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B. Sc. Engg. (CEE)/1st Semester 15 December, 2023 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

Semester Final Examination **Winter Semester: 2022 - 2023**
Course Number: Math 4153 **Full Marks: 150**
Course Title: Differential Calculus, Integral Calculus & Matrix **Time: 3.0 Hours**

There are 6 **(Six)** questions. **Answer all questions.** Programmable calculators are not allowed. Do not write on this question paper. The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the brackets.

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1. Find the following integrals:

   (a) $\int \frac{dx}{\cos 3x - \cos x}$ (08) (CO3) (PO2)

   (b) $\int \frac{(3x-2)dx}{\sqrt{3+2x-4x^2}}$ (08) (CO3) (PO2)

   (c) $\int \log(x+\sqrt{x^2+a^2})dx$ (09) (CO3) (PO2)

2. (a) Find a reduction formula for $I_n = \int (\cos^{-1}x)^n\, dx$ (07) (CO3) (PO2)

   (b) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)(2+\sin x)}\, dx$ (08) (CO3) (PO2)

   (c) Prove that: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\, dx = \pi(\frac{\pi}{2}-1)$, (by using properties) (10) (CO3) (PO2)

3. (a) Express the following integral in terms of Gamma functions: (09) (CO3) (PO2)

   $$\int_0^1 x^m\,(1-x^n)^p\, dx.$$

   (b) Evaluate: $\int_{\pi/6}^{\pi/4}\int_0^{4\sin 2\theta} r\sin\theta\cos\theta\, dr d\theta$ (08) (CO3) (PO2)

   (c) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (08) (CO3) (PO2)

4. (a) Explain the transpose of the product of the two matrices is the product in reverse order of their transposes, i,c; $(AB)' = B'A'$. (07) (CO4) (PO1)

(b) Define the orthogonal matrix. Verify that (08) (CO4) (PO1)

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & 1 \end{bmatrix}$$ is orthogonal

(c) Find the rank of the following matrix $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (10) (CO4) (PO1)

by using elementary row transformation.

5. (a) Find the inverse of the following matrix (09) (CO4) (PO1)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$ by algebraic method.

(b) Determine for what values of λ and μ the following equations have (i) no solution, (ii) a unique solution and (iii) infinite number of solution (10) (CO4) (PO1)

$$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$$

(c) Show that every non-Singular matrix can be expressed as a product of elementary matrices. (06) (CO4) (PO1)

6. (a) Give the statement of Cayley-Hamilton theorem. Using the Cayley-Hamilton theorem find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ (08) (CO4) (PO1)

(b) Find the eigenvalues and eigenvectors of the matrix (17) (CO4) (PO1)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$