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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

WINTER SEMESTER, 2022-2023

DURATION: 1 HOUR 30 MINUTES

FULL MARKS: 75

Math 4541: Multivariable Calculus and Complex Variables

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer all 3 (three) questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) What is an analytic function, and how can we derive the necessary condition for a function $f(z)$ to be analytic? 8
(CO1)
(PO1)
- b) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative. 8
(CO2)
(PO1)
- c) Define a harmonic function and conjugate harmonic function. Find the harmonic conjugate function of the function $u(x, y) = 2x(1 - y)$. 9
(CO2)
(PO1)
2. a) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express $f(z)$ in terms of z . 8
(CO2)
(PO1)
- b) Evaluate $\oint_C (z - z^2) dz$, where C is the upper half of the circle $|z - 2| = 3$. 8
What is the value of the integral if C is the lower half of the above given circle? (CO2)
(PO1)
- c) A 75-N weight is suspended by two wires, as shown in Figure 1. Find the forces F_1 and F_2 acting in both wires. 9
(CO2)
(PO1)

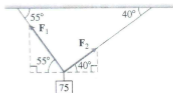


Figure 1: Force distribution for Question 2.c)

3. a) Find the volume of the box (parallelepiped) determined by $u = i + 2j - k$, $v = -2i + 3k$, and $w = 7j - 4k$. 8
(CO2)
(PO1)
- b) Find the distance from $S(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$. 8
(CO2)
(PO1)

- c) In computer graphics and perspective drawing, we need to represent objects seen by the eye in space as images on a two-dimensional plane. Suppose that the eye is at $E(x_0, 0, 0)$ as shown in Figure 2, and that we want to represent a point $P_1(x_1, y_1, z_1)$ as a point on the YZ -plane. We do this by projecting P_1 onto the plane with a ray from E . The point P_1 will be portrayed as the point $P(0, y, z)$. The problem for us as graphics designers is to find y and z given E and P_1 .
- Write a vector equation that holds between \overrightarrow{EP} and $\overrightarrow{EP_1}$. Use the equation to express y and z in terms of $x_0, x_1, y_1,$ and z_1 .
 - Test the formulae obtained for y and z in part (i) by investigating their behavior at $x_1 = 0$ and $x_1 = x_0$ and by seeing what happens as $x_0 \rightarrow \infty$. What do you find?

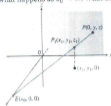


Figure 2: Projection diagram for Question 3.c)