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B.Sc. in EEE, 1st Semester. December 23, 2023

09:00 am - 12:00 pm

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Semester Final Examination — Winter Semester, A. Y. 2022-2023
Course No.: Math 4121 — Time: 3 Hours
Course Title: Mathematics I — Full Marks: 150

There are 6 (six) questions. Answer all 6 (six) questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets

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1. a) Explain first derivative test, second derivative test and higher derivative test for the point of inflection of a function. Determine the point of inflection of the function $y = (5x-4)^3$ using first derivative. (12) (CO3) (PO1)

   b) Mention the different indeterminate forms. Explain the way of evaluating these forms. State and prove L'Hospitals theorem. (13) (CO2) (PO1)

2. a) If $u = F(x^2+y^2+z^2) f(xy+yz+zx)$ then find the value of (12) (CO2) (PO1)

   $$(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z}$$

   b) Find the radius of curvature of the curve (13) (CO1) (PO1)

   $y = \frac{1}{2}a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ at the point (0, a)

3. a) Solve the following: (12) (CO1) (PO1)

   i) $\int \frac{x+1}{\sqrt{4+8x-5x^2}}dx$ ii) $\int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}}dx$

   b) Find the reduction formula for (13) (CO1) (PO1)

   $I_m = \int_0^{\frac{\pi}{2}} \cos^m x dx$ hence find Wallis's formula for $\int_0^{\frac{\pi}{2}} \cos^m x dx$

4. a) Evaluate the following definite integrals (12) (CO2) (PO1)

   i) $\int_0^{\frac{\pi}{2}} \ln\sqrt[6]{\cos x}dx$ ii) $\int_0^{a} x^8\sqrt{a^2-x^2}\, dx$

b) Show that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ hence find the value of $B(2, \frac{1}{2})$. (13) (CO2) (PO1)

5. a) Evaluate $\int_1^3 \dfrac{e^{\sqrt{2x-1}}}{\sqrt{x+2}}\, dx$ by Simpson's rule taking 10 subintervals. (12) (CO1) (PO1)

b) Find the length of the loop of the curve $3ay^2 = x(x-a)^2$. (13) (CO2) (PO1)

6. a) Find the area included between the curve $y^2(a-x) = x^3$ and its asymptote. (12) (CO3) (PO1)

b) The arc of the asteroid $x = a\cos^3\theta,\ \ y = a\sin^3\theta$ from $\theta = 0$ to $\theta = \dfrac{\pi}{4}$ revolves about x-axis. Find the surface area of the solid generated. (13) (CO3) (PO1)