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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF BUSINESS AND TECHNOLOGY MANAGEMENT

Mid-Semester Examination

Winter Semester, A. Y. 2022-2023

Course No. : Math 4361

Time : 1.5 hours

Course Title : Mathematics

Full Marks : 75

Answer all 3 (three) questions. All questions carry equal marks. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

1. a) Define limit of $f(x)$ at x_0 . Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x^3} - x^3)$. 5 (CO1) (PO1)
- b) Define continuity of $f(x)$ at x_0 . For what value of k and m the following function is continuous everywhere? 7 (CO1) (PO1)

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \leq 2 \\ 2x^2 + x + 7, & x \leq -1 \end{cases}$$

- c) Find the derivative of (i) $y = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}}$ (ii) $y = (2x^2 - 1)^{\sin x}$ 13 (CO1) (PO1)

2. a) Show that $f(x) = \sin x (1 + \cos x)$ has a relative maxima at $x = \frac{\pi}{3}$. 10 (CO1) (PO1)

- b) State L Hospital's rule. Evaluate (i) $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^x$ (ii) $\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$ 7 (CO1) (PO1)

- c) State Taylor's theorem. Expand $f(x) = \sin x$ in power of $\left(x - \frac{\pi}{2}\right)$ with remainder term. 8 (CO1) (PO1)

3. a) Define successive differentiation. Find the nth derivative of $f(x) = (ax + b)^n$. 8 (CO1) (PO1)

- b) State and prove Leibnitz theorem. 7 (CO1) (PO1)

- c) If $y = (\sin^{-1} x)^2$, the by Leibnitz's theorem show that 10 (CO1) (PO1)

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$