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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2022-2023**

DURATION: 3 HOURS **FULL MARKS: 150**

CSE 6261: Advanced Probability and Stochastic Processes

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 8 (eight)** questions. Figures in the right margin indicate full marks of questions.

1. There are two servers available to process n jobs. All the jobs are ready at the beginning. Initially, each server begins work on one job each. Whenever a server completes work on a job, that job leaves the system and the server begins processing a new job (provided that there are still jobs waiting to be processed). Assume that T_1 denotes the departure time of the first departing job, T_i denotes the inter-departure time of the i-th job (i.e., the difference of departure times of the i-th and $(i-1)$-th job), $i = 2, 3, \dots, n$, and T denotes the time until all jobs have been processed. Suppose the time that it takes server j to process a job is exponentially distributed with rate μ_j where $j = 1, 2$.
   a) Find the expected time to process the first departing job (i.e., the departure time of the first job), $E[T_1]$. 5
   b) Find the expected inter-departure time of the n-th job, $E[T_n]$. 12
   c) Find the expected time to process all n jobs, $E[T]$. 8

2. In order to study the behavior of communication networks for transmission of speech, speaker activity is modeled as a Markov chain as shown in Figure 1. The interpretation of the states is as follows: State 0 indicates silence; State 1 indicates unvoiced speech; and State 2 represents voiced speech. The transition rates are given to be: $r_{01} = 1$, $r_{10} = 1.5$, $r_{12} = 2$, and $r_{21} = 3$.

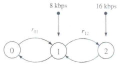

Figure 1: Markov Chain for Question 2.

   a) Determine the limiting-state probabilities of this model. 10
   b) If unvoiced speech is transmitted at 8 kbps and voiced speech is transmitted at 16 kbps, find the average bit rate produced by the transmitter. 5

3. a) Let X and Y be independent random numbers from the interval $(0, 1)$. Find the joint probability density function of $U = 2\ln X$ and $V = 2\ln Y$. 10
   b) Let X and Y be two positive independent continuous random variables with the probability density functions $f(x)$ and $f(y)$, respectively. Find the probability density function of $U = X/Y$. 10

   Hint: Let $V = X$; and find the joint probability density function of U and V. Then calculate the marginal probability density function of U.

4. An absent-minded professor has four umbrellas that she uses when commuting from home to office and back. Among these four umbrellas, some are at home and some in the office. If it rains and an umbrella is available in her current location, she takes it. If it does not rain, she always forgot to take an umbrella. It may happen that all umbrellas are in one place (home or office), the professor is at the other place, and if it rains at the time of his commuting then she must get wet.

   a) Suppose that it rains with probability p each time she commutes, independently of other times. Find the steady-state probability that she gets wet on a given day. 10

   b) Assume that according to the current estimate $p = 0.6$ in Gazipur in the rainy season. How many umbrellas should the professor have so that, if she follows the strategy above, the probability that she gets wet is less than 0.1? 10

5. Consider N nodes in a wireless LAN with a single base station. Each of the N nodes has exactly one packet to be transmitted to the base station. To coordinate with each other and avoid collisions the following simple rule protocol is used:

   - Time is slotted - transmissions are only initiated at the beginning of a slot.
   - Every node transmits in each slot with a probability of p and does not transmit with a probability $(1 - p)$.
   - When exactly one node transmits in a slot, there is a success, and that node will never have to transmit again.
   - When more than one node transmits in the same slot, there is a collision - each of the nodes colliding will simply retry with the same probability p until they succeed.

   a) Find the expected number of slots until all nodes transmit successfully when $N = 5$. 8

   b) Find the expected number of slots until all nodes transmit successfully when $N = k$. 7

6. Consider an HMM with two states 1 and 2 and that emits two symbols: A and B. The state transition diagram is shown in Figure 2.

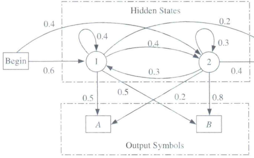

Figure 2: State transition diagram of the HMM in Question 6.

   a) Use the Viterbi algorithm to obtain the most likely state sequence that produced the observation sequence "BAAB". 10

   b) Estimate the probability that the sequence "BAAB" was emitted by the preceding system. 10

7. A coin that comes up heads with probability p is continuously flipped until the pattern T, T, H appears. That is, you stop flipping when the most recent flip lands head and the two immediately preceding it land tails. Assume that X denotes the number of flips made. Find $E[X]$. 15

8. The joint PMF of two discrete random variables X given Y is given in Table 1.

Table 1: PMF, $P_{XY}(x, y)$ for Question 8.

$P_{XY}(x, y)$	a	b	c
1	1/3	1/12	1/12
2	1/12	0	1/24
3	1/24	1/3	0

for $S_X = \{1, 2, 3\}$ and $S_Y = \{a, b, c\}$.

a) Find $H[X]$. 10

b) Find $I[X; Y]$ 10