

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination

Winter Semester: 2022 - 2023

Course Number: Math 4311/Math 4599

Full Marks: 75

Course Title: Vector Analysis, Multivariable Calculus and Complex Variables

Time: 1.5 Hours

There are 3 (Three) questions. Answer all of them. The symbols have their usual meanings. Programmable calculators are not allowed. The figures in the right margin indicate full marks.

1. a) Consider the vectors $a = i - 3j + 2k$, $b = 2i - 4j - k$, $c = 3i + 2j - k$. [12] CO2
(i) Find the vector component of a in the directions of b and perpendicular to b . PO1
(ii) Determine whether the vectors a , b , c , are coplanar or not.
- b) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \cdot dr$ from $(0,0,0)$ to $(1,1,1)$ along the following paths C: (i) $x = t$, $y = t^2$, $z = t^3$. (ii) the straight lines from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ then to $(1,1,1)$. [13] CO1
PO1
2. a) Show that $F = (2xy + z^2)i + x^2j + 3xz^2k$ is a conservative force field. Find the scalar potential ϕ . Then use ϕ to find the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$. [12] CO3
PO2
- b) (i) Solve $z^6 + 1 = \sqrt{3}i$. [7] CO1
(ii) Discuss the Riemann surface for the function $z^{1/5}$. [6] PO1
3. a) Consider the transformation $w = \ln z$. [10] CO1
Show that PO1
(i) circles with center at the origin in the z plane are mapped into lines parallel to the v axis in the w plane,
(ii) lines or rays emanating from the origin in the z plane is mapped into lines parallel to the u axis in the w plane,
(iii) the z plane is mapped into a strip of width 2π in the w plane. Illustrate the results graphically.
- b) (i) Determine whether the following function u is harmonic or not. If yes, find the conjugate harmonic function v and express $f(z) = u + iv$ as an analytic function of z . [10] CO1
PO1
$$u(x, y) = e^{-2xy} \sin(x^2 - y^2).$$

(ii) For the function $f(z) = \frac{\cos z}{(z+i)^3}$. Locate and name the singularities. [5]