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B.Sc. Engg. CSE 7th Semester | 10 October 2023 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION | WINTER SEMESTER, 2022-2023

DURATION: 1 HOUR 30 MINUTES | FULL MARKS: 75

Math 4741: Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) i. Define mutually exclusive events and exhaustive events. Give an example of two events in an experiment of tossing two coins, which are mutually exclusive but not exhaustive. — 4 (CO1) (PO1)

   ii. What is the probability that a leap year selected at random will have 53 Mondays? — 4 (CO1) (PO1)

   b) i. Suppose an urn contains 5 white, 6 red, and 4 blackballs. If a student picks two balls at random, find out the probability that both balls are red. Find the probability of one white and one black ball. — 4 (CO1) (PO1)

   ii. Three students A, B, and C are asked to solve a problem by writing a C++ program. The chances of solving are $\frac{2}{3}, \frac{1}{2}$, and $\frac{1}{3}$ respectively. What is the probability that the problem will be solved? — 4 (CO1) (PO1)

   c) i. Two dice are rolled. What is the probability that at least one is a six? If the two faces are different, what is the probability that at least one is a six? — 4 (CO1) (PO1)

   ii. The dice game craps is played as follows. The player throws two dice, and if the sum is seven or eleven, then the player wins. If the sum is two, three, or twelve, then the player loses. If the sum is anything else, the player continues throwing until he/she either throws that number again (in which case he/she wins) or throws a seven (in which case he/she loses). Calculate the probability that the player wins. — 5 (CO1) (PO1)

2. a) i. Suppose we roll two fair dice. Let E_1 denote the event that the sum of the dice is six, E_2 be the event that the sum of the dice equals seven, and F denote the event that the first die equals four. Is E_1 independent of F? Is E_2 independent of F? — 4 (CO1) (PO1)

   ii. A bag contains 7 red and 5 black balls; another bag contains 5 red and 8 black balls. A ball is drawn from the first bag and without noticing its color, it is put in the second bag. Then a ball is drawn from the second bag. Find the probability that the ball drawn is red in color. — 4 (CO1) (PO1)

b) i. An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident involving a scooter driver is 0.01, a car driver is 0.03 and a truck driver is 0.15. If one of the insured persons meets with an accident, what is the probability that he/she is a car driver? 5 (CO1) (PO1)

ii. Ram speaks truth 2 out of 3 times and Shyam 4 out of 5 times; they agree in an assertion that from a bag containing 6 balls of different colors, a red ball has been drawn. Find the probability that the statement is true. 5 (CO1) (PO1)

c. Suppose that an airplane engine will fail, when in flight, with probability $1 - p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane? 7 (CO3) (PO2)

3. a) An urn contains five red, three orange, and two blue balls. Two balls are randomly selected.

i. What is the sample space of this experiment? 1 (CO1) (PO1)

ii. Let X represent the number of orange balls selected. What are the possible values of X? Also, calculate its corresponding probability. 2 (CO1) (PO1)

iii Calculate $E[X]$ and $Var[X]$ 3 (CO2) (PO1)

b) Let the number of accidents occurring in front of IUT gate each day, X, be a Poisson random variable with parameter λ. The mass function of Poisson random variable is:

$$p(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

i. Define the Poisson random variable with mass function $p(X = x)$, where $x = 0,1, \dots, \infty$. Also, show that $\sum_{x=0}^{\infty} p(x) = 1$. 4 (CO2) (PO1)

ii. Derive the expected number of road accident, i.e. $E[X]$. 3 (CO2) (PO1)

iii If the parameter $\lambda = 1$, find the probability that no accident occurs today. 2 (CO2) (PO1)

c) i. A computer store owner figures that 50 percent of the customers entering his store will purchase a laptop, 20 percent will purchase a Mac, and 30 percent will just be browsing. If five customers enter his store on a certain day, what is the probability that two customers purchase Macs, one customer purchases a laptop, and two customers purchase nothing? 5 (CO3) (PO2)

ii. Suppose there are 25 different types of coupons and each time one obtains a coupon, it is equally likely to be any one of the 25 types. Compute the expected number of different types that are contained in a set of 10 coupons. 5 (CO1) (PO1)