Program: B. Sc. Engg. (ME/IPE)
Semester: 1st Semester

Date: December 15, 2023(Afternoon)
Time: 1:30 pm – 4:30 pm

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Final Semester Examination
Course Number: Math 4111
Course Title: Modelling with calculus and ODE

Winter Semester: 2022 - 2023
Full Marks: 150
Time: 3.0 Hours

There are **6 (six)** questions. Answer **all** questions. The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in brackets.

1. a) (i) Analyze and sketch a graph of the function **[15] CO1 PO1**

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

(ii) Label the function in a(i): any intercepts, relative extrema, points of inflection, and asymptotes.

b) A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch presented in Fig. Q1(b). What should the dimensions of the page be so that the least amount of paper is used? **[10] CO1 PO1**

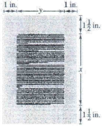

Fig. Q1(b)

2. a) A function is given below: **CO1 PO1**

$$f(x) = \frac{x^2 - 3x - 4}{x - 2}$$

(i) find the critical numbers of $f(x)$, if any. **[3]**

(ii) find the open intervals on which the function is increasing or decreasing. **[6]**

(iii) apply the First Derivative Test to identify all relative extrema. **[4]**

b) The measured radius of a ball bearing is 0.7 inch, as shown in Fig. Q2(b) below. The measurement is correct to within 0.01 inch. Estimate the propagated error in volume V of the ball bearing. **[12]** **CO1** **PO1**

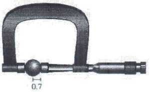

Fig. Q2(b)

3\. a) Let $y = f(x)$ be a curve. (i) Derive an integral formula to calculate the length of the said curve. Hence (ii) find the length of the curve $y = \sqrt{9-x^2}$, $0 \le x \le 3$ illustrated in Fig. Q3(a), (iii) verify your answer by noting that the curve is a part of a circle. **[15]** **CO2** **PO2**

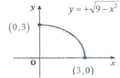

Fig. Q3(a)

b) The portion of the curve $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, is an arc of the circle $x^2 + y^2 = 4$. This curve is rotated about the x-axis as presented in Fig. Q3(b). Apply integral technique to evaluate the area of the resulting surface. **[10]** **CO2** **PO2**

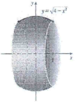

Fig. Q3(b)

4\. a) Test the exactness of the following differential equation and apply appropriate technique to solve $\left(x^2 + y^2 + x\right)dx + (xy)dy = 0$ **[10]** **CO3** **PO1**

b) Suppose that in the simple circuit presented in Fig. Q4(b) the resistance is 6Ω and the inductance is 2H. If a generator produces a variable voltage of $E(t)=2t^2$ volts, and the switch is closed when $t=0$ so the current starts with $I(0)=0$. Apply differential equation technique (i) to compute $I(t)$, (ii) the current after 5s and (iii) the limiting value of the current. [15] CO3 PO1

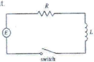

Fig. Q4(b)

5\. a) Apply Bernoulli's technique to solve the initial value problem [10] CO3 PO1

$$\frac{dy}{dx}+\frac{y}{x}=(\ln x)y^2,\ y(1)=1.$$

b) According to Newton's Law of cooling, the rate of change of temperature T satisfies the equation [15] CO3 PO1

$$\frac{dT}{dt}=-k(T-T_s)\ ,$$

where T_s is the ambient temperature, k is a constant, t is the time in minutes. If you place an object in a room with temperature 10^0C and you observe that the temperature of the object drops from 90^0C to 30^0C in 20 minutes. Apply at least two methods of first order ordinary differential equations to determine the temperature of that object after 10 minutes.

6\. a) Solve the following differential equation using undetermined coefficient [12] CO3 PO2

$$y''-3y'-4y=-8e^t\cos 2t$$

b) A 10 lb mass stretches a spring 2". The mass is displaced an additional 2" and then set in motion with initial upward velocity of 1 ft/sec. (i) Determine position of mass at any later time. (ii) Also find period, amplitude, and phase of the motion. [13] CO3 PO2