# ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT) <br> ORGANISATION OF ISLAMIC COOPERATION (OIC) DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING 

Semester Final Examination<br>Course Number: EEE 4505<br>Course Title: Computational Methods in Engineering

Winter Semester: 2022-2023
Full Marks: 150
Time: 3 Hours

There are 06 (Six) questions. Answer all questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in brackets. Assume reasonable value(s) for missing data if any.

1. a) Find the DC operation point of the tunnel diode in the circuit of Fig. 1(a) between $\mathrm{V}=0.04 \mathrm{~V}$ to $\mathrm{V}=0.07 \mathrm{~V}$ using the bisection method. Given that
$I_{D}=\left(17.76 \mathrm{~V}-103.79 \mathrm{~V}^{2}+229.62 \mathrm{~V}^{3}-226.31 \mathrm{~V}^{4}+82.72 \mathrm{~V}^{5}\right) \mathrm{mA}$.


Fig. 1(a)
b) The voltage $\mathrm{V}_{\mathrm{mp}}$ at which the output of a solar cell gives maximum power is given by

$$
\begin{equation*}
e^{\left(\frac{q V_{m p}}{k_{B} T}\right)\left(1+\frac{q V_{m p}}{k_{B^{T}}}\right)=e^{\left(\frac{q V_{o q}}{k_{B}^{T}}\right)}} \tag{12}
\end{equation*}
$$

where $V_{O C}$ is the open circuit voltage, $T$ is the temperature in Kelvin, $q=$ $1.6022 \times 10^{-19} \mathrm{C}$ is the charge on an electron, and $k_{B}=1.3806 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ is the Boltzmann's constant. For $V_{o c}=0.5 \mathrm{~V}$ and room temperature $\mathrm{T}=297 \mathrm{~K}$, use fixedpoint iteration to determine the voltage $\mathrm{V}_{\mathrm{mp}}$ at which the power output of the solar cell is a maximum with a starting value of $\mathrm{V}_{\mathrm{mp}}=0.5 \mathrm{~V}$.

Question 1: OR
a) The Manning equation can be written for a rectangular open channel as,

$$
\begin{equation*}
Q=\frac{\sqrt{S}(B H)^{5 / 3}}{n(B+2 H)^{2 / 3}} \tag{13}
\end{equation*}
$$

where $\mathrm{Q}=$ flow $\left(\mathrm{m}^{3} / \mathrm{s}\right), \mathrm{S}=$ slope $(\mathrm{m} / \mathrm{m}), \mathrm{B}=$ channel width $(\mathrm{m}), \mathrm{H}=$ depth $(\mathrm{m})$, and $\mathrm{n}=$ the Manning roughness coefficient. Using a fixed-point iteration scheme find H . Given $\mathrm{Q}=5, \mathrm{~S}=0.0002, \mathrm{~B}=20$, and $\mathrm{n}=0.03$.
b) The modified secant method is an alternative approach that involves a fractional perturbation of the independent variable to estimate the derivative used in the NewtonRaphson method. Use this modified method to determine the real root of $x^{3.6}=75$, within $\varepsilon_{s}=0.1 \%$ using an initial guess of $x_{0}=3.5$ and perturbation $\delta=0.01$.
2. a) The following system of equations is designed to determine concentrations (the c 's in $\mathrm{gm}^{3}$ ) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides are in gday),

$$
\begin{aligned}
15 c_{1}-3 c_{2}-c_{3} & =3800 \\
-3 c_{1}+18 c_{2}-6 c_{3} & =1200 \\
-4 c_{1}-c_{2}+12 c_{3} & =2350
\end{aligned}
$$

Determine concentrations $c_{1}, c_{2}$ and $c_{3}$ in a series of coupled reactors Using LU decomposition method.

## Question 2(a) OR

a) The loop current equations of a fictitious electrical circuit are given below. Use the Gauss-Seidel method with relaxation $(\lambda=1.3)$ to solve the following system to a tolerance of $\varepsilon_{s}=5 \%$. If necessary, rearrange the equations to achieve convergence.

$$
\begin{aligned}
2 I_{1}-6 I_{2}-I_{3} & =-38 \\
-3 I_{1}-I_{2}+7 I_{3} & =-34 \\
-8 I_{1}+I_{2}-2 I_{3} & =-20
\end{aligned}
$$

b) The voltage across the resistor R during the charging phase of a $2 \mu \mathrm{~F}$ capacitor is measured every 4 seconds as shown below.

| $\mathrm{t}(\mathrm{s})$ | 0 | 4 | 8 | 12 | 16 | 20 | 24 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~V}_{\mathrm{R}}$ | 10.0 | 8.5 | 5.0 | 3.5 | 2.4 | 1.6 | 1.1 |

Choose an appropriate model for the measurement and use the least square curve fitting method to estimate the value of $R$.
3. a) The open-circuit voltage $E_{g}(V)$ of a $D C$ generator is measured for four values of field current Ie (A) as shown in the table below.

| $\mathrm{I}(\mathrm{A})$ | 1.0 | 1.5 | 2.0 | 2.5 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Eg}(\mathrm{V})$ | 180 | 210 | 215 | 217 |

Determine Newton's divided difference interpolation polynomial and hence find the value of $\mathrm{Eg}(\mathrm{V})$ at $\mathrm{l}=1.75$ (A).

## Question 3(a) OR

a) The transient response of an R-L circuit with step input is measured and tabulated 35,

| $t$ (s) | 0 | 0.25 | 0.75 | 1.25 | 1.75 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| i(A) | 0 | 0.28 | 0.57 | 0.68 | 0.75 |

Fit the data to a function of the form $i(t)=A\left(1-e^{-b t}\right)$. Estimate the inductance of the circuit after one iteration using Gauss Newton Method if $\mathrm{R}=1.5 \Omega$.
b) The current flowing through an underdamped circuit is given as $i(t)=5 e^{-1.25 t} \sin (2 \pi t)$ for $0 \leq t \leq \frac{T}{2}$ and $i(t)=0$ for $\frac{T}{2}<t \leq T$. Where $T=1 \mathrm{~s}$. Calculate the RMS value of current using the two-point Gauss-Legendre formula.
4. a) Refer to the electrical circuit shown in Fig. 4 (a). Derive two simultaneous ODE relating $i(t)$ and $v(t)$. After having initial conditions determined, solve the equations using Euler's method with step size $t=0.005 \mathrm{~s}$. Show five time steps of your calculation.


Fig. 4(a)
b) Faraday's law characterizes the voltage drop across an inductor as

$$
V_{L}=L \frac{d i}{d t}
$$

Estimate the voltage drop at $t=0.2 \mathrm{~s}$ using Backward Difference, Forward Difference and Centered Difference methods of $O\left(h^{2}\right)$ correct from the following data for inductance of 4 H .

| $\mathrm{u}(\mathrm{s})$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.5 | 0.70 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{i}(\mathrm{A})$ | 0.0 | 0.16 | 0.32 | 0.56 | 0.84 | 2.0 |

5. a) A practical resistor may not always obey Ohm's law. For example, the voltage drop may be nonlinear, and the circuit dynamics for a source free R-L circuit is described by a relationship such as,

$$
L \frac{d i}{d t}+R\left(i-i^{3}\right)=0
$$

Solve for $i(t)$ numerically if $L=2 H, R-1.5 \Omega$ and $i(0)=0.5$ A from $t=0$ to 1 s with a step size $=0.2 \mathrm{~s}$.
b) The Poisson's equation in one dimensional electrostatic problem is given as,

$$
\begin{equation*}
\frac{d^{2} V}{d x^{2}}=-\frac{\rho_{v}}{\varepsilon} \tag{10}
\end{equation*}
$$

Where $\rho_{y}=$ charge density. Use the finite-difference technique with $\Delta \mathrm{x}=2$ to determine V for a wire where $\mathrm{V}(0)=1000, \mathrm{~V}(8)=0, \varepsilon=2, \mathrm{~L}=8$, and $\rho_{v}=30$.

6 a) Derive the implicit scheme of the Euler's method for a first order ODE of the form $\frac{d y}{d t}+a y=0$. Mention the advantages and disadvantages of implicit methods in solving a system of stiff differential equations.
b) Derive the Newton-Raphson formula to find the optimum (Maximum/minimum) point of an objective function.
Pressure measurements are taken at certain points behind an airfoil over time. These data best fit the curve $p=6 \cos (t)-1.5 \sin (t)$. Using Newton Raphson method find the minimum pressure. Use an initial value $t_{0}=2.0 \mathrm{~s}$. Show four iterations.
c) Define golden ratio. Tlustrate its application in searching the optimum value of an objective function in one dimensional unconstraint optimization problem.

