



ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION
DURATION: 1 HOUR 30 MINUTES

WINTER SEMESTER, 2022-2023
FULL MARKS: 75

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.
 Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Gaussian elimination usually involves doing row operations on matrices. But your task is to do column operation on the matrix A : 10
(CO3)
(PO1)

$$A = \begin{bmatrix} \boxed{-1} & -3 & 2 \\ 2 & \boxed{7} & 5 \\ -1 & 0 & 3 \end{bmatrix}$$

Do appropriate column operations on matrix A so that there is no non-zero entry to the right of the single box. Then do column operations so that there is no non-zero entries to the right of double box. Show all necessary steps for these column operations.

- b) Formulate the appropriate matrices that can be multiplied with the matrix A to achieve the same outcome as Question 1.a) How would you multiply these matrices with A ? What factorization of A does this produce? 10
(CO3)
(PO1)
- c) If X is an invertible square matrix, what can you imply about $C(X)$ and $N(X)$? 5
(CO1)
(PO1)
2. a) Suppose A is a 6×6 matrix with following entries: 5 +
5 + 3
(CO2)
(PO1)

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

- i. What is the rank of A ?
- ii. Find a basis for $N(A)$.
- iii. How many solutions would it have for any right hand side, b ?
- b) Consider the following system of equations: 12
(CO1)
(PO1)
- $$\begin{aligned} x + 2y - 2z &= b_1 \\ 2x + 5y - 4z &= b_2 \\ 4x + 9y - 8z &= b_3 \end{aligned}$$

Under what condition on b is this solvable? Find all solutions x, y, z when it is solvable.

3. a) Compute the 3 by 3 matrices
- M_1, M_2, M_3
- where:

$$M_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 \end{bmatrix} \quad M_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 4 \end{bmatrix}$$

Use elimination to solve $Ax = b$, where $A = M_1 + M_2 + M_3$ and $b = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$.

- b) Suppose A is a 4 by 5 matrix and $s = (4, 5, 2, 1, 0)$ is the only special solution for $Ax = 0$.
- Describe how $C(A)$ and $N(A)$ would look like.
 - What is the exact Row-reduced Echelon form R of A ?

3 + 7

(CO1)

(PO1)