10

Name of the Program: B. Sc. (ME/IPE/BScTE 2Y) **Date: 15/12/2023**
Semester: 3rd Sem./2nd Sem. **Time: 9:00 am -12:00noon**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES (NSc)

Semester Final Examination **Winter Semester: A.Y. 2022-2023**
Course Code: Math 4311/Math 4599 **Full Marks: 150**
Course Title: Vector Analysis, Multivariable Calculus and Complex Variables **Time: 3 Hours**

Answer all the questions. Marks of each question and corresponding CO and PO are written in the brackets. The symbols used have their usual meaning.

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1. (a) Use a line integral to find the area of the region enclosed by the asteroid $x = a\cos^3\varphi,\ y = a\sin^3\varphi\ (0 \le \varphi \le 2\pi)$ **[13] CO2 PO1**

   (b) Use Green's Theorem to evaluate $\int_C x^2y\,dx + x\,dy$ along the triangular path C with vertices (0,0), (1,0) and (1,2) described in the positive direction. **[12] CO2 PO1**

2. (a) Evaluate $\iint_S A\cdot n\,dS$, where $\mathbf{A} = (x + y^2)i - 2x\,\mathbf{j} + 2yz\,\boldsymbol{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. **[13] CO2 PO1**

   (b) Evaluate $\iint_S \boldsymbol{F}\cdot\boldsymbol{n}dS$, where $\mathbf{F} = 4xz\boldsymbol{i} - y^2\boldsymbol{j} + yz\boldsymbol{k}$ and S is the surface of the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0$ and $z = 1$. **[12] CO2 PO1**

3. (a) Use Divergence Theorem to find the outward flux of the vector field $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\,\mathbf{j} + z^3\mathbf{k}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$. **[12] CO2 PO2**

   (b) Use Stokes' theorem to find the work performed by the force field $F(x, y, z) = x^2i + 4xy^3\,j + y^2xk$ on a particle that traverses the rectangle C with corners (0,0,0), (1,0,0), (1,3,3), and (0,3,3). **[13] CO2 PO2**

4. (a) Construct a Riemann surface for the function $z^{1/3}$. **[10] CO3 PO1**

   (b) Determine whether the following function u is harmonic or not. If yes, find the conjugate harmonic function v and express $f(z) = u + iv$ as an analytic function of z. **[8] CO3 PO1**

$$u(x, y) = e^x \sin y.$$

(c) Expand $f(z)=\frac{z}{(z-1)(2-z)}$ in a Laurant series valid for $|z-1|>1$. [7] CO3 PO1

5. (a) State Cauchy's theorem for complex integration. Verify Cauchy's theorem for the function $f(z)=z^3-iz^2-5z+2i$ if C is the circle $|z|=1$. [10] CO3 PO1

(b) Determine the region of the w plane into which the region bounded by $x=1, y=1$, and $x+y=1$ is mapped by the transformation $w=z^2$. [8] CO3 PO1

(c) Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz$ where C is the circle $|z|=4$. [7] CO3 PO1

6. (a) Evaluate $\int_0^{2\pi}\frac{1}{3-2\cos\theta+\sin\theta}d\theta$. [15] CO3 PO2

(b) Find a bilinear transformation that maps the upper half of the z plane into the unit circle in the w plane in such a way that $z=i$ is mapped into $w=0$ while the point at infinity is mapped into $w=-1$. (10) CO3 PO2