ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — SUMMER SEMESTER, 2021-2022

DURATION: 3 HOURS — FULL MARKS: 150

CSE 4803: Graph Theory

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) A Binary Tree is defined as a tree with three or more vertices in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three. Given a binary tree with n vertices, derive the equations to determine the number of vertices with degree one and degree three. — 7 (CO1) (PO1)

   b) German chemist August Wilhelm von Hofmann was the first to introduce molecular models into organic chemistry. He found out that Alkane (C_mH_n) is a molecule in which every carbon atom has four bonds, every hydrogen atom has one bond, and no sequence of bonds forms a cycle. He displayed the molecular models of Alkanes in London's Royal Institution at a Friday Evening Discourse. Hofmann modeled the molecules in such a way that $n = 2m+2$. However, the chemists present in the discourse were skeptical about the formulation. Defend the formulation of Hofmann using graph theoretic concepts. — 8 (CO3) (PO3)

   c) A network of five cities (a, b, c, d, and e) are shown in Figure 1. The cities are indicated using circles. The cost of building a bidirectional road between two cities are shown near the dashed lines connecting them. — 10 (CO1) (PO1)

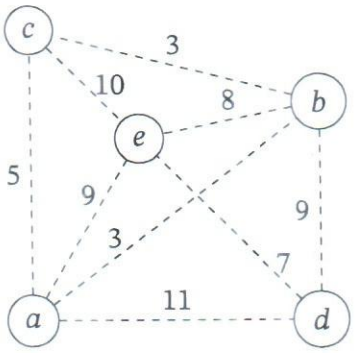

Figure 1: City Network for Questions 1.c)

Determine the least cost of connecting all the cities using Kruskal's Algorithm.

2. Consider the following statements: — 5 × 6 (CO1) (PO1)

   a) Every regular graph of degree $d(d \geq 3)$ is nonseparable.

   b) A planar graph has a cut-vertex if and only if its dual has a cut-vertex.

   c) A graph with n vertices and vertex connectivity k must have at least $kn/2$ edges.

   d) If G is a 2-connected simple planar graph with minimum degree 3, then its dual graph G^* is simple.

   e) Every strongly connected directed graph is an Euler digraph.

   For each statement, state whether you agree with it or not. If you do, provide a brief justification. If you do not, provide a counterexample with a brief explanation.

3. As shown in Figure 2, the country, AjobDesh, has 9 cities (A, B, C, D, E, F, H, S, and T) indicated using circles. The cities are connected via bidirectional roads depicted using lines. The travel time for each road is a positive integer as shown near the road. The capital of the country is located in city S, and the historic home town of the President is in city T.

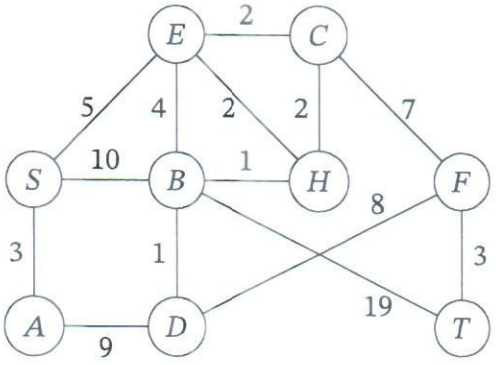

Figure 2: Map of AjobDesh for Question 3.

Once a year, Mr. President visits his historic home town T, for which his motorcade passes along some path from S to T. Since the president is a very busy man, he sends a scout who surveys several roads starting from S, leading up to T to determine the path to be taken by Mr. President.

To survey and identify the path, the scout runs different search algorithms. When deciding the next city to visit, the scout breaks ties in lexicographic order. He does not visit the same city more than once. He stops scouting as soon as he lands in city T.

For each of the following search algorithms, determine the order in which the scout will visit the cities and the path to be taken by the President:

a) Depth-First Search

b) Breadth-First Search

c) Djikstra's Algorithm

4. Four users, Abir (A), Maliha (M), Naim (N), and Shahriar (S), are sending data via a single coaxial cable that uses Frequency Division Multiple Access (FDMA) to send data through a single communication channel. It divides the bandwidth of the channel into six separate non-overlapping frequency sub-channels, namely C_1, C_2, C_3, C_4, C_5, and C_6. FDMA allocates each subchannel to a separate user. Users can send data through a subchannel by modulating it on a carrier wave at the subchannel's frequency.

The distribution of the subchannels depends on the modulation capacity of the devices being used. Abir's laptop can modulate the data in the frequency of C_2 or C_5. Maliha's desktop can modulate in the frequency of C_2 or C_5. Naim's tablet can modulate in the frequency of C_1, C_2, C_3, C_4, or C_6. Shahriar's smartphone can modulate in the frequency of C_2 or C_5. Your goal is to find a way to allocate frequency bands to the users while ensuring no interference between different users.

a) Formulate the problem as a graph and draw it by appropriately representing the entities and their relationships as vertices and edges respectively. 4 (CO2) (PO2)

b) Can each user be assigned a unique subchannel? If not, determine the maximum number of users that can be assigned. 16 (CO3) (PO3)

a) There are three houses H_1, H_2, and H_3, each to be connected to each of the three utilities — water (W), gas (G), and electricity (E) — by means of conduits.

i. Show that it is not possible to make such connections without any crossovers of the conduits. 10 (CO1) (PO1)

ii. Draw the graph corresponding to the problem showing the connections with minimum number of crossings. 5 (CO2) (PO2)

b) The graph shown in Figure 3 is called Petersen's Graph, a 3-regular graph with 10 vertices and 15 edges. There is no 3 or 4-length circuit in the graph. 12 (CO1) (PO1)

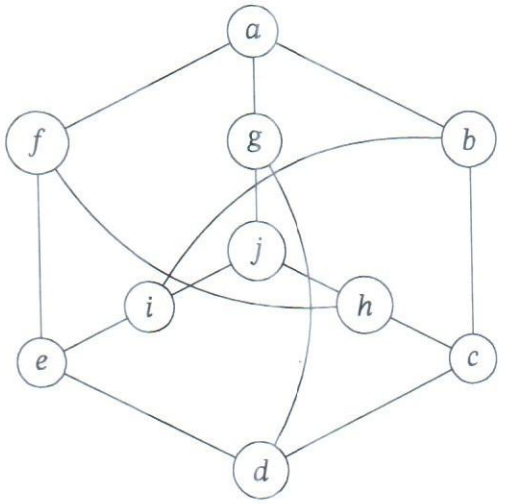

Figure 3: Petersen's Graph for Question 5.b)

Determine whether the graph is planar or not.

6. a) Consider that there are four axioms, A_1, A_2, A_3, and A_4. The derivation of A_1 depends on A_2 and A_4. The derivation of A_2 depends on A_3. The derivation of A_3 depends on A_1. The derivation of A_4 depends on A_2 and A_3. This type of circular reasoning is known as a fallacy.

i. Formulate the problem as a graph and draw it by appropriately representing the entities and their relationships as vertices and edges respectively. 4 (CO2) (PO2)

ii. Determine the axiom(s) that must be derived independent of the other axiom(s) to resolve the fallacy. You need to minimize the number of derivations. 11 (CO1) (PO1)

b) A de Bruijn sequence of order n on a size-k alphabet A is a cyclic sequence in which every possible length-n string on A occurs exactly once as a contiguous substring. Such a sequence is denoted by $B(k, n)$ and has length k^n, which is also the number of distinct strings of length n on A. Each of these distinct strings, when taken as a substring of $B(k, n)$, must start at a different position. Therefore, $B(k, n)$ must have at least k^n symbols. De Bruijn sequences are optimally short with respect to the property of containing every string of length n at least once. The sequence can be constructed by taking an Euler line of an $(n-1)$-dimensional de Bruijn graph. The sequence is often used for key generation in Stream Cipher. 11+4 (CO1) (PO1)

i. Construct $B(2, 4)$ for $A = \{0, 1\}$.

ii. Construct a spanning arborescence from the Euler line derived in Question 6.b)i.