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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2022-2023**
DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

Math 4543: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) i. Define numerical methods. Briefly explain their necessity. 2 (CO2) (PO1)

ii. How do numerical approaches differ from analytical approaches to solving a problem? 2 (CO2) (PO1)

b) i. Suppose, R is an arbitrary number. Prove that the Newton-Raphson formula for finding the inverse of the square root of R, which is mathematically denoted as $\frac{1}{\sqrt{R}}$, is as shown in Equation 1. 5 (CO2) (PO1)

$$x_{i+1} = \frac{1}{2}\left(x_i + \frac{1}{Rx_i}\right) \qquad (1)$$

ii. What are the advantages of formulating $\frac{1}{\sqrt{R}}$ like Equation 1 using the Newton-Raphson method? 2 (CO3) (PO1)

c) The Silver Ratio, often denoted by the Greek letter δ (delta), is the ratio of two quantities a and b with $a > b > 0$, if the ratio $\frac{2a+b}{a} = \frac{a}{b}$. 14 (CO2) (PO1)
The constant δ is equal to the positive root of the quadratic equation $x^2 = 2x + 1$, which is

$$x_{root} = \delta = 1 + \sqrt{2} = 2.41421356237\ldots$$

Apply the Regula Falsi method, also known as the False Position method, to estimate the value of the Silver Ratio δ. The initial upper and lower guesses are $x_l = 2$ and $x_u = 3$ respectively. Demonstrating the step-by-step mathematical procedure, conduct 4 iterations, and find the relative approximate error ($|\epsilon_a|\%$) and the number of significant digits that are at least correct (m) at the end of each iteration. Draw Table 1 on your answer script and fill it out after performing the necessary calculations.

Table 1: The relevant values obtained in the answer of Question 1.c)

Iteration	x_l	x_u	x_r	$\lvert\epsilon_a\rvert\%$	m	$f(x_r)$
1	2	3		—	—	
2						
3						
4						

2. a) Define Truncation Error. What are the different avenues through which Truncation Error can be introduced? Explain with proper examples. (CO) (PO1)

b) The Taylor series for a function $f(x)$ is — 15 (CO2) (PO1)

$$f(x+h) = f(x) + f'(x)\frac{h}{1!} + f''(x)\frac{h^2}{2!} + f'''(x)\frac{h^3}{3!} + f''''(x)\frac{h^4}{4!} + f'''''(x)\frac{h^5}{5!} + \dots$$

Using the Taylor series (with at least the first 5 terms), derive the Maclaurin series of

$$f(x) = \sec(x)$$

c) The Maclaurin series of the exponential function e^x is —

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

i. Why is the exponential function e^x categorized as a Transcendental function? 1 (CO4) (PO1)

ii. Using the Remainder Theorem, establish the bounds of the truncation error in the representation of $e^{1.69}$ if only the first 5 terms of the series are used. 4 (CO2) (PO1)

3. a) Differentiate between Interpolation and Extrapolation. With suitable real-world examples, briefly write when each of the methods is applicable. 4 (CO3) (PO1)

b) Suppose, there is an unknown function $f(x)$ and you are given 3 points $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$ through which $f(x)$ passes. Now, in order to approximate $f(x)$, you decide to use the Newton's Divided Difference method of interpolation and obtain a Quadratic Interpolant $f_2(x)$. Derive this 2nd order Newton's Divided Difference polynomial $f_2(x)$. 8 (CO4) (PO1)

c) i. Briefly explain the necessity of Spline Interpolation. 2 (CO3) (PO1)

ii. Suppose you want to perform Spline interpolation on $n+1$ data points $((x_0, f(x_0))$, $(x_1, f(x_1)), \dots, (x_n, f(x_n))$ using Cubic splines of the form $f(x) = ax^3 + bx^2 + cx + d$. How many unknowns and how many equations would you have to deal with? Mention how you would obtain those equations and derive them. 5 (CO4) (PO1)

d) A thermistor, also known as thermal resistor, is a semiconductor type of resistor whose resistance is strongly dependent on temperature. To measure temperature using a thermistor, the manufacturers provide users with a Temperature (T) vs. Resistance (R) calibration curve. If we measure the resistance of the thermistor using an Ohmmeter, then we can refer to the aforementioned curve and determine the corresponding temperature value. Figure 1 and Table 2 portrays multiple recorded observations involving a thermistor. 6 (CO2) (PO1)

Table 2: Temperature T of a thermistor as a function of its resistance R for Question 3.d)

Resistance, R (Ω)	**Temperature, T** ($^\circ C$)
1101	25.113
911.3	30.131
636	40.12
451.1	50.128

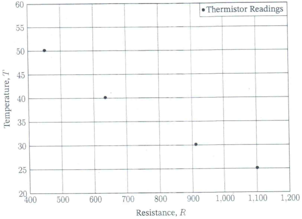

Figure 1: Temperature T vs. Resistance R for Question 3.d)

Determine the value of the temperature T when the thermistor resistance is $R = 754.8\Omega$ using the Lagrangian method of interpolation and a second order polynomial.