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B.Sc. in EEE, 1st Semester December 22, 2023(**Time:**)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Semester Final Examination
Course No.: Math 4123
Course Title: Matrix and Differential Equations

Winter Semester, A. Y. 2022-2023
Time: 3 Hours
Full Marks: 150

There are **6 (six)** questions. Answer all **6 (six)** questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

1(a) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and use this theorem to find the followings: **15 (Co1, Po2)**
(i) A^{-1},
(ii) $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

(b) Solve: $\frac{d^5y}{dx^5} - \frac{7(d^4y)}{dx^4} + \frac{20(d^3y)}{dxy} - \frac{40(d^2y)}{dx^2} + \frac{64dy}{dx} - 48y = 0.$ **10 (Co3, Po2)**

2(a) Using the matrix method to find the currents $i_1(t)$ and $i_2(t)$ in an electrical network containing resistances $R_1 = 12$ ohms, $R_2 = 8$ ohms, inductors $L_1 =$ 4 henry, $L_2 = 4$ henry and $E = 400 \sin t$ volt. The currents $i_1(t)$ and $i_2(t)$ are initially zero. **16 (Co5, Po2)**

(b) Solve: $(D_x^2 - 4D_xD_y + 4D_y^2)z = e^{2x+y} + 4\cos(3x + 2y)$. **9 (Co4, Po2)**

3(a) Solve: $[D^3 + 3D^2 + D - 5)]y = 2e^x \cos^2 x$. **13 (Co3, Po2)**

(b) Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \ln x$. **12 (Co3, Po2)**

4(a) Solve: $[(x+3)D^2 - (2x+7)D + 2]y = (x+3)^2e^x$ by the method of factorization of the operator. **13 (Co3, Po2)**

(b) Find the partial differential equation by eliminating the arbitrary functions from the equation $\varphi(x - y + z, x^2 + 2y^2 - 3z^2) = 0$. **12 (Co1, Po2)**

5(a) Solve the wave equation **15 (Co5, Po2)**

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial x^2}$$

under the conditions: $v(0,t) = v(\pi, t) = 0$,

$\left(\frac{\partial v}{\partial t}\right)_{t=0} = 0, v(x,0) = \sin 3x, 0 < x < \pi.$

(b) Solve: $(xz^3 + x^2yz)p - (yz^3 + xy^2z)q = x^4$. **10 (Co4, Po2)**

6(a) Derive the Laplace equation in polar coordinate from Cartesian coordinate. **13 (Co4, Po2)**

(b) Find the complete integral, singular integral, and general integral of **12 (Co4, Po2)**

$$px^2 + 2qxy = 2zx + pq.$$