04 October, 2023

B.Sc Engg.(M)/5th Sem
B.Sc. Engg. (IPE)/5th Sem

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
**DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING**

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| Mid Semester Examination | Winter Semester: A.Y. 2022-2023 |
| Course No.: Math 4511 | Time: 1 hour and 30 minutes |
| Course Title: Numerical Analysis | Full Marks: 75 |

**There are 3 (Three) Questions. Answer all of them.**
Marks in the Margin indicate full marks. Programmable calculators are not allowed.
**Assume reasonable values for any missing data(if any).**

1. a) The *Manning equation* can be written for a rectangular open channel as [18] CO1, PO4/PO2,

$$Q = \frac{\sqrt{S}(BH)^{5/3}}{n(B+2H)^{2/3}}$$

Where, $Q$ = flow [m$^3$/s], $S$ = slope [m/m], $H$ = depth [m], $B$ = width [m] and $n$ = the Manning roughness coefficient. Solve this equation for $H$. Given $Q = 5$, $S = 0.0002$, $B = 20$, and $n = 0.03$.

Use three iterations of the **Newton-Raphson method** to determine your answer. Determine the *approximate relative error* after each iteration.

b) Consider the following *optimization* problem. [07] CO1, PO4/PO2

$$f(\mathbf{x}) = 5x_1^2 + x_2^2 + 4x_1x_2 - 14x_1 - 6x_2 + 20$$

It's corresponding graph is given in Figure 1. Implement **Gradient Descent method** with $\eta$=0.09 to find optimal **x**, perform two iterations.

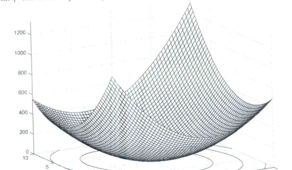


Figure 1: Graph of the function stated in Question 1(b)

2. The equations of the curve and the ellipse, which are shown in the Figure 2, are given by [25] CO1, PO4/ PO2

$$f_1(x,y) = y - \frac{1}{2}\left(e^{\frac{x}{2}} + e^{\frac{-x}{2}}\right) = 0$$

$$f_2(x,y) = 9x^2 + 25y^2 - 225 = 0$$

Use **Newton's method** to determine the point of intersection of the curves that resides in the first quadrant of the coordinate system.
Perform three iterations.

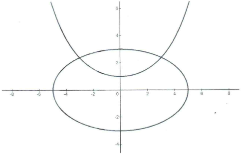


Figure 2: Plot of functions stated in Question 2

3. a) The deflection of a uniform beam subject to a linearly increasing distributed load can be computed as (13) CO1, PO4/ PO2

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x)$$

Given that $L$ = 600 cm, $E$ = 50,000 kN/cm$^2$, $I$ = 30,000 cm$^4$, and $w_0$ = 2.5 kN/cm, determine the point of maximum deflection using the **golden-section** *search*. Perform three iterations with initial guesses of $x_l$ = 0 and $x_u$ = $L$.

b) The following data present the power of a diesel engine at different engine speeds: (12) CO1, PO4/ PO2

| Engine Speed (rpm) | 1200 | 1500 | 2000 | 2500 |
|---|---|---|---|---|
| Engine Power (hp) | 65 | 130 | 185 | 225 |

Forming the **Lagrange Interpolating Polynomial** among the data points, estimate the engine power at speed of 2300 rpm.