ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2022-2023**
DURATION: 3 HOURS **FULL MARKS: 150**

CSE 4703: Theory of Computing

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Explain if each of the following assertion is correct or incorrect. 3×5 (CO1) (PO1)
   i. Membership problem in context-free language is decidable.
   ii. M_1 is a deterministic finite automaton (DFA) while M_2 is a non-deterministic finite automaton (NFA). The problem of determining the equivalence of language generated by M_1 and M_2 i.e. $L_1(M_1) = L_2(M_2)$ is an undecidable problem.
   iii. If $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and $L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$ are two language, then $L_1 \cup L_2$ is context-free language.
   iv. The set of all irrational numbers in the interval $(0, 1)$ is infinitely countable.
   v. A Turing machine (TM) can compute anything a desktop PC can, although it might take more time

   b) Programming languages allow comments to appear between delimiters such as /# and #/. Build a memory-free state machine that can recognize and accept all valid delimited comment strings. A string must start with "/#" and end with "#/", without any intervening "#/". For simplicity, assume the alphabet, $\sum = \{a, b, /, \#\}$. 10 (CO1) (PO2)

2. a) Determine the regular expression for the language L given as follows. 10 (CO1) (PO1)

$$L = \{a^n b^m \mid m + n = even \text{ and } m, n \geq 0\}, \text{ where } \sum = \{a, b\}$$

   b) Let G be a context-free grammar given below. 15 (CO2) (PO1)

$$S \rightarrow AB \mid SS \mid a$$
$$A \rightarrow BS \mid CD \mid b$$
$$B \rightarrow DD \mid b$$
$$C \rightarrow DE \mid a \mid b$$
$$D \rightarrow a$$
$$E \rightarrow SS$$

   Determine whether the string ***'abaab'*** is the member of the $L(G)$ using the CYK algorithm (in your answer you need to the show table).

3. a) Design a context-free grammar for the language L given as follows on alphabet, $\sum = \{a, b\}$. 10 (CO2) (PO2)

$$L = \{a^n b^m \mid m \geq n\} \cup \{a^n b^{2n+1} \mid n \geq 1\}$$

   b) Design state diagram of pushdown automata (PDA) for the language L defined as follows. 10 (CO2) (PO2)

$$L = \{a^n b^{m+n} c^n \mid m, n \geqq 1\}, \text{ where } \sum = \{a, b, c\}$$

   c) When simulating an NFA on a Turing machine (TM) to determine whether a string w is accepted or rejected by the NFA, you may encounter certain challenges. What are these challenges, and how can you solve them? 5 (CO2) (PO2)

4. a) Alan Turing's Turing machine (TM) is a robust theoretical model for solving problems. A general-purpose computer is more accurately represented by a TM, which is like a finite automaton but has infinite memory. However, Turing machines cannot solve all problems. Answer the following question about TM. $5 + 6 + 4$ (CO1) (PO1)

   i. Give a formal definition of TM.

   ii. Give the precise mathematical expression of the transition function of a variety of TM types, such as deterministic TM, non-deterministic TM, and multi-tape TM.

   iii. The original Alan's TM model and its all variants have the same power—justify this statement.

   b) Design a Turing machine (TM) that takes as input two messages, m_1 and m_2, of equal length and compares whether the m_1 and m_2 messages are identical. The tape initially contains $m_1\#m_2$, where '#' is a tape symbol that is used as the separator. Assume the message contains the alphabet, $\sum = \{a, b\}$. 10 (CO2) (PO2)

5. a) Mr. Robel has been assigned to develop a Turing machine (TM) that takes a DFA as an input and verifies whether the DFA accepts no strings, indicating that it recognizes an empty language. 5+10 (CO3) (PO2)

   i. Is the construction of such a TM machine possible?

   ii. Prove that emptiness testing for regular language that is recognized by DFA is a decidable problem.

   b) Consider the language $A_{\varepsilon-CFG}$ is defined as follows. 10 (CO3) (PO2)

$$A_{\varepsilon-CFG} = \{<G> \mid G\ is\ a\ context-free\ grammar(CFG)\ that\ generates\ string\ \varepsilon\}.$$

   Show that language $A_{\varepsilon-CFG}$ is decidable.

6. a) Consider the language A_{TM} is defined as follows. 10 (CO3) (PO2)

$$A_{TM} = \{<M, w> \mid M\ is\ a\ Turing\ machine\ (TM)\ and\ M\ accepts\ w\ \}.$$

   Show that language A_{TM} is undecidable.

   b) It is proven that a language is Turing-recognizable if and only if some enumerator (E) enumerates it. To prove this claim, we run Turing machine M for i steps on each input string, $s_1, s_2, ..., s_i$ and print out the string s_j, if M accepts it. Why do we not use the following simpler algorithm for the forward direction of the proof? As before, $s_1, s_2, ...$ is a list of all strings in $\sum^*$. 10 (CO3) (PO2)

   $E =$ "*Ignore the input.*

   1. *Repeat the following for* $i = 1, 2, 3, ...$
   2. *Run* M *on* s_i.
   3. *If it accepts, print out* s_i."

   c) You are given two classes of problems, labeled as X and Y, together with a Turing machine M. How can you determine if X and Y are decidable or turing-recognizable? 5 (CO3) (PO2)