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Program: B. Sc. Engg. (IPE)
Semester: 7th

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Mid-Semester Examination | **Winter Semester: 2022-2023**
Course Number: IPE 4785 | **Full Marks: 75**
Course Title: Reliability and Maintenance Engineering | **Time: 1 hour and 30 mins**

Answer all the 4 (four) questions below. The distribution of marks and the CO-PO mapping are given in brackets. Necessary formula and table are attached.

Q1. Draw a step curve and indicate MTTF, MTTR, mean time between failures (MTTB) and availability. **[3]** (CO1, PO1)

Q2. A washing machine is advertised as having more than a 9-yr life. If the following is its PDF, **[22]** (CO1, PO1)

$$f(t) = 0.1\,(1 + 0.05t)^{-3} \qquad t \geq 0$$

(i) Find is the reliability of the machine for the third year?
(ii) Determine the design life if a 0.90 reliability is required.
(iii) Find the hazard rate function. Is it increasing or decreasing?
(iv) Determine its reliability for the next 7 years if it has survived a 1.5-yr warranty period.
(v) Find its MTTF before the warranty period?
(vi) Find its MTTF after the warranty period assuming it has still survived?

Q3. A hydraulic system experiences chance (CFR) failures with an MTTF of 1250 hr. Find the following: **[25]** (CO2, PO1)

(i) The reliability for a 210-hr mission
(ii) The design life for a 0.90 reliability
(iii) The median time to failure
(iv) If a second, redundant (and independent) component is added, find again the reliability for a 210-hr mission and determine how much the reliability has been improved due to the addition of the redundant component. What will be the redundant system MTTF?
(v) If there are three spare components available, what will be the reliability for the 210-hr mission?
(vi) If the component has a guaranteed life of 100-hr, what will be the design life for a 0.90 reliability?

Q4. A jet engine consists of four components in series. Each rectifier has a Weibull failure distribution with β equal to 1.9. However, they have different characteristic lifetimes given by 10,500 hr, 13,500 hr, 18,500 hr, and 21,500 hr. **[25]** (CO2, PO1)

(i) Find the MTTF of the engine.

(ii) Determine the design life of the engine corresponding to a reliability of 0.90.

(iii) Find the B1.2 life.

(iv) If the jet engine is new, determine the probability of a component failure on an 812-hr trip.

(v) If the engine (with the components) has had 2000 hr of burn-in period accomplished, what is the probability of a component failure during the next 3000 hr of use?

(vi) If all the components have equal scale parameter of 15,000 hr, find the reliability of the engine.

FAILURE DISTRIBUTION

$F(t) = \int_0^t f(t')\,dt'$; $R(t) = \int_t^{\infty} f(t')\,dt'$

$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$; $\text{MTTF} = E(T) = \int_0^{\infty} t f(t)\,dt$

$\text{MTTF} = \int_0^{\infty} R(t)\,dt$; $\sigma^2 = \int_0^{\infty} t^2 f(t)\,dt - (\text{MTTF})^2$

$\lambda(t) = \frac{-dR(t)}{dt} \cdot \frac{1}{R(t)} = \frac{f(t)}{R(t)}$; $R(t) = \exp\left[-\int_0^t \lambda(t')\,dt'\right]$

$R(t \mid T_0) = \frac{R(T_0 + t)}{R(T_0)} = \exp\left[-\int_{T_0}^{T_0+t} \lambda(t')\,dt'\right]$ $\quad \text{MTTF} = \int_0^{\infty} R(t)\,dt$

INDEFINITE INTEGRAL

$\int (a + bt)^n\,dt = \frac{(a + bt)^{n+1}}{(n + 1)b} \qquad n \neq -1$

DEFINITE INTEGRAL

$\int_0^{\infty} t^n e^{-at}\,dt = \frac{n!}{a^{n+1}} \qquad a > 0$, n a positive integer

CFR MODEL

$R(t) = \exp\left[-\int_0^t \lambda\,dt'\right] = e^{-\lambda t}, \quad t \geq 0$; $F(t) = 1 - e^{-\lambda t}$; $f(t) = -\frac{dR(t)}{dt} = \lambda e^{-\lambda t}$

$\text{MTTF} = \frac{1}{\lambda}$; $\sigma^2 = \frac{1}{\lambda^2}$; $t_R = -\frac{1}{\lambda}\ln R$

$t_{\text{med}} = \frac{0.69315}{\lambda} = 0.69315\ \text{MTTF}$; $R(t \mid T_0) = e^{-\lambda t} = R(t)$

FAILURE MODES

$R(t) = \prod_{i=1}^{n} R_i(t)$; $\lambda(t) = \sum_{i=1}^{n} \lambda_i(t)$

$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} 1/\text{MTTF}_i}$

$R(t) = \exp\left[-\int_0^t \lambda\,dt'\right] = \exp[-\lambda t]$

IDENTICAL COMPONENTS

$\lambda = n\lambda_1$; $\text{MTTF} = \frac{1}{n\lambda_1}$

FAILURE ON DEMAND

$\lambda_{\text{eff}} = \frac{t_I}{t_I + t_O}\lambda_I + \frac{t_O}{t_I + t_O}\lambda_O + \frac{p}{t_I + t_O}$

REPETITIVE LOADING

$R_n = e^{-np}$; $R(t) = e^{-(p/\Delta t)t} = e^{-\lambda t}$

RELIABILITY BOUNDS

$e^{-\lambda_L t} \geq R(t) \geq e^{-\lambda_U t}$

TWO PARAMETER EXPONENTIAL DISTRIBUTION

$R(t) = e^{-\lambda(t - t_0)}$; $\text{MTTF} = t_0 + \frac{1}{\lambda}$; $t_{\text{med}} = t_0 + \frac{0.69315}{\lambda}$ $\quad t_R = t_0 + \frac{\ln R}{-\lambda}$; $\sigma = 1/\lambda.$

POISSON PROCESS

$p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad n = 0, 1, 2, \ldots$ $\quad F_{Y_k}(t) = 1 - e^{-\lambda t}\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!}$; $R_S(t) = \sum_{n=0}^{s} p_n(t)$

REDUNDANT SYSTEM

$R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$; $\lambda(t) = \frac{\lambda(1 - e^{-\lambda t})}{(1 - 0.5e^{-\lambda t})}$; $\text{MTTF} = \frac{1.5}{\lambda}$

WEIBULL DISTRIBUTION

$$\lambda(t) = at^b \; ; \; \lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \; ; \; R(t) = e^{-(t/\theta)^\beta}$$

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-(t/\theta)^\beta} \; ; \; \text{MTTF} = \theta\,\Gamma\left(1 + \frac{1}{\beta}\right) \; ; \; \sigma^2 = \theta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

$$\Gamma(x) = (x-1)! \; ; \; t_R = \theta(-\ln R)^{1/\beta} \; ; \; t_{0.50} = \theta(0.69315)^{1/\beta}$$

$$t_{\text{mode}} = \begin{cases} \theta(1 - 1/\beta)^{1/\beta} & \text{for } \beta > 1 \\ 0 & \text{for } \beta \le 1 \end{cases}$$

BURN-IN SCREENING

$$R(t \mid T_0) = \exp\left[-\left(\frac{t+T_0}{\theta}\right)^\beta + \left(\frac{T_0}{\theta}\right)^\beta\right]$$

FAILURE MODES

$$\lambda(t) = \beta t^{\beta-1}\left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^\beta\right]; \; \theta = \left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^\beta\right]^{-1/\beta} ; \; \lambda(t) = \frac{n\beta}{\theta^\beta}(t)^{\beta-1}$$

IDENTICAL COMPONENTS

$$R(t) = \exp\left[-n\left(\frac{t}{\theta}\right)^\beta\right]$$, shape parameter β, scale parameter $\theta/n^{1/\beta}$.

THREE PARAMETER WEIBULL

$$R(t) = \exp\left[-\left(\frac{t-t_0}{\theta}\right)^\beta\right]; \; \lambda(t) = \frac{\beta}{\theta}\left(\frac{t-t_0}{\theta}\right)^{\beta-1}; \; \text{MTTF} = t_0 + \theta\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$t_{\text{med}} = t_0 + \theta(0.69315)^{1/\beta} \; ; \; t_R = t_0 + \theta(-\ln R)^{1/\beta}$$

REDUNDANCY WITH WEIBULL

$$R_s(t) = 1 - [1 - R(t)]^2 ; \; R_s(t) = 2e^{-(t/\theta)^\beta} - e^{-2(t/\theta)^\beta}$$

$$\text{MTTF} = \theta\Gamma\left(1+\frac{1}{\beta}\right)\left(2 - 2^{-1/\beta}\right) ; \; \lambda_s(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}\frac{2 - 2e^{-(t/\theta)^\beta}}{2 - e^{-(t/\theta)^\beta}}$$

TABLE A.9
Gamma function

x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$
1.01	.99433	1.51	.88659	2.01	1.00427	2.51	1.33875
1.02	.98884	1.52	.88704	2.02	1.00862	2.52	1.34830
1.03	.98355	1.53	.88757	2.03	1.01306	2.53	1.35798
1.04	.97844	1.54	.88818	2.04	1.01758	2.54	1.36779
1.05	.97350	1.55	.88887	2.05	1.02218	2.55	1.37775
1.06	.96874	1.56	.88964	2.06	1.02687	2.56	1.38784
1.07	.96415	1.57	.89049	2.07	1.03164	2.57	1.39807
1.08	.95973	1.58	.89142	2.08	1.03650	2.58	1.40844
1.09	.95546	1.59	.89243	2.09	1.04145	2.59	1.41896
1.10	.95135	1.60	.89352	2.10	1.04649	2.60	1.42962
1.11	.94740	1.61	.89468	2.11	1.05161	2.61	1.44044
1.12	.94359	1.62	.89592	2.12	1.05682	2.62	1.45140
1.13	.93993	1.63	.89724	2.13	1.06212	2.63	1.46251
1.14	.93642	1.64	.89864	2.14	1.06751	2.64	1.47377
1.15	.93304	1.65	.90012	2.15	1.07300	2.65	1.48519
1.16	.92980	1.66	.90167	2.16	1.07857	2.66	1.49677
1.17	.92670	1.67	.90330	2.17	1.08424	2.67	1.50851
1.18	.92373	1.68	.90500	2.18	1.09000	2.68	1.52040
1.19	.92089	1.69	.90678	2.19	1.09585	2.69	1.53246
1.20	.91817	1.70	.90864	2.20	1.10180	2.70	1.54469
1.21	.91558	1.71	.91057	2.21	1.10785	2.71	1.55708
1.22	.91311	1.72	.91258	2.22	1.11399	2.72	1.56964
1.23	.91075	1.73	.91467	2.23	1.12023	2.73	1.58237
1.24	.90852	1.74	.91683	2.24	1.12657	2.74	1.59528
1.25	.90640	1.75	.91906	2.25	1.13300	2.75	1.60836
1.26	.90440	1.76	.92137	2.26	1.13954	2.76	1.62162
1.27	.90250	1.77	.92376	2.27	1.14618	2.77	1.63506
1.28	.90072	1.78	.92623	2.28	1.15292	2.78	1.64868
1.29	.89904	1.79	.92877	2.29	1.15976	2.79	1.66249
1.30	.89747	1.80	.93138	2.30	1.16671	2.80	1.67649
1.31	.89600	1.81	.93408	2.31	1.17377	2.81	1.69068
1.32	.89464	1.82	.93685	2.32	1.18093	2.82	1.70506
1.33	.89338	1.83	.93969	2.33	1.18819	2.83	1.71963
1.34	.89222	1.84	.94261	2.34	1.19557	2.84	1.73441
1.35	.89115	1.85	.94561	2.35	1.20305	2.85	1.74938
1.36	.89018	1.86	.94869	2.36	1.21065	2.86	1.76456
1.37	.88931	1.87	.95184	2.37	1.21836	2.87	1.77994
1.38	.88854	1.88	.95507	2.38	1.22618	2.88	1.79553
1.39	.88785	1.89	.95838	2.39	1.23412	2.89	1.81134
1.40	.88726	1.90	.96177	2.40	1.24217	2.90	1.82736
1.41	.88676	1.91	.96523	2.41	1.25034	2.91	1.84359
1.42	.88636	1.92	.96877	2.42	1.25863	2.92	1.86005
1.43	.88604	1.93	.97240	2.43	1.26703	2.93	1.87673
1.44	.88581	1.94	.97610	2.44	1.27556	2.94	1.89363
1.45	.88566	1.95	.97988	2.45	1.28421	2.95	1.91077
1.46	.88560	1.96	.98374	2.46	1.29298	2.96	1.92814
1.47	.88563	1.97	.98769	2.47	1.30188	2.97	1.94574
1.48	.88575	1.98	.99171	2.48	1.31091	2.98	1.96358
1.49	.88595	1.99	.99581	2.49	1.32006	2.99	1.98167
1.50	.88623	2.00	1	2.50	1.32934	3.00	2