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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2022-2023**

DURATION: 3 HOURS **FULL MARKS: 150**

CSE 4549: Simulation and Modeling

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

1. Consider a system where customers arrive following the exponential distribution with a mean of 3 minutes. There is a uniform distribution of customers, between 2 and 4, waiting outside when the system opens. It takes 15 seconds to walk from the front door to the queue. There are two different types of customers. A *regular* customer and a *repeat* customer. There are approximately 25% repeat customers.

   There is one queue which feeds to two different order taking clerks. Repeat customers have priority in the queue. That means whenever a repeat customer arrives, s/he immediately goes to the head of the queue unless there is already a repeat customer in the queue. In that case, the customer enters the queue behind the last repeat customer. One of the two clerks is new and is not as capable as the other. The order taking time for the experienced clerk is exponentially distributed with a mean of 5 minutes. The order taking time for the inexperienced clerk is exponentially distributed with a mean of 6 minutes.

   Once a customer is finished placing her/his order, s/he waits until it is complete. In the meantime, the order is placed in an order processing queue where a single order processing clerk processes the order. The order processing time is exponentially distributed with mean 3 minutes. When the order is processed, it is placed on a 100 feet conveyor that travels at 200 feet per minute. When the order arrives at the end of the conveyor, the customer picks up her/his order, and the system time ends. The customer then walks out of the store which takes 15 seconds.

   You are asked to develop a discrete event system simulation program to collect statistics on the time average number in queue for each of the two queues, the average utilization rate for each of the three clerks, and the average system time for each of the two customer types.

   a) State the set of events and the set of state variables for the simulation model. Draw an event graph to justify that you stated the minimum number of events required for the system. 3 + 3 (CO1) (PO2)

   b) Mention the relation between the events and the state variables, and how the events are changing the system states. Assume the simulation is terminated by an event. 3 + 3 (CO1) (PO2)

   c) Write down the state equation(s) and the output equation(s) of the simulation model. The state equation(s) is/are expected to reflect the change in each of the state variables with the occurrence of events. 5 + 3 (CO1) (PO2)

   d) Draw separate flow charts of the event routines (i.e., the event handling functions) for each of the events of the simulation model mentioned in Question 1.a). 20 (CO1) (PO2)

      Note that, from the event handling functions you can, in general, call other functions only, which may be shared by other event handling functions as well. Further, draw the flow charts of the functions that are called from the event handling functions.

2. a) Consider a random variable X which has PDF given by 10 (CO2) (PO1)

$$f_X(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1, \\ 2 - x & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

This distribution is called a triangular distribution with endpoints at 0 and 2, and mode at 1. Develop a random variate generator for this random variable.

b) Consider the following symmetric distribution on [-1, +1]: 10 (CO2) (PO1)

$$f_X(x) = \begin{cases} x + 1 & \text{if } -1 \leq x \leq 0, \\ -x + 1 & \text{if } 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Develop a random variate generator for this random variable using the convolution method. Further, generate 3 random values for the random variable. Assume necessary pseudo random numbers for the generator.

3. The number of vehicles arriving at the northwest corner of an intersection in a 5-minute period between 7:00 A.M. and 7:05 A.M. was monitored for five workdays over a 20-week period. Table 1 shows the resulting data. The first entry in the table indicates that there were 12 5-minute periods during which zero vehicles arrived, 10 periods during which one vehicle arrived, and so on. The number of vehicles is a discrete random variable.

Table 1: Number of Arrivals in a 5-Minute Period for Question 3.

Arrivals per Period	Frequency	Arrivals per Period	Frequency
0	12	6	7
1	10	7	5
2	19	8	5
3	17	9	3
4	10	10	3
5	8	11	1

a) Find the summary statistics of data – *lexis ratio* and *skewness*. Also, from the summary statistics, comment on the possible distribution(s). 4 + 3 (CO2) (PO1)

b) Make an appropriate histogram of the data. From the histogram, determine a fitted distribution of the data and justify the selection of the fitted distribution. 4 + 3 (CO2) (PO1)

c) With the help of a frequency comparison graph, find the similarities between the fitted distribution and the true underlying distribution (from which the data has been collected). 7 (CO2) (PO1)

d) Apply the Chi-square test to these data for the following hypotheses test: 9 (CO2) (PO1)
H_0: The random variable has the fitted distribution.
H_1: The random variable does not have the fitted distribution.

4. a) Assume you are simulating a computer network to measure the average delay of packets, θ. To estimate θ, you run the simulation 20 times and generated 20 independent mean delays of packets. Assume the values obtained are $8+8$ (CO3) (PO4)

102	113	131	107	114
95	133	145	139	117
93	111	124	122	136
141	119	122	151	143

i. Construct a 90% confidence interval for the estimated value of θ.

ii. Find the number of additional simulation-runs to be necessary if you want to be 99% certain that your final estimate of θ is correct within ± 3.25.

b) You have simulated two systems for 7 replications. The sample means of the response time for system 1 are 100, 105, 110, 108, 102, 112 and 98 milliseconds. For system 2 they are 100, 105, 110, 108, 100, 110 and 95 milliseconds. Find the 90% and 95% confidence intervals for the difference of the mean response times of the two systems. Explain which system has a better response time based on the measured confidence intervals. 9 (CO3) (PO4)

5. Consider a barbershop that can hold only three customers, one in service and two in waiting. Additional customers are turned away when the system is full. On the average, a customer arrives at the barbershop for every half an hour and the barber usually spends 20 minutes to provide service for one customer. You need to model the barbershop as an $M/M/m/K$ ($m = 1$, and $K = 3$) queueing system. Note that, the probability that there are n customers in a $M/M/1$ queueing system is given by

$$P_n = \frac{\lambda_{n-1}\lambda_{n-2}\dots\lambda_1\lambda_0}{\mu_n\mu_{n-1}\dots\mu_2\mu_1\left(1+\sum_{n=1}^{\infty}\frac{\lambda_{n-1}\lambda_{n-2}\dots\lambda_1\lambda_0}{\mu_n\mu_{n-1}\dots\mu_2\mu_1}\right)}, n \geq 1$$

a) Derive the formula for finding the probability that there is/are n customer(s), $n = 0, 1, 2, 3$, in the system. 10 (CO1) (PO2)

b) Find the fraction of time the barber is busy. 5 (CO1) (PO2)

c) Find the rate of customers being turned away due to the fact that there is no space available. 5 (CO1) (PO2)

d) Find the average waiting time of the customers. 5 (CO1) (PO2)

6. Consider a single product conventional inventory system. Suppose you are asked to compare the results for $(s, S) = (20, 40)$, which is called model 1, vs. $(s, S) = (20, 100)$, which is called model 2, where n denotes minimum order threshold (i.e., an order is placed, if the inventory level is less than s) and S denotes the maximum inventory capacity. There are three sources of randomness: interdemand times, demand sizes, and delivery lags. $5+5$ (CO1) (PO2)

With the help of a timing diagram and necessary random numbers show that a single stream of random numbers might result in a poor synchronization (or a lack of synchronization). Further, show that the use of 3 streams of random numbers (one for each of the sources of randomness) may not solve the synchronization problem completely.

Critical points $t_{\nu,\gamma}$ for the t distribution with ν df, and z_γ for the standard normal distribution
$\gamma = P(T_\nu \le t_{\nu,\gamma})$, where T_ν is a random variable having the t distribution with ν df; the last row, where $\nu = \infty$, gives the normal critical points satisfying $\gamma = P(Z \le z_\gamma)$, where Z is a standard normal random variable

	γ															
ν	0.6000	0.7000	0.8000	0.9000	0.9333	0.9500	0.9600	0.9667	0.9750	0.9800	0.9833	0.9875	0.9900	0.9917	0.9938	0.9950
1	0.325	0.727	1.376	3.078	4.702	6.314	7.916	9.524	12.706	15.895	19.043	25.452	31.821	38.342	51.334	63.657
2	0.289	0.617	1.061	1.886	2.456	2.920	3.320	3.679	4.303	4.849	5.334	6.205	6.965	7.665	8.897	9.925
3	0.277	0.584	0.978	1.638	2.045	2.353	2.605	2.823	3.182	3.482	3.738	4.177	4.541	4.864	5.408	5.841
4	0.271	0.569	0.941	1.533	1.879	2.132	2.333	2.502	2.776	2.999	3.184	3.495	3.747	3.966	4.325	4.604
5	0.267	0.559	0.920	1.476	1.790	2.015	2.191	2.337	2.571	2.757	2.910	3.163	3.365	3.538	3.818	4.032
6	0.265	0.553	0.906	1.440	1.735	1.943	2.104	2.237	2.447	2.612	2.748	2.969	3.143	3.291	3.528	3.707
7	0.263	0.549	0.896	1.415	1.698	1.895	2.046	2.170	2.365	2.517	2.640	2.841	2.998	3.130	3.341	3.499
8	0.262	0.546	0.889	1.397	1.670	1.860	2.004	2.122	2.306	2.449	2.565	2.752	2.896	3.018	3.211	3.355
9	0.261	0.543	0.883	1.383	1.650	1.833	1.973	2.086	2.262	2.398	2.508	2.685	2.821	2.936	3.116	3.250
10	0.260	0.542	0.879	1.372	1.634	1.812	1.948	2.058	2.228	2.359	2.465	2.634	2.764	2.872	3.043	3.169
11	0.260	0.540	0.876	1.363	1.621	1.796	1.928	2.036	2.201	2.328	2.430	2.593	2.718	2.822	2.985	3.106
12	0.259	0.539	0.873	1.356	1.610	1.782	1.912	2.017	2.179	2.303	2.402	2.560	2.681	2.782	2.939	3.055
13	0.259	0.538	0.870	1.350	1.601	1.771	1.899	2.002	2.160	2.282	2.379	2.533	2.650	2.748	2.900	3.012
14	0.258	0.537	0.868	1.345	1.593	1.761	1.887	1.989	2.145	2.264	2.359	2.510	2.624	2.720	2.868	2.977
15	0.258	0.536	0.866	1.341	1.587	1.753	1.878	1.978	2.131	2.249	2.342	2.490	2.602	2.696	2.841	2.947
16	0.258	0.535	0.865	1.337	1.581	1.746	1.869	1.968	2.120	2.235	2.327	2.473	2.583	2.675	2.817	2.921
17	0.257	0.534	0.863	1.333	1.576	1.740	1.862	1.960	2.110	2.224	2.315	2.458	2.567	2.657	2.796	2.898
18	0.257	0.534	0.862	1.330	1.572	1.734	1.855	1.953	2.101	2.214	2.303	2.445	2.552	2.641	2.778	2.878
19	0.257	0.533	0.861	1.328	1.568	1.729	1.850	1.946	2.093	2.205	2.293	2.433	2.539	2.627	2.762	2.861
20	0.257	0.533	0.860	1.325	1.564	1.725	1.844	1.940	2.086	2.197	2.285	2.423	2.528	2.614	2.748	2.845
21	0.257	0.532	0.859	1.323	1.561	1.721	1.840	1.935	2.080	2.189	2.277	2.414	2.518	2.603	2.735	2.831
22	0.256	0.532	0.858	1.321	1.558	1.717	1.835	1.930	2.074	2.183	2.269	2.405	2.508	2.593	2.724	2.819
23	0.256	0.532	0.858	1.319	1.556	1.714	1.832	1.926	2.069	2.177	2.263	2.398	2.500	2.584	2.713	2.807
24	0.256	0.531	0.857	1.318	1.553	1.711	1.828	1.922	2.064	2.172	2.257	2.391	2.492	2.575	2.704	2.797
25	0.256	0.531	0.856	1.316	1.551	1.708	1.825	1.918	2.060	2.167	2.251	2.385	2.485	2.568	2.695	2.787
26	0.256	0.531	0.856	1.315	1.549	1.706	1.822	1.915	2.056	2.162	2.246	2.379	2.479	2.561	2.687	2.779
27	0.256	0.531	0.855	1.314	1.547	1.703	1.819	1.912	2.052	2.158	2.242	2.373	2.473	2.554	2.680	2.771
28	0.256	0.530	0.855	1.313	1.546	1.701	1.817	1.909	2.048	2.154	2.237	2.368	2.467	2.548	2.673	2.763
29	0.256	0.530	0.854	1.311	1.544	1.699	1.814	1.906	2.045	2.150	2.233	2.364	2.462	2.543	2.667	2.756
30	0.256	0.530	0.854	1.310	1.543	1.697	1.812	1.904	2.042	2.147	2.230	2.360	2.457	2.537	2.661	2.750
40	0.255	0.529	0.851	1.303	1.532	1.684	1.796	1.886	2.021	2.123	2.203	2.329	2.423	2.501	2.619	2.704
50	0.255	0.528	0.849	1.299	1.526	1.676	1.787	1.875	2.009	2.109	2.188	2.311	2.403	2.479	2.594	2.678
75	0.254	0.527	0.846	1.293	1.517	1.665	1.775	1.861	1.992	2.090	2.167	2.287	2.377	2.450	2.562	2.643
100	0.254	0.526	0.845	1.290	1.513	1.660	1.769	1.855	1.984	2.081	2.157	2.276	2.364	2.436	2.547	2.626
∞	0.253	0.524	0.842	1.282	1.501	1.645	1.751	1.834	1.960	2.054	2.127	2.241	2.326	2.395	2.501	2.576

Critical points $\chi^2_{\nu,\gamma}$ for the chi-square distribution with ν df

$\gamma = P(Y_\nu \leq \chi^2_{\nu,\gamma})$ where Y_ν has a chi-square distribution with ν df; for large ν, use the approximation for $\chi^2_{\nu,\gamma}$ in Sec. 7.4.1

	γ						
ν	0.250	0.500	0.750	0.900	0.950	0.975	0.990
1	0.102	0.455	1.323	2.706	3.841	5.024	6.635
2	0.575	1.386	2.773	4.605	5.991	7.378	9.210
3	1.213	2.366	4.108	6.251	7.815	9.348	11.345
4	1.923	3.357	5.385	7.779	9.488	11.143	13.277
5	2.675	4.351	6.626	9.236	11.070	12.833	15.086
6	3.455	5.348	7.841	10.645	12.592	14.449	16.812
7	4.255	6.346	9.037	12.017	14.067	16.013	18.475
8	5.071	7.344	10.219	13.362	15.507	17.535	20.090
9	5.899	8.343	11.389	14.684	16.919	19.023	21.666
10	6.737	9.342	12.549	15.987	18.307	20.483	23.209
11	7.584	10.341	13.701	17.275	19.675	21.920	24.725
12	8.438	11.340	14.845	18.549	21.026	23.337	26.217
13	9.299	12.340	15.984	19.812	22.362	24.736	27.688
14	10.165	13.339	17.117	21.064	23.685	26.119	29.141
15	11.037	14.339	18.245	22.307	24.996	27.488	30.578
16	11.912	15.338	19.369	23.542	26.296	28.845	32.000
17	12.792	16.338	20.489	24.769	27.587	30.191	33.409
18	13.675	17.338	21.605	25.989	28.869	31.526	34.805
19	14.562	18.338	22.718	27.204	30.144	32.852	36.191
20	15.452	19.337	23.828	28.412	31.410	34.170	37.566
21	16.344	20.337	24.935	29.615	32.671	35.479	38.932
22	17.240	21.337	26.039	30.813	33.924	36.781	40.289
23	18.137	22.337	27.141	32.007	35.172	38.076	41.638
24	19.037	23.337	28.241	33.196	36.415	39.364	42.980
25	19.939	24.337	29.339	34.382	37.652	40.646	44.314
26	20.843	25.336	30.435	35.563	38.885	41.923	45.642
27	21.749	26.336	31.528	36.741	40.113	43.195	46.963
28	22.657	27.336	32.620	37.916	41.337	44.461	48.278
29	23.567	28.336	33.711	39.087	42.557	45.722	49.588
30	24.478	29.336	34.800	40.256	43.773	46.979	50.892
40	33.660	39.335	45.616	51.805	55.758	59.342	63.691
50	42.942	49.335	56.334	63.167	67.505	71.420	76.154
75	66.417	74.334	82.858	91.061	96.217	100.839	106.393
100	90.133	99.334	109.141	118.498	124.342	129.561	135.807