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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2022-2023**

DURATION: 3 HOURS **FULL MARKS: 150**

CSE 4303: Data Structures

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Professor Bunyan thinks he has discovered a remarkable property of Binary Search Trees (BST). Suppose that the search for key k in a BST ends up in a leaf. Consider 3 sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Professor Bunyan claims that any three keys $a \in A$, $b \in B$, and $c \in C$ must satisfy $a \leq b \leq c$. Provide a counterexample disproving the professor's claim. 6 (CO1) (PO1)

   b) Suppose a Priority Queue is implemented using a balanced BST. Discuss how the following requirements can be satisfied: 3×3 (CO1) (PO1)
      i. Extract the element with the highest priority
      ii. Insert an element with an arbitrary priority
      iii. Modify the priority of an existing element

   c) Prove that, the process of building a Max-heap from an arbitrary set of numbers can be done in linear time. 10 (CO3) (PO2)

2. a) Professor Marley hypothesizes that he can obtain substantial performance gains by modifying the separate chaining scheme to keep each list in sorted order. How does the professor's modification affect the running time for successful searches, unsuccessful searches, insertions, and deletions? 5 (CO3) (PO2)

   b) Supposed a hash table is maintained by adopting an open addressing technique as the collision resolution strategy. Identify the potential problem(s) of deleting an element from the table and propose solution(s) to improve the situation. 5 (CO3) (PO2)

   c) Consider inserting the keys $\{10, 22, 31, 4, 15, 28, 17, 88, 59\}$ into a hash table of length $m = 11$, where the hash function is defined as $h(x) = x \% m$. Illustrate the steps of inserting the keys using linear probing, quadratic probing, and double hashing. Utilize a secondary hash function $h_2(x) = 1 + (x \% (m-1))$ where necessary. 5×3 (CO2) (PO1)

3. a) Suppose the Disjoint Set data structure is applied to determine the connected components of an undirected graph $G = (V, E)$, where $V = \{a, b, c, d, e, f, g, h, i, j, k\}$. Showing detailed steps, list the vertices of each connected component by processing the edges of E in the order $(d,i), (f,k), (g,i), (b,g), (a,h), (i,j), (d,k), (b,j), (d,f), (g,j)$, and (a,e). 12 (CO2) (PO1)

   b) To implement *union-by-height*(i, j), the heights of the trees containing i and j need to be compared. When the tree is modified by a union operation, how much time will it take to modify the heights of all the nodes that need to be modified? Assume the heights are stored with each node. 6 (CO3) (PO2)

4. a) Suppose N integers are stored in an array. To find the maximum XOR value between two elements of this array, one obvious way is to compute the XOR of every pair of elements and identify the maximum, resulting in an algorithm of $O(N^2)$ time complexity. Propose an improved approach using the Trie data structure and simulate considering the array as $\{9, 3, 10, 5, 25\}$. 10+5 (CO3) (PO2)

b) What is the benefit of using the Segment Tree data structure? Explain the concept of lazy propagation in the context of a segment tree. 10 (CO1) (PO1)

5. a) How can topological sort be used to detect cycles in a directed graph? 5 (CO1) (PO1)

b) Suppose we wish to search a linked list of length n, where each element contains a key k representing a long character string. To find a query string from this list, one obvious approach is to check every node by comparing the characters, requiring $O(kn)$ time in the worst case. Propose a better idea without using any additional data structure. 5 (CO3) (PO2)

c) Define Data Structures. Mention one use case for each of the following data structures: Stack, Deque, Linked List, Priority Queue, AVL Tree, Hash Table, Trie, and Graph. $2+8$ (CO1) (PO1)

d) Assume some numbers are stored in a Linked List (LL). Mention the worst-case running time for the operations mentioned in Table 1. 24×0.5 (CO1) (PO1)

	unsorted, singly LL	sorted, singly LL	unsorted, doubly LL	sorted, doubly LL
Search (key)				
Insert (x)				
Delete (x)				
Successor (x)				
Predecessor (x)				
Minimum ()				
Maximum ()				

Table 1: Incomplete worst-case run time table for Question 5.d)

6. a) Showing detailed steps, find the distance (d) and predecessor (π) values for each vertex that result from running a Breadth-first search (BFS) on the directed graph given in Figure 1. Use vertex q as the source. The BFS procedure will consider the vertices in alphabetical order and assume that each adjacency list is ordered alphabetically. 16 (CO2) (PO1)

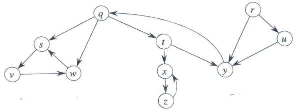

Figure 1: A directed graph for Question 6.a)

b) Suppose Depth-first Search is applied on a graph G, and the discovery and finishing time for every vertex are stored using the attributes d & f, respectively. Justify that an edge (u, v) is: 3×3 (CO1) (PO1)

i. A tree edge or forward edge if and only if $u.d < v.d < v.f < u.f$

ii. A back edge if and only if $v.d \leq u.d < u.f \leq v.f$

iii. A cross edge if and only if $v.d < v.f < u.d < u.f$