12

B.Sc. in EEE, 3rd Semester December 15, 2023 **(Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Semester Final Examination — Winter Semester, A. Y. 2022-2023
Course No.: Math 4321 — Time; 3.0 Hours
Course Title: Transform Technique and Linear Algebra — Full Marks: 150

There are 06 **(Six)** questions. Answer all 06 **(Six)** questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

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1. a) Experimental values of y give the displacement of a machine part for the rotation x of a flywheel. Evaluate y in a Fourier series up to second harmonics. 15 (CO2, PO2)

x:	0^0	60^0	120^0	180^0	240^0	300^0	360^0
y:	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

   b) Define Laplace Transform of a periodic function. Find the Laplace transform of the function $F(t) = \begin{cases} cost\,, 0 < t < \pi \\ 0\,,\ \pi < t < 2\pi \end{cases}$. 03+07=10 [CO1, PO1)

2. a) Define Beta and Gamma functions. Show that $L\left\{t^{-\frac{1}{2}}\right\} = \frac{\Gamma(1/2)}{s^{\frac{1}{2}}}$ and use it to prove $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$. 04+05+04 =13 (CO2, PO2)

   b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$ by using the statement of the convolution theorem. 12 (CO2, PO2)

3. a) Find the Laplace transform of the ramp function and draw the graph of the ramp function. 08+02=10 (CO1, CO2, PO2)

   b) Solve the IVP $y''(t) + a^2y(t) = sinbt, a \neq b$, with the conditions $y(0) = 1$, and $y'(t) = 0$ by using the derivative of Laplace transform. 15 (CO2, PO2)

4. a) Discuss the basic concept of linear algebra. Write the importance of linear algebra in the field of electrical engineering. 03+04=07 (CO1, PO1)

   b) Suppose $u = (2, -3, 8, -7)$, and $v = \left(\frac{1}{2}, -\frac{1}{6}, \frac{5}{6}, \frac{1}{6}\right)$ are any two vectors in $\mathbb{R}^4$. 18 (CO1, PO1)

      i) Discuss whether they are unit vector or not, if not find its unique unit vector.

ii) Find the angle between them.

iii) Find the projection of u on v and v on u.

5.	a)	If C be any curve represented by $F(t) = (t^2,\ 3t-2,\ t^3,\ t^2+5)$ in $\mathbb{R}^4$, where $0 \le t \le 4$. Find (i) the initial and terminal points of the curve C and (ii) the unit tangent vector T to the curve C when $t = 2$.	12 (CO2, PO2)
	b)	Define vector space and linear combination of a vector space over a scalar field K in $\mathbb{R}^n$. Find the values of λ, for which the vectors $(1, \lambda, 5)$ in $\mathbb{R}^3$ is a linear combination of the vector $(1, -3, 2)$ and $(2, -1, 1)$.	05+08=13 (CO1, PO1)
6.	a)	Discuss the linear dependence and independence of a vector space. Determine whether the matrices $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ are linearly dependent or independent.	03+06=09 (CO2, PO2)
	b)	Test the following transformations or mappings are linear or not: i) $T: \mathbb{R}^3 \to \mathbb{R}$ define by $T(x, y, z) = 2x - y + 4z$ ii) $T: \mathbb{R}^3 \to \mathbb{R}^3$ define by $T(x, y, z) = (x + y, -x - y, z)$	16 (CO1, PO1)