ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2022-2023**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 100**

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 4 (four)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) Find the equation of the straight lines bisecting the angle between the straight lines given by: 8 (CO1) (PO1)

$$ax^2 + 2hxy + by^2 = 0$$

b) Find the condition that the general equation of the second degree 9 (CO1) (PO1)

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

may represent a pair of straight lines.

c) Prove that the equation: 8 (CO2) (PO1)

$$3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$$

represents two straight lines and find the angle between them.

2. a) For the function f graphed in Figure 1, find : 7×1 (CO2) (PO1)

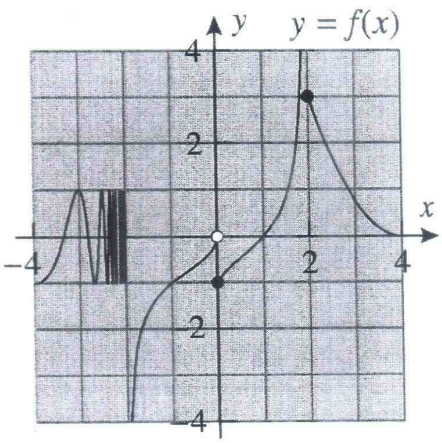

Figure 1: Graph of f Question 2.a)

i. $\lim_{x \to -2^-} f(x)$

ii. $\lim_{x \to -2^+} f(x)$

iii. $\lim_{x \to 0^-} f(x)$

iv. $\lim_{x \to 0^+} f(x)$

v. $\lim_{x \to 2^-} f(x)$

vi. $\lim_{x \to 2^+} f(x)$

vii. The vertical asymptotes of the graph of f.

b) Sketch the graph of the following equations by translating and reflecting the graph of $y = \frac{1}{x}$ and $y = \sqrt{x}$ respectively: 2 × 3 (CO2) (PO1)

i. $y = 2 - \frac{1}{x+1}$

ii. $y = 3 - \sqrt{x+1}$

c) Find the natural domain and range of the following functions: 3 × 4 (CO2) (PO1)

i. $f(x) = \frac{1}{(x-1)(x-3)}$

ii. $f(x) = \sqrt{x^2 - 5x + 6}$

iii. $f(x) = \frac{1}{1-\sin x}$

3. a) Find the values of the constants k and m, if possible, that will make the function f continuous everywhere. 6 (CO2) (PO1)

$$f(x) = \begin{cases} x^2 + 5, & \text{if } x > 2 \\ m(x+1) + k, & \text{if } -1 < x \leq 2 \\ 2x^3 + x + 7, & \text{if } x \leq -1 \end{cases}$$

b) During the first 40 s of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches a height of $s = 0.3t^3$ ft. 4 × 3 (CO3) (PO1)

i. How high does the rocket travel in 40 s?

ii. What is the average velocity of the rocket during the first 40 s?

iii. What is the average velocity of the rocket during the first 1000 ft of its flight?

iv. What is the instantaneous velocity of the rocket at the end of 40 s?

c) Show that 7 (CO2) (PO1)

$$f(x) = \begin{cases} x^2 + 2, & \text{if } x \leq 1 \\ x + 2, & \text{if } x > 1 \end{cases}$$

is continuous but not differentiable at x=1. Sketch the graph of f.

4. a) On a sunny day, a 50 ft flagpole casts a shadow that changes with the angle of elevation of the sun as shown in Figure 2. Let s be the length of the shadow and θ the angle of elevation of the sun. Find the rate at which the length of the shadow is changing with respect to θ when $\theta = 45°$. Express your answer in units of feet/degree. 7 (CO3) (PO1)

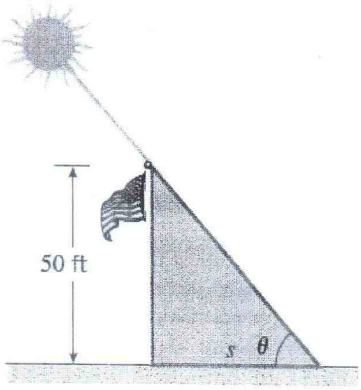

Figure 2: Shadow's orientation for Question 4.a)

b) Sketch the graph of the equation 8

$$y = x^3 - 3x + 2,$$ (CO2) (PO1)

and identify the locations of the intercepts, relative extrema, and inflection points.

c) For the function given by 5×2

$$f(x) = \frac{x - 2}{(x^2 - x + 1)^2},$$ (CO2) (PO1)

find the following :

i. the intervals on which f is increasing,

ii. the intervals on which f is decreasing,

iii. the open intervals on which f is concave up,

iv. the open intervals on which f is concave down, and

v. the x-coordinates of all inflection points.