

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)  
ORGANISATION OF ISLAMIC COOPERATION (OIC)

**DEPARTMENT OF BUSINESS AND TECHNOLOGY MANAGEMENT**

Semester Final Examination

Winter Semester, A. Y. 2022-2023

Course No. : Math 4101

Time : 3 hours

Course Title : Mathematics I

Full Marks : 150

Answer all 6 (six) questions. All questions carry equal marks. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

1. a) i. Define rank of a matrix. 09 (CO3)  
(PO2)
- ii. Find the rank of  $A = \begin{pmatrix} 2 & 1 & 0 & -1 & 3 \\ 1 & 2 & 1 & 2 & 0 \\ 0 & 3 & 1 & 1 & 1 \\ -1 & -5 & -3 & -7 & 3 \end{pmatrix}$
- b) Given  $A = \begin{pmatrix} 2 & 9 & 1 \\ 5 & 7 & 3 \\ 1 & 2 & 8 \end{pmatrix}$ . Calculate  $A^2 - 5A$ . 08 (CO3)  
(PO2)
- c) i. What is the inverse of a matrix? 08 (CO3)
- ii. When is a matrix invertible? (PO2)
- iii. Find the inverse of  $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{pmatrix}$ .
2. a) i. Write down the coefficient matrix of the following system of equations: 12.5 (CO3)
- $x - 2y + z = -1$  (PO3)
- $2x - 3z = 2$
- $x + y - 2z = 0$
- ii. Hence solve the system of equations by finding the inverse of the coefficient matrix.
- b) Solve the following linear equations: 12.5 (CO3)  
(PO4)
- $3x + 2y - z = 20$
- $2x + 3y + 6z = 70$
- $x - y + 6z = 41$

3. a) i. State Cayley-Hamilton theorem. 12.5 (CO3)  
 ii. Given  $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ , Verify Cayley-Hamilton Theorem for A. (PO4)
- b) Given  $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$ . Use Cayley-Hamilton theorem to find  $A^{-1}$ . 12.5 (CO3)  
 (PO4)
4. a) Determine the value of  $\lambda$  so that the following system of linear equations has (i) a unique solution, (ii) more than one solution, (iii) no solution 12.5 (CO4)  
 (PO4)
- $$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + \lambda z &= 3 \\ x + \lambda y + 3z &= 2 \end{aligned}$$
- b) Solve the following system of linear equations: 12.5 (CO4)  
 (PO4)
- $$\begin{aligned} x_1 + x_2 &= -3 \\ 2x_1 - 2x_2 - x_3 &= -8 \\ 4x_1 - x_3 &= -14 \\ x_1 - 3x_2 - x_3 &= -5 \end{aligned}$$
5. a) A diet is to contain at least 80 units of carbohydrate, 50 units of fat and 120 units of protein. Two foods  $F_1$  and  $F_2$  are available:  $F_1$  costs Tk. 3 per unit and  $F_2$  costs Tk. 5 per unit. A unit of food  $F_1$  contains 2 units of carbohydrate, 2 units of fat and 3 units of protein, and a unit of food  $F_2$  contains 5 units of carbohydrate, 1 unit of fat and 4 units of protein. 12.5 (CO3)  
 (PO4)
- Formulate the linear programming (LP).
  - Find the minimum cost for a diet that consists of a mixture of these two foods and meets the minimum nutrition requirements.
- b) For the following LP 12.5 (CO3)  
 (PO4)
- $$\begin{aligned} \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & -x_1 + 3x_2 \leq 9 \\ & x_1 \leq 8 \\ & x_1 + 2x_2 \leq 16 \\ & x_1, x_2 \geq 0 \end{aligned}$$
- Solve it graphically.
  - How many basic solutions are there? How many are feasible?
  - Find the basis and basic variables for each feasible corner point.

6. a) i. What is probability? 08 (CO4)  
ii. Discuss the properties of probability. (PO5)  
iii. Two dice are thrown together. Let A be the event "getting 6 on the 1<sup>st</sup> die" and B be the event "getting 2 on the 2<sup>nd</sup> die". Are the events A and B independent?
- b) Suppose three coins are tossed. What is the probability of getting 3 Heads? 08 (CO4)  
(PO5)
- c) i. Karim is picking a vowel from the set of English alphabets. What is the probability of taking the vowel from the alphabets? 09 (CO4)  
(PO5)  
ii. A bag contains 3 red balls and 5 black balls. Fatema is selecting a ball randomly from the bag. What is the probability that a ball drawn is red?