Program: B. Sc. Engg. (ME/IPE) Date: October 6, 2023
Semester: 1st Semester Time: 2:30 pm – 4:00 pm

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination Winter Semester: 2022 - 2023
Course Number: Math 4111 Full Marks: 75
Course Title: Modelling with calculus and ODE Time: 1.5 Hours

There are **3 (three)** questions. Answer **all** questions. The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the brackets.

1. a) (i) An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides shown in Fig. Q1(a). Write volume V as a function of x, the length of the corner squares and find the domain of the function $V(x)$. [5] CO1 PO1

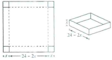

Fig. Q1(a)

(ii) Sketch a graph of the following function and discuss its continuity at $x = 0$. [8]

$$f(x)=\begin{cases} -2x+1; & x<0 \\ 0 \;\; ; & 0\le x<1 \\ 2x-1 \; ; & x\ge 1 \end{cases}$$

b) (i) Graph of $h(x)=\left|-\dfrac{x}{2}\right|+x^2$ is shown in Fig. Q1(b). Find the limit of h (if it exists) using the graph at point $x = 1$ and $x = 2$. If the limit does not exist,explain why. [5] CO1 PO2

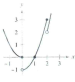

Fig. Q1(b)

(ii) Applying Sandwich theorem, evaluate the following limit. [7]

$$\lim_{x\to\infty}\left(\frac{2x^2+\cos 3x}{3x^2+5}\right)$$

2. a) (i) Determine the slope of the graph of $3\left(x^2+y^2\right)^2=100xy$ at the point (3, 1). [8] CO1 PO2

A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants in the stack emissions modeled by the following function.

$$C(p)=\frac{80,000p}{100-p};\ 0\le p<100.$$

(ii) Find the cost of removing q% of the pollutants where q is the last two digits of your student ID. [2]

(iii) Find the limit of C as p approaches 100 from the left and interpret its meaning. [3]

b) The endpoints of a movable rod of length 1 meter have coordinates (x, 0) and (0, y) presented in Fig. Q3(b). The position function of the end on the x-axis is $x(t)=\frac{1}{2}\sin\frac{\pi t}{6}$, where t is the time in seconds. CO1 PO2

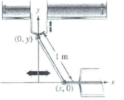

Fig. Q3(b)

(i) Find the time of one complete cycle of the rod. [2]

(ii) What is the lowest point reached by the end of the rod on the y-axis? [3]

(iii) Find the speed of the y-axis endpoint when the x-axis endpoint is $(\frac{1}{4},0)$ [7]

3. a) Suppose the velocity in meters/second of an object moving along a line is given by the function $v(t)=t^3-6t$, where $0\le t\le 3$. Approximate the displacement of the object by dividing the time interval [0, 3] into 6 sub-intervals of equal length. CO2 PO1

(i) Estimate the velocity by a constant equal to the value of v evaluated at the right endpoints of the subinterval. [5]

(ii) Set up an expression for $\int_0^3 (t^3-6t)\,dt$ as a limit of sums. [4]

(iii) Use a computational algebra system to evaluate the expression. [3]

b) (i) Let R be the region bounded by loop of the curve $a^2y^2=x^3(2a-x)$. Find the area of the region R. [7] CO2 PO1

(ii) Apply the integral formula to find the volume of the solid generated by revolving the regions bounded by $y=3\sin 2x,\ 0\le x\le\frac{\pi}{2}$ about the x-axis. [6]