximation formula for $f''(x_i)$. 7 (CO1) (PO1)

ii. Find $f'(1.5)$ and $f''(1.5)$. 2×3 (CO2) (PO1)

b) Prove whether Central Difference Approximation produces less error than Forward Difference or Backward Difference Approximation. Explain why Forward Difference and Backward Difference formulae may still be needed even though Central Difference formula produces less error than both. $7 + 5$ (CO3) (PO1)

2. a) i. Rewrite the following third-order differential equation as a set of simultaneous first-order differential equations: 5 (CO1) (PO1)

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-t}$$

$$y(0) = 1$$

$$\frac{dy}{dx}(0) = 2$$

$$\frac{d^2y}{dx^2}(0) = 2$$

ii. For the differential equation in Question 2.a)i, find $y(0.75)$ using Euler's method. Consider step size, $h = 0.25$. 14 (CO2) (PO1)

b) Show how the variables are transformed when linearization is applied on Exponential function, Power function and Growth function. 2×3 (CO1) (PO1)

3. a) Explain how LU Decomposition is more computationally efficient for solving linear system compared to Inverse matrix multiplication or Gaussian Elimination.

b) Suppose you are given the time vs velocity data for two separate missiles in Table 2. (CO2) (PO1)

Table 2: Time vs Velocity data for two missiles for Question 3b)

t	V_1	V_2
0	0	0
10	227.04	152.35
15	362.78	316.62
20	517.35	502.71
22.5	602.97	620.25
30	901.67	978.87

Using Direct method of Interpolation with a third order polynomial, find the velocity of both missiles at $t = 21$ seconds. For solving the simultaneous equations, you are not allowed to use inverse matrix multiplication or Gaussian Elimination more than once.

4. a) i. What is the geometrical interpretation of Eigenvalues and Eigenvectors for a given matrix? Explain with an example. 6 (CO1) (PO1)

ii. Provide mathematical reasoning as to whether $\lambda = 3$ is one of the four eigenvalues of the following matrix A: 12 (CO2) (PO1)

$$A = \begin{bmatrix} 4 & 3 & 6 & 3 \\ 3 & 4 & 6 & 2 \\ 4 & 2 & 3 & 2 \\ 2 & 3 & 6 & 5 \end{bmatrix}$$

b) Find the eigenvalues and eigenvectors of the following matrix B: 7 (CO2) (PO1)

$$B = \begin{bmatrix} 6 & -3 \\ 1 & 4 \end{bmatrix}$$

5. a) Find the following integral using Simpson's 3/8 Rule: 13 (CO2) (PO1)

$$I = \int_0^{100} f(x)\,dx$$

where,

$$f(x) = \begin{cases} 0, & 0 < x < 30 \\ -9.1688 \times 10^{-6}x^3 + 2.796 \times 10^{-3}x^2 - 0.28487x + 9.6778, & 30 < x < 172 \end{cases}$$

b) Solve the integral given in Question 5.a) using Euler's method considering step size, $h = 25$. 12 (CO2) (PO1)

When a voltage input of 20V is provided, the speed of a motor is obtained by the following differential equation: 12 (CO2) (PO1)

$$20 = 0.02\frac{dv}{dt} + 0.06v$$

If the initial speed of the motor was 0, then find the speed at $t = 0.8$ using Runge-Kutta 4^{th} order method taking step size, $h = 0.4$.

b) For drawing road networks from aerial images, light intensities are measured at different pixel locations. The following intensities are given as a function of pixel location in Table 3: 13 (CO2) (PO1)

Table 3: Pixel location vs Intensity data for Question 6.b)

Pixel (k)	**Intensity** (y)
-3	119
-2	165
-1	231
0	243
1	244
2	214
3	136

Suppose we want to perform polynomial regression on the given data using a 2^{nd} order polynomial:

$$y = a_0 + a_1k + a_2k^2$$

Find the corresponding polynomial model parameters.