B.Sc Engg.(M)/5th Sem
B.Sc. Engg. (IPE)/5th Sem

23 December, 2023
Morning (9.00AM-12.00 PM)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

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ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Semester Final Examination
Course No.: Math 4511
Course Title: Numerical Analysis

Winter Semester: A.Y. 2022-2023
Time: 3 hours
Full Marks: 150

There are 6 (Six) Questions. Answer all of them.

Marks in the Margin indicate full marks. Programmable calculators are not allowed.

Assume reasonable values for any missing data(if any).

Necessary formulas are provided

1. The location $\bar{x}$ of the centroid of an arc (**Figure. 1**) of a circle is given by: (25) CO1, PO2

$$\bar{x} = \frac{r\,\sin\alpha}{\alpha}$$

Determine the angle α for which

$$\bar{x} = \frac{3r}{4}.$$

First, derive the equation that must be solved and then determine the root, using the **Bisection Method**. Start with $a = 0.5$ and $b = 1.5$, and carry out the first four iterations. Determine the *approximate relative error* after each iteration.

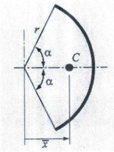

Figure. 1

2. Determine the roots of the following simultaneous nonlinear equations using **Newton's method** (25) CO1, PO2

$$y = -x^2 + x + 0.75$$
$$y + 5xy = x^2$$

Employ Initial guesses of $x = y = 1.2$

3. An object can be projected upward at a specified velocity. If it is subject to linear drag, its altitude as a function of time can be computed as

$$z = z_0 + \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)\left(1 - e^{-\left(\frac{c}{m}\right)t}\right) - \frac{mg}{c}t$$

C
PO

Where z = altitude (m) above the earth's surface (defined as $z = 0$), z_0 = the initial altitude (m), m = mass (kg), c = a linear drag coefficient (kg/s), v_0 = initial velocity (m/s), and t = time (s). Note that for this formulation, positive velocity is considered to be in the upward direction. Given the following parameter values: $g = 9.81$ m/s^2, $z_0 = 100$ m, $v_0 = 55$ m/s, $m = 80$ kg, and $c = 15$ kg/s, the equation can be used to calculate the jumper's altitude. Determine the time and altitude of the peak elevation with the **golden-section search.** Perform 3 iterations with initial guesses of t_l = 0 and t_u = 10 s.

4. A plane is being tracked by radar, and data are taken every second in polar coordinates θ and r. (25) CO2, PO4

t, s	200	202	204	206	208	210
θ, rad	0.75	0.72	0.7	0.68	0.67	0.66
r, m	5120	5370	5560	5800	6030	6240

At 206 seconds, use the **centered finite-difference** (second order correct) to find the vector expressions for velocity and acceleration The velocity and acceleration given in polar coordinates are

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

5. The Head Severity Index (HSI) measures the risk of head injury in a car crash. It is calculated by: (25) CO2, PO4

$$HSI = \int_0^t [\,a(t)]^{2.5}\,dt$$

Where $a(t)$ is the normalized acceleration (acceleration in m/s^2 divided by 9.81 m/s^2) and t is time in seconds during a crash. The acceleration of a dummy head measured during a crash test is given in the following table

t, s	0	5	10	15	20	25	30	35	40	45	50	55	60
a, m/s^2	0	3	8	20	33	42	40	48	60	12	8	4	3

Determine the HIS.

i) Use the composite **trapezoidal method**
ii) Use the composite **Simpson's 1/3 method.**
iii) Use the composite **Simpson's 3/8 method**

6. Consider the following first-order ODE: (25) CO3, PO2

$$\frac{dy}{dx} = \frac{x^2}{y}$$

From $x = 0$ to $x = 2.1$ with $y(0) = 2$.

i) Solve with **Euler's explicit method** using $h = 0.7$.
ii) Solve with the classical **fourth-order Runge-Kutta method** using $h = 0.7$.

Formula sheet

into Eq. (23.2) to yield

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{2h^2}h + O(h^2)$$

or, by collecting terms,

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2) \tag{23.5}$$

Notice that inclusion of the second-derivative term has improved the accuracy to $O(h^2)$. Similar improved versions can be developed for the backward and centered formulas as well as for the approximations of the higher derivatives. The formulas are summarized in Figs. 23.1 through 23.3 along with all the results from Chap. 4. The following example illustrates the utility of these formulas for estimating derivatives.

FIGURE 23.1
Forward finite-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.

	Error
First Derivative	
$f'(x_i) = \dfrac{f(x_{i+1}) - f(x_i)}{h}$	$O(h)$
$f'(x_i) = \dfrac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$	$O(h^2)$
Second Derivative	
$f''(x_i) = \dfrac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$	$O(h)$
$f''(x_i) = \dfrac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2}$	$O(h^2)$
Third Derivative	
$f'''(x_i) = \dfrac{f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i)}{h^3}$	$O(h)$
$f'''(x_i) = \dfrac{-3f(x_{i+4}) + 14f(x_{i+3}) - 24f(x_{i+2}) + 18f(x_{i+1}) - 5f(x_i)}{2h^3}$	$O(h^2)$
Fourth Derivative	
$f''''(x_i) = \dfrac{f(x_{i+4}) - 4f(x_{i+3}) + 6f(x_{i+2}) - 4f(x_{i+1}) + f(x_i)}{h^4}$	$O(h)$
$f''''(x_i) = \dfrac{-2f(x_{i+5}) + 11f(x_{i+4}) - 24f(x_{i+3}) + 26f(x_{i+2}) - 14f(x_{i+1}) + 3f(x_i)}{h^4}$	$O(h^2)$

FIGURE 23.2
Backward finite-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.

	Error
First Derivative	
$f'(x_i) = \dfrac{f(x_i) - f(x_{i-1})}{h}$	$O(h)$
$f'(x_i) = \dfrac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$	$O(h^2)$
Second Derivative	
$f''(x_i) = \dfrac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$	$O(h)$
$f''(x_i) = \dfrac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2}$	$O(h^2)$
Third Derivative	
$f'''(x_i) = \dfrac{f(x_i) - 3f(x_{i-1}) + 3f(x_{i-2}) - f(x_{i-3})}{h^3}$	$O(h)$
$f'''(x_i) = \dfrac{5f(x_i) - 18f(x_{i-1}) + 24f(x_{i-2}) - 14f(x_{i-3}) + 3f(x_{i-4})}{2h^3}$	$O(h^2)$
Fourth Derivative	
$f''''(x_i) = \dfrac{f(x_i) - 4f(x_{i-1}) + 6f(x_{i-2}) - 4f(x_{i-3}) + f(x_{i-4})}{h^4}$	$O(h)$
$f''''(x_i) = \dfrac{3f(x_i) - 14f(x_{i-1}) + 26f(x_{i-2}) - 24f(x_{i-3}) + 11f(x_{i-4}) - 2f(x_{i-5})}{h^4}$	$O(h^2)$

FIGURE 23.3
Centered finite-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.

	Error
First Derivative	
$f'(x_i) = \dfrac{f(x_{i+1}) - f(x_{i-1})}{2h}$	$O(h^2)$
$f'(x_i) = \dfrac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$	$O(h^4)$
Second Derivative	
$f''(x_i) = \dfrac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$	$O(h^2)$
$f''(x_i) = \dfrac{-f(x_{i+2}) + 16f(x_{i+1}) - 30f(x_i) + 16f(x_{i-1}) - f(x_{i-2})}{12h^2}$	$O(h^4)$
Third Derivative	
$f'''(x_i) = \dfrac{f(x_{i+2}) - 2f(x_{i+1}) + 2f(x_{i-1}) - f(x_{i-2})}{2h^3}$	$O(h^2)$
$f'''(x_i) = \dfrac{-f(x_{i+3}) + 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$	$O(h^4)$
Fourth Derivative	
$f''''(x_i) = \dfrac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$	$O(h^2)$
$f''''(x_i) = \dfrac{-f(x_{i+3}) + 12f(x_{i+2}) - 39f(x_{i+1}) + 56f(x_i) - 39f(x_{i-1}) + 12f(x_{i-2}) - f(x_{i-3})}{6h^4}$	$O(h^4)$