(30)

M.Sc. Engg./Ph.D. **October 04, 2023**
2:30 pm – 4:00 pm

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Mid-Semester Examination **Winter Semester: 2022 - 2023**
Course Number: EEE 6413 **Full Marks: 75**
Course Title: Engineering Optimization **Time: 1 Hour 30 Minutes**

There are **04 (four)** questions. Answer any **03 (three)** questions. The symbols have their usual meanings. Marks of each question are written in the brackets in right margin.

1.a) Write the statement of an optimization problem. **(05)**

b) State a constrained and an unconstrained optimization problem. Briefly explain Design Vector, Design Constraints, Constraint Surface, Objective Function, Objective Function Surfaces with necessary equation and suitable diagram as per necessity. **(10)**

c) A retail store stocks and sells three different models of TV sets. The store cannot afford to have an inventory worth more than \$45 000 at any time. The TV sets are ordered in lots. It costs $\$a_j$ for the store whenever a lot of TV model j is ordered. The cost of one TV set of model j is c_j. The demand rate of TV model j is d_j units per year. The rate at which the inventory costs accumulate is known to be proportional to the investment in inventory at any time, with $q_j = 0.5$, denoting the constant of proportionality for TV model j. Each TV set occupies an area of $s_j = 0.40$ m^2 and the maximum storage space available is 90 m^2. The data known from the past experience are given below. **(10)**

	TV model j		
	1	2	3
Ordering cost, a_j (\$)	50	80	100
Unit cost, c_j (\$)	40	120	80
Demand rate, d_j	800	400	1200

Formulate the problem of minimizing the average annual cost of ordering and storing the TV sets.

2.a) With suitable diagram, illustrate different types of extreme point **(05)**

b) i) By using a graphical method, solve the optimization problem **(12)**

$$\text{minimize } f(x) = x_1^2 + x_2 + 4$$
$$\text{subject to: } c_1(x) = -x_1^2 - (x_2 + 4)^2 + 16 \geq 0$$
$$c_2(x) = x_1 - x_2 - 6 \geq 0$$

ii) Indicate the feasible region.
iii) Is the optimum point constrained?

c) Point $x* = [2\ 4]^T$ is a local minimizer of the problem **(8)**

$$\text{minimize } f(x) = \frac{1}{4}[x_1^2 + 4x_2^2 + 4(3x_1 + 8x_2) + 100]$$
$$\text{subject to: } x_1 = 2, x_2 \geq 0$$

i) Find the feasible directions.
ii) Check if the second-order necessary conditions are satisfied.

3.a) Define definiteness of Matrices? Why and how they are important and related to an optimization problem? **(5)**

b) State the Sylvester's criterion definiteness of a matrix. **(5)**

c) Determine the nature of the quadratic function: **(6)**

$$f(x) = 9x_1^2 + 2x_1x_2 + 7x_1x_3 + 8x_2^2 + 6x_2x_3 + 5x_3^2$$

d) Find the dimensions of a box of largest volume that can be inscribed in a sphere of radius ***r*** using the method of constrained variation. **(9)**

4.a) What are the disadvantages of the method of direct substitution and constrained variation in optimization problem with equality constraints? **(04)**

b) Formulate the method of Lagrange multiplier for problems with equality constraint for a simple case of two variables and one constraint. Expand the formulation to explain the necessary condition for a general problem. **(09)**

c) Find the dimensions of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to $A_0 = 24\pi$. Use the method of Lagrange multiplier with necessary and sufficient conditions. **(12)**