

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

Semester: Mid Semester Examination
Course No.: MATH 4353
Course Title: Laplace Transformation, Series, PDE

Winter Semester: 2022-2023
Full Marks: 75
Time: 1.5 hours

There are 3 (three) sets of questions. Answer all of them. The figures in the right margin indicate full marks. COs and POs are also specified in the right margin of the questions. The symbols have their usual meaning.

	CO	PO	Full Marks
1. a) Define Laplace Transform a periodic function.	1	1	(2)
Using the definition find $\mathcal{L}\{f(t)\}$, where $f(t)$ is the half-wave rectified sinusoidal function defined by $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$	2	2	(7)
b) Use $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)du$, where $\mathcal{L}\{f(t)\} = F(s)$ to show. $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt = \log\left(\frac{5}{3}\right)$.	2	2	(8)
c) Apply the definition of error function to compute the value of $\mathcal{L}\{erf\sqrt{t}\}$.	3	2	(8)
2. a) Apply Convolution theorem to find $\mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.	3	2	(6)
b) Compute $\mathcal{L}^{-1}\left\{\frac{s+5}{(s-1)(s^2+1)}\right\}$ using Heaviside expansion formula.	2	2	(7)
c) Apply Laplace transform to solve the following IVP: (i) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \sin t$ ($t \geq 0$) subject to the conditions $x(0) = x'(0) = 0$. (ii) $ty''(t) + y'(t) + ty(t) = 0; y(0) = 1, y'(0) = 0$.	3	2	(12)

3. a) Develop the Fourier series for the square wave function $f(t)$ shown in Fig. 1: 1 1 (10)

2 2 (3)



Fig. 1 A square wave with period $2a$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

b) Find the Fourier series of the function $f(x) = [x]$, $-2 \leq x \leq 2$ and 2 2 (6)

$$f(x+4) = f(x).$$

c) Determine half-range sine series for the function $f(x) = \pi x - x^2$ valid in 2 2 (6)

$$0 \leq x \leq \pi.$$