ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — WINTER SEMESTER, 2022-2023

DURATION: 3 HOURS — FULL MARKS: 200

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 8 (eight)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Find the relative extrema of the function $f(x) = 3x^5 - 5x^3$. 5 (CO2) (PO1)

   b) Locate the critical points for the function $f(x) = \sqrt[3]{(x^2 - 25)}$, and identify which critical points are stationary. 7 (CO2) (PO1)

   c) A baseball diamond is a square whose sides are 90 ft long shown in Figure 1. Suppose that a player running from second base to third base has a speed of 30 ft/s at the instant when he is 20 ft from third base. At what rate is the player's distance from home plate changing at that instant? 13 (CO3) (PO2)

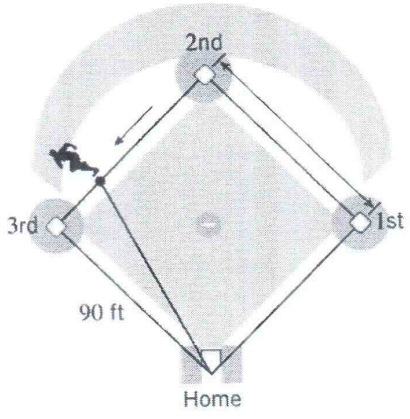

Figure 1: Baseball diamond for Question 1.c)

2. a) The direction cosines of a moving line in two adjacent positions are l, m, n and $l + \delta l, m + \delta m, n + \delta n$. Show that the small angle $\delta\theta$ between the positions is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$. 12 (CO1) (PO1)

   b) If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, respectively. Show that the cosines of the line perpendicular to them both are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$. 13 (CO1) (PO1)

3. a) Find the equation of the plane through the points $(2, 2, 1)$ and $(9, 3, 6)$ and perpendicular to the plane $2x + 6y + 3z = 9$. 12 (CO2) (PO1)

   b) A variable plane is at a constant distance p from the origin and meets the axis in A, B, C. Through A, B, C planes are drawn parallel to the co-ordinates planes. Show that the locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. 13 (CO2) (PO1)

4. a) Prove that the equation: 12 (CO2) (PO1)

$$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$$

   represents a pair of parallel lines.

b) If the pair of straight lines $x^2 - 2axy - y^2 = 0$ and $x^2 - 2bxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that $ab = -1$. 13 (CO2) (PO2)

5. a) Find the radical centre of the following three circles: 6 (CO1) (PO1)

$$x^2 + y^2 + x + 2y + 3 = 0,$$
$$x^2 + y^2 + 2x + 4y + 5 = 0,$$
$$x^2 + y^2 - 7x - 8y - 9 = 0.$$

b) Find the equations of the circle through the intersections of the circles: 6 (CO1) (PO1)

$$x^2 + y^2 + 5x + 6y + 7 = 0,$$
$$x^2 + y^2 + 4x + 3y + 2 = 0,$$

whose centre lies on the line $x + 3y + 5 = 0$.

c) The line joining the origin to the point $P(2, 3)$ is the diameter of a circle which cuts the lines $5x^2 - 12xy + 3y^2 = 0$ at Q and R. Find the combined equation of the lines PQ and PR. 13 (CO1) (PO1)

6. a) Find the n^{th} derivative of $\tan^{-1}\frac{x}{a}$. 12 (CO2) (PO1)

b) If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ and deduce that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. Also find $y_n(0)$. 13 (CO1) (PO1)

7. a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$. 8 (CO3) (PO1)

b) Find an equation for the tangent line to the Folium of Descartes at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$. 8 (CO3) (PO1)

c) At what point(s) in the first quadrant is the tangent line to Folium of Descartes horizontal? 9 (CO3) (PO1)

8. a) Find the absolute maximum and minimum values of $f(x) = 4x^2 - 12x + 10$ on the closed interval $[1, 2]$, and state where those values occur. 6 (CO2) (PO1)

b) If $f(x) = x^2 - 8x + 15$, verify that for the hypotheses of Rolle's Theorem are satisfied on the interval $[3, 5]$, and find all value(s) of c in that interval that satisfy the conclusion of the theorem. 6 (CO2) (PO1)

c) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides represented in Figure 2. What size should the squats be to obtain a box with the largest volume? 13 (CO3) (PO2)

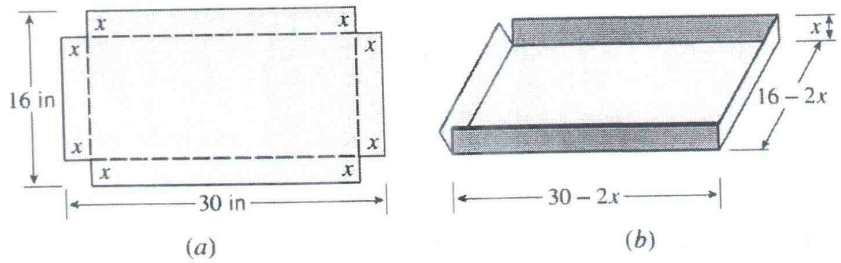

Figure 2: Cardboard box diagram for Question 8.c)