ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2022-2023**

DURATION: 3 HOURS **FULL MARKS: 150**

Math 4541: Multivariable Calculus and Complex Variable

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Find the domain and range of the function $f(x, y) = 100 - x^2 - y^2$ and plot the level curves $f(x, y) = 0$, $f(x, y) = 51$, and $f(x, y) = 75$ in the domain of f in the plane. 8 (CO1) (PO1)

   b) Show that 8 (CO1) (PO2)

$$f(x, y) = \begin{cases} 2xy/(x^2 + y^2), & (x, y) \neq (0, 0) \\ 0 & , \ (x, y) = (0, 0) \end{cases}$$

   is continuous at every point except the origin.

   c) Consider the circuit diagram given in Figure 1. If resistors of R_1, R_2 and R_3 *ohms* are connected in parallel to make an $R-ohm$ resistor, the value of R can be found from the Equation (1) 9 (CO1) (PO1)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (1)$$

   Find the value of $\delta R / \delta R_2$ when $R_1 = 30, R_2 = 45$, and $R_3 = 90$ *ohms*.

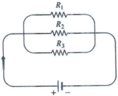

Figure 1: Circuit configuraiton for Question 1.c

2. a) Find the curvature and torsion for the helix 16 (CO1) (PO1)

$$r(t) = (a \cos t)i + (a \sin t)j + btk, \quad where \ a, b \geq 0 \ and \ a^2 + b^2 \neq 0$$

   b) An object of mass m travels along the parabola $y = x^2$ with a constant speed of 10 *units/sec*. What is the force on the object due to its acceleration at $(0, 0)$? At $(2^{\frac{1}{2}}, 2)$? Write your answers in terms of i and j. 9 (CO2) (PO1)

3. a) Find the angle between the velocity and acceleration vector of $r(t) = (3t + 1)i + 23tj + t^2k$ at time $t = 0$. Create a graph for the vector function given by $r(t) = (\cos t)i + (\sin t)j + tk$. 8 (CO1) (PO1)

b) Find the point on the curve $r(t) = (12 \sin t)i - (12 \cos t)j + 5tk$. at a distance 13p units along the curve from the point $(0, -12, 0)$ in the direction opposite to the direction of increasing arc length. 8 (CO1) (PO1)

c) Suppose we do not know the path of a hang glider, but only its acceleration vector $a(t) = -(3 \cos t)i - (3 \sin t)j + 2k$. We also know that initially (at time $t = 0$) the glider departed from the point $(4, 0, 0)$ with velocity $v(0) = 3j$. Find the glider's position as a function of t. 9 (CO2) (PO2)

4. a) Find $\delta w / \delta r$ when $r = 1,\ s = -1$ if $w = (x + y + z)^2$, $x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$ 8 (CO1) (PO1)

b) Let $f(x, y) = (x - y)(x + y)$. Find the directions u and the values of $D_u f(1, -1)$ for which $D_u f(1, -1)$ is largest. 8 (CO1) (PO1)

c) Find the tangent plane and normal line of the level surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$. 9 (CO1) (PO1)

5. a) Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 4y - x^2 - y^2$ on the triangular region in the first quadrant bounded by the lines $x = 0,\ y = 0$ and $y = 9 - x$. 16 (CO1) (PO1)

b) Find the dimensions of the closed right circular cylindrical can of smallest surface area whose volume is $16\pi\ cm^3$. 9 (CO1) (PO1)

6. a) Find the volume of the prism whose base is the triangle in the $xy - plane$ bounded by the $x-axis$ and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$. 8 (CO1) (PO1)

b) Using double integral find the area of the region R enclosed by the parabola $y = x^2$ and the line$y = x + 2$. 8 (CO1) (PO1)

c) Find the volume of the wedge like solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the $x - axis$. 9 (CO2) (PO1)