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BSc. Engg (CEE)/1st Semester **6th October 2023 (Morning)**

Islamic university of technology (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING (CEE)

Mid Term Examination **Winter semester: 2022-2023**

Course no: Math- 4153 **Full Marks: 75**

Course Title: Differential Calculus, Integral Calculus & Matrix **Time: 1.5 Hours**

There are 3 (three) Questions. Answer all. Programmable Calculators are not allowed. Do not write on this question, paper. The figures in the right margin indicate full marks and corresponding Co and PO. The symbols have their usual meaning.

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1. a) By applying (δ, ϵ) definition, show that $lim_{x\to\frac{1}{2}} \frac{4x^2-1}{2x-1} = 2$ **(7) (CO1,PO1)**

and find the value of δ when $\epsilon = 0.01$

b) A function f (x) is defined as follows: **(8) (CO1,PO1)**

$$f(x) = \begin{cases} x, \; when \;\; 0 \le x < \frac{1}{2} \\ 1-x, when \;\; \frac{1}{2} \le x < 1 \end{cases}$$

Test the Continuity and differentiability of the function f (x) at $x = \frac{1}{2}$

c) If $x = \sin\left(\frac{lny}{m}\right)$, then show that: **(10) (CO1,PO1)**

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

And find the value of y_n, when x = 0.

2. a) Evaluate the following limits, $\lim_{x\to 0} \left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$. **(6) (CO1,PO1)**

b) Obtain Taylor's expansion of $\tan^{-1}\frac{y}{x}$ about (1,1) upto and including second degree terms. **(8) (CO1,PO1)**

c) A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal squares of side x meters are removed from the corners. The sides of this sheet are now turned up to form an open rectangular box. Find approximately, the height of the box, such that the volume of the box is maximum. **(11) (CO1,PO1)**

3. a) If $x\cos\alpha + y\sin\alpha = p$ touch the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$, Show that **(8) (CO2,PO2)**

$$(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$$

b) Find the radians of curvature of the Witon of Agnesi **(8) (CO2,PO2)**

$y^2 = \frac{4a^2(2a-x)}{x}$, where the curve meets x–axis.

c) If $u = f(x^2 + 2yz, y^2 + 2xz)$, find **(9) (CO1,PO1)**

$$(y^2 - xz)\frac{\partial u}{\partial u} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z}.$$