B.Sc. Eng. (CEE)/ 3rd Sem. 22 December 2023 (Afternoon)

# ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
## ORGANISATION OF ISLAMIC COOPERATION (OIC)
## DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

**Semester: Final Examination** — **Winter Semester: 2022-2023**
**Course No.: MATH 4353** — **Full Marks: 150**
**Course Title: Laplace Transformation, Series, PDE** — **Time: 3 hours**

There are **6 (six)** sets of questions. Answer **all** of them. The figures in the right margin indicate full marks. **CO**s and **PO**s are also specified in the right margin of the questions. The symbols have their usual meaning.

| | | | CO | PO | Marks |
|---|---|---|---|---|---|
| 1. | a) | State sufficient condition for the existence of Laplace Transform. | 1 | 1 | (2) |
| | | Using the property $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$ find $\mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$. | 3 | 2 | (7) |
| | b) | Apply Heaviside expansion formula to compute $\mathcal{L}^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$. | 3 | 2 | (8) |
| | c) | Solve the following initial value problem by using Laplace Transform: $(D^2 + 3D + 2)y = exp(-t); where\ y(0) = 4, y'(0) = 1.$ | 2 | 2 | (8) |
| 2. | a) | Explain analytical form of Fourier series with convergence and points of discontinuity. | 1 | 1 | (4) |
| | b) | Find the Fourier series expansion of $f(x) = \begin{cases} -\pi; -\pi < x < 0 \\ x; 0 < x < \pi \end{cases}$ | 2 | 2 | (10) |
| | | Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}$. | 2 | 2 | (3) |
| | c) | Use suitable formula for half-range series to find cosine series of $f(x) = x$ valid in $0 < x < 2$. | 3 | 2 | (8) |
| 3. | a) | Develop complex form of Fourier series for a function $f(t)$. | 1 | 1 | (4) |
| | b) | Find the Fourier transform of the gate function $f(t)$ defined by $f(t) = \begin{cases} 1\ for\ \lvert t\rvert < a \\ 0\ for\ \lvert t\rvert > a \end{cases}$ | 2 | 2 | (5) |
| | | and hence compute $\int_{-\infty}^{\infty} \frac{\sin a\omega\, \cos\omega t}{\omega} d\omega$. | 2 | 2 | (3) |

| | | | | | |
|---|---|---|---|---|---|
| | c) | Define finite Fourier sine transform. | 1 | 1 | (2) |
| | | Hence apply the transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $u(0,t) = 0, u(6,t) = 0$ *and* $u(x,0) = 2x$. | 3 | 2 | (11) |
| 4. | a) | Find the partial differential equation from $z = ax^3 + by^3 + ab$ by eliminating arbitrary constants. | 2 | 2 | (5) |
| | b) | Write down an algorithm to solve first order linear partial differential equation by Lagrange's method. | 1 | 1 | (4) |
| | | Hence apply the algorithm to solve $(y+z)p + (z+x)q = x + y$. | 3 | 2 | (6) |
| | c) | Use Charpit's method to compute the complete integral of $2zx - px^2 - 2qxy + pq = 0$. | 3 | 2 | (10) |
| 5. | | Solve the following partial differential equations: | | | |
| | a) | $(D_x^3 - 2D_x^2 D_y - D_x D_y^2 + 2D_y^3)z = e^{x+y}$. | 2 | 2 | (8) |
| | b) | $(D_x^2 + 2D_x D_y + D_y^2 - 2D_x - 2D_y)z = cos(x + 2y)$. | 2 | 2 | (9) |
| | c) | $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = x^2 y$ | 2 | 2 | (8) |
| 6. | a) | Derive the Rodrigue's formula $P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n}(x^2 - 1)^n$. | 2 | 2 | (10) |
| | b) | Use Rodrigue's formula to find Legendre Polynomials $P_0(x), P_1(x), P_2(x)$ *and* $P_3(x)$. | 3 | 2 | (4) |
| | c) | Prove that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer. | 3 | 2 | (11) |