B.Sc. Engg. (EE), 5th Semester | December 15, 2023 | 9:00 AM – 12:00 PM

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Semester Final Examination | Winter Semester, A.Y. 2022-2023
Course No.: EEE 4501 | Full Marks: 150
Course Title: Electromagnetic Fields and Waves | Time: 3 Hours

There are **05 (five)** questions. Answer all **05 (five)** questions. Marks for parts of the questions and corresponding CO and PO are indicated in the right margin. Programmable calculators are not allowed. Do not write on this question paper. Symbols carry their usual meanings.

1. a) i) Three point charges are located in the $z = 0$ plane: a charge $+Q$ at point $(-1, 0)$, a charge $+Q$ at point $(1, 0)$, and a charge $-2Q$ at point $(0, 1)$. Determine the electric flux density at $(0, 0)$. **8 (CO3) (PO2)**

ii) A dielectric sphere $\varepsilon_1 = 2\varepsilon_0$ is buried in a medium with $\varepsilon_2 = 6\varepsilon_0$. Given that $\mathbf{E}_2 = 10\sin\theta\,\mathbf{a}_r + 5\cos\theta\,\mathbf{a}_\theta$ in the medium, calculate $\mathbf{E}_1$ and $\mathbf{D}_1$ in the dielectric sphere. **7 (CO3) (PO2)**

b) Infinite line $x = 3$, $z = 4$ carries 16 nC/m and is located in free space above the conducting plane $z = 0$. **7+8 (CO3) (PO2)**
(i) Find $\mathbf{E}$ at $(2, -2, 3)$.
(ii) Calculate the induced surface charge density on the conducting plane at $(5, -6, 0)$.

2. a) A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find $\mathbf{H}$ everywhere. **15 (CO3) (PO2)**

b) A -2 mC charge starts at point $(0, 1, 2)$ with a velocity of $5\mathbf{a}_x$ m/s in a magnetic field $\mathbf{B} = 6\mathbf{a}_y$ Wb/m^2. Determine the position and velocity of the particle after 10 s, assuming that the mass of the charge is 1 gram. Describe the motion of the charge. **6+6+3 (CO3) (PO2)**

3. a) i) The plane $z = 0$ separates air ($z \geq 0$, $\mu = \mu_0$) from iron ($z \leq 0$, $\mu = 200\mu_0$). Given that **10 (CO3) (PO2)**

$$\mathbf{H} = 10\mathbf{a}_x + 15\mathbf{a}_y - 3\mathbf{a}_z \text{ A/m}$$

in air, find $\mathbf{B}$ in iron and the angle it makes with the interface.

ii) The magnetic field in a material space ($\mu = 15\mu_0$) is given by **5 (CO3) (PO2)**

$$\mathbf{B} = 4\mathbf{a}_x + 12\mathbf{a}_y \text{ mWb/m}^2.$$

Calculate the energy stored in region $0 < x < 2$, $0 < y < 3$, $0 < z < 4$.

b) Explain why and how Maxwell's fourth equation for static field ($\nabla \times \mathbf{H} = \mathbf{J}$) is to be modified for time varying field. From that discussion define displacement current density. **(12+3) (CO4) (PO2)**

4. a) i) A conductor located at $0 < y < 1.6$ m moves with velocity $2\mathbf{a}_x$ m/s in a magnetic field, $\mathbf{B} = 10\cos\beta y\,\mathbf{a}_z$ Wb/m^2 where β is a constant. Determine the induced voltage. **8 (CO4) (PO2)**

ii) An ac voltage source is connected across the plates of a parallel-plate capacitor so that $\mathbf{E} = 25\sin(10^3 t)\mathbf{a}_z$ V/m. Calculate the total current crossing a 2 m × 5 m area placed perpendicular to the electric field. Assume that the capacitor is air filled. **7 (CO4) (PO2)**

b) An antenna radiates in free space and 15 (CO4) (PO2)

$$\mathbf{H}\,(r,\theta,\phi,t) = \frac{12\sin\theta}{r}\cos(2\pi \times 10^8 t - \beta r)\,\mathbf{a}_\theta \text{ mA/m.}$$

Find the corresponding **E** (in time domain) in terms of β using phasor algebra.

5. a) i) The magnetic field intensity of a uniform plane wave in a good conductor ($\varepsilon = \varepsilon_o$, $\mu = \mu_o$) is given by 10 (CO5) (PO2)

$$\mathbf{H} = 20e^{-12z}\cos(2\pi \times 10^6 t + 12z)\,\mathbf{a}_y \text{ mA/m.}$$

Find the conductivity, intrinsic impedance and the corresponding **E** field.

ii) In free space, $\mathbf{E} = 40\cos(\omega t - 10z)\mathbf{a}_y$ V/m. Find the total average power passing through a circular disk of radius 1.5 m in the $z = 0$ plane. 5 (CO5) (PO2

b) Classify polarization of EM fields. Explain each type of polarization using appropriate diagrams and equations. State and explain the Poynting theorem using equation and diagram. 12+3 (CO5) (PO2)

Rectangular to Cylindrical

Variable change
$$\begin{cases} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{cases}$$

Component change
$$\begin{cases} A_\rho = A_x \cos \phi + A_y \sin \phi \\ A_\phi = -A_x \sin \phi + A_y \cos \phi \\ A_z = A_z \end{cases}$$

Cylindrical to Rectangular

Variable change
$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}\left(\dfrac{y}{x}\right) \begin{cases} \sin \phi = \dfrac{y}{\sqrt{x^2 + y^2}} \\ \cos \phi = \dfrac{x}{\sqrt{x^2 + y^2}} \end{cases} \\ z = z \end{cases}$$

Component change
$$\begin{cases} A_x = A_\rho \dfrac{x}{\sqrt{x^2 + y^2}} - A_\phi \dfrac{y}{\sqrt{x^2 + y^2}} \\ A_y = A_\rho \dfrac{y}{\sqrt{x^2 + y^2}} + A_\phi \dfrac{x}{\sqrt{x^2 + y^2}} \\ A_z = A_z \end{cases}$$

Rectangular to Spherical

Variable change
$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

Component change
$$\begin{cases} A_r = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi \\ \quad + A_z \cos \theta \\ A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi \\ \quad - A_z \sin \theta \\ A_\phi = -A_x \sin \phi + A_y \cos \phi \end{cases}$$

Spherical to Rectangular

Variable change
$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \cos^{-1} \dfrac{z}{\sqrt{x^2 + y^2 + z^2}} \begin{cases} \cos \theta = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}} \\ \sin \theta = \dfrac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} \end{cases} \\ \phi = \tan^{-1}\left(\dfrac{y}{x}\right) \begin{cases} \cos \phi = \dfrac{x}{\sqrt{x^2 + y^2}} \\ \sin \phi = \dfrac{y}{\sqrt{x^2 + y^2}} \end{cases} \end{cases}$$

Component change
$$\begin{cases} A_x = \dfrac{A_r x}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{A_\theta xz}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}} - \dfrac{A_\phi y}{\sqrt{x^2 + y^2}} \\ A_y = \dfrac{A_r y}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{A_\theta yz}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}} + \dfrac{A_\phi x}{\sqrt{x^2 + y^2}} \\ A_z = \dfrac{A_r z}{\sqrt{x^2 + y^2 + z^2}} - \dfrac{A_\theta \sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} \end{cases}$$

VECTOR DERIVATIVES

Cartesian Coordinates (x, y, z)

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (r, θ, ϕ)

$$\mathbf{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & (r\sin\theta)\,A_\phi \end{vmatrix}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\mathbf{a}_\theta$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\mathbf{a}_\phi$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$