Program: B. Sc. In ME, 6th Semester
Semester: Summer

Date: 5 May, 2023
Time: 10:00 a.m.– 01:00 p.m.

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Final-Semester Examination
Course Number: ME 4637
Course Title: Computational Mechanics

Summer Semester: 2021 - 2022
Full Marks:150
Time: 3 Hours

There are **6 (Six)** questions. Answer **all** of them.

The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the brackets. **Assume reasonable value for any missing data.**

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1. Consider the following 2D *Diffusion* equation with a *source term* **(25) (CO2) (PO5/PO4, K3/K5, P3/P5)**

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = 3xy$$

Solve the equation with *Finite Differences* in the following grid (**Figure 1**).

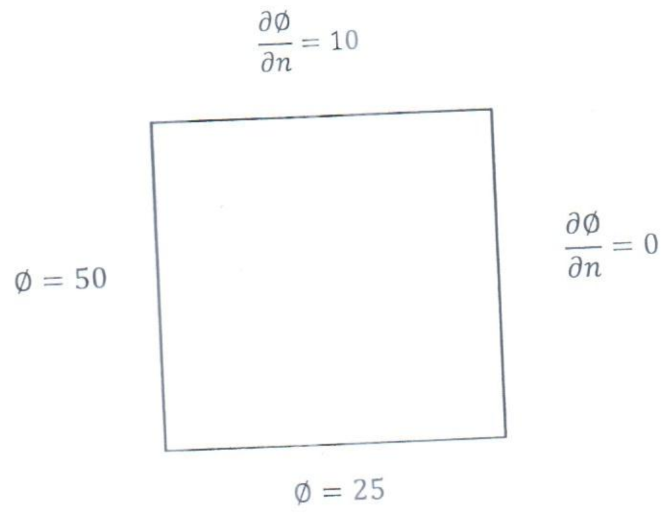

Figure 1

2. Following is the mathematical model of an *Unsteady Convection-Diffusion* phenomenon. **(30) (CO2) (PO5/PO4, K3/K5, P3/P5)**

$$\frac{\partial \emptyset}{\partial t} + 50\frac{\partial \emptyset}{\partial x} + \frac{\partial \emptyset}{\partial y} = 3\left(\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2}\right) + 3xy$$

Starting from zero initial condition, compute the time evaluation of $\emptyset(x, y, t)$ using *Finite Differences*

3. A vibrating String can be modeled by the following *time dependent differential equation*: **(20) (CO2) (PO5/PO4, K3/K5, P3/P5)**

$$\frac{\partial \emptyset}{\partial t} = 3\frac{\partial^2 \emptyset}{\partial x^2}$$

<u>Boundary Conditions:</u>

1. $\emptyset(x = 0, t) = \cos(\pi t)$
2. $\frac{d\emptyset}{dx} = 2, at\ x = 1, t$

<u>Initial Condition:</u>

$\emptyset(x, t = 0) = 0$

Find the *Finite Difference* solution with *Implicit Euler*.
Use time step size of 0.1 s. Also, grid will have 5 points with $\Delta X = 0.25$.

4. Write a MATLAB Code to obtain *Finite volume* solutions to 2D *Diffusion* equation. **(25) (CO2) (PO5/PO4, K3/K5, P3/P5)**

5. Consider the following 1D equation: **(25) (CO2) (PO5/PO4, K3/K5, P3/P5)**

$$5\frac{d\emptyset}{dx} - 8\frac{d^2\emptyset}{dx^2} = 3x,\ 0 \leq x \leq 1$$

<u>Boundary Conditions:</u>

3. *Dirichlet* Boundary Condition at left, $\emptyset(0) = 10$
4. *Neumann* Boundary Condition at right i.e. Flux of the solution at $x = 1$ is 3

Solve on a *Finite Volume* Domain (**Figure 2**) with 4 cells

Figure 2

6. Consider the following *Differential* equation. **(25) (CO2) (PO5/PO4, K3/K5, P3/P5)**

$$10\frac{\partial \emptyset}{\partial x} + 75\frac{\partial \emptyset}{\partial y} = 3\left(\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2}\right)$$

Solve the equation in a rectangular domain using *Finite Differences*. Use *4th Order scheme* for *convection derivatives.*
Assume *Dirichlet* Boundary condition at each edge.

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