Program: BSc. Eng. (ME/IPE)
Semester: 5th Semester

Date: 9 December 2023
Time: 09:00 am – 12:00 pm (morning)

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Semester Final Examination — Winter Semester, A.Y. 2022-2023
Course Number: IPE 4531 — Full Marks : 150
Course Title: Probability and Statistics — Time : 3 hours

There are 6 **(Six)** questions. Answer all of them. he symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the right column. Assume reasonable values if required. Selected formulas and charts are provided at the end of the question.

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1 Write down two features of chi square analysis that differentiate it from others. Four brands of light bulbs are being considered for use in the final assembly area of the KP plant in Gazipur. The director of purchasing asked for samples of 200 from each manufacturer. The numbers of acceptable and unacceptable bulbs from four manufacturers are shown below. At the 0.05 significance level, is there a difference in the quality of the bulbs? Justify the answers. — 25 CO2 PO3

Quality of the bulb	Manufacturer				
	A	B	C	D	Total
Unacceptable	27	35	34	24	120
Acceptable	13	15	26	26	80
Total	40	50	60	50	200

2 a A researcher would like to conduct a survey to determine the mean electricity consumption cost of a family in a year in the rural area of a country. The question is, how many families should be sampled? In a pilot sample of 10 families, the standard deviation of the sample was \$100. The sponsor of the survey wants you to use the 95 percent confidence level. The estimate is to be within \$50. Determine the number of families to be interviewed. — 6 CO2 PO3

b In a TV game show, a contestant selects one of three doors, behind one of the doors there is a price, and behind the other two doors there are no prizes. After a contestant selects a door, the game show host opens one of the remaining doors, and reveals there is no prize behind it. The host then asks the contestant whether he wants to switch to other door or stick to the door he is about to open. Using probability analysis, determine the optimal choice for the contestant. — 7 CO2 PO3

c The average life of a certain type of water pumps is 9 years with a standard deviation of 2 years. The Manufacturer replaces free all pumps that fail while under guarantee. If the manufacturer is willing to replace only 6% of the pumps that fail, how long a guarantee should be offered? Assume that the life time of a pump follows a normal distribution — 6 CO2 PO3

d According to Journal of Mechanical and Industrial Engineering Chronicle, — 6

approximately 30% of all pipework failures in power plants are caused by operator error. Determine the probability that out of the next 12 pipework failures at least 1 are due to operator error. CO2 PO3

3 a Sunsang produces budget AC. The management wants to know if there is a difference in proportions of urban and rural people who like their AC with .05 significance level. A random sample of 100 urban people revealed 20 liked the AC well enough to purchase it. Similarly, a sample of 200 rural people revealed 100 liked the AC well enough to make a purchase 10 CO2 PO3

b PQ real estate wishes to compare the two companies they use to appraise the value of residential homes. Accordingly it selected a sample of 4 residential properties and scheduled both firms for an appraisal. The results, reported in Thousand $, are provided in the table. At the .05 significance level, can we conclude there is a difference in the mean appraised values of the homes? 15 CO2 PO3

Name of Home	Company A	Company B
Maxell	156	151
Oracle	222	207
Azure	162	157
Castle	141	147

4 a A Professor had the 22 students in the class of Introduction to Statistics class. They rated his performance as Excellent, Good, Fair, or Poor. A graduate student collected the ratings and assured the students that Professor would not receive them until after course grades had been sent to the Registrar's office. The rating (ie., the treatment) a student gave the professor was matched with his or her course grade, which could range from 0 to 100. The sample information is reported below. Is there a difference in the mean score of the students in each of the four rating categories? Use the 0.01 significance level. 20 CO3 PO3

Graduate Grades			
Excellent	Good	Fair	Poor
98	85	70	69
90	68	73	70
85	77	76	72
80	83	78	65
	88	80	74
		68	64
		65	

b Write a short note on Markov Chain with schematic illustration. 5 CO3 PO3

5 a Differentiate between Type I error and Type II error. AP washers Ltd. produces washers of a certain diameter. From a day's production a sample of 4 washers is selected randomly from the production line and their diameters (in mm) are recorded. The average diameter and range of this sample (of size 4) are computed and recorded. The Quality Control Engineer collected this type of samples in 10 days in the month of November and the findings are shown in table below. From this table, draw the *X-bar* and *R* chart. After finalizing the control charts, in a 15 CO2 PO3

given day, four(4) washers are randomly selected with the diameter(mm) of 9.125, 8.789, 10.01, and 9.60. Now using the control charts, comment.

Day	Average diameter of the sample(mm)	Rang e, *R*
1	10.769	0.050
2	10.730	0.016
3	10.718	0.040
4	10.728	0.014
5	10.730	0.029
6	10.720	0.020
7	10.711	0.038
8	10.713	0.026
9	10.718	0.008
10	10.789	0.032

b The demand for refrigerators over the past 4 years and the corresponding population in a small town are shown below. Using regression method, forecast the demand of the refrigerators in future (i.e. 2024) if the population becomes 61,000. Also find the coefficient of correlation, coefficient of determination and comment on the significance of the found value of these two coefficients. 10 CO2 PO3

Year	Refrigerator's demand (no of refrigerators)	Population (no of people)
2020	2600	25920
2021	5100	29333
2022	5400	32000
2023	5600	33500

6 a LNK Lawn Care, Inc. manufactures and assembles lawn mowers that are shipped to dealers. Two different procedures have been proposed for mounting the engine on the frame of the lawnmower. It was decided to conduct a time and motion study for these two procedures. A sample of five employees was timed using the Welles method and six using the Atkins method. The results, in minutes, are shown below. Is there a difference in the mean mounting times? Use the 0.10 significance level. 15 CO2 PO3

Wells	Atkins
2	4
4	7
8	6
3	8
3	4
	5

b Write down the difference between standard hypothesis testing with Chi square test and ANOVA. The MK hardware chain claims that the waiting time of customers for service is normally distributed, with a mean of 3 minutes and a standard deviation of 1 minute. The quality assurance department found in a sample of 50 customers at the Boardbazar outlet that the mean waiting time was 2.75 minutes. At the 0.1 significance level, can we conclude that the mean waiting time is different from 3 minutes? 10 CO2 PO3

Formulas:::

$$E = z\frac{s}{\sqrt{n}}$$

Median:

$$s = \sqrt{\frac{n\sum_{i}^{n}(f_i X_i^{\,2}) - (\sum_{i}^{n} f_i X_i)^2}{n-1}}$$

$$M_d = L + \left(\frac{N_{\frac{2}{2}} - n_b}{n_w}\right) i \qquad i = R / (1 + 3.322 \log n)$$

$$b(x; n, P) = {}^{n}C_x * P^x * (1 - P)^{n-x}$$

$$P(x; \mu) = (e^{-\mu}) * (\mu^x) / x!$$

$$h(x; N, n, k) = [\,{}^{k}C_x\,] * [\,{}^{N-k}C_{n-x}\,] / [\,{}^{N}C_n\,]$$

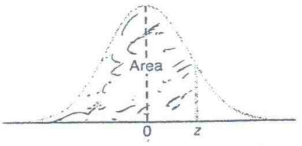


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

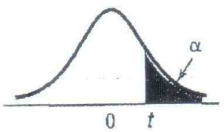
Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

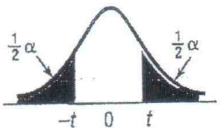
t chart

Confidence Interval

Left-tailed test



Right-tailed test



Two-tailed test

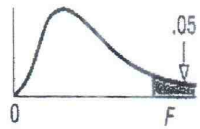
	Confidence Intervals					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test, α					
df	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850

21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Factors for control chart

Number of Items in Sample	Chart for Averages	Chart for Ranges		
	Factors for Control Limits	Factors for Central Line	Factors for Control Limits	
n	A_2	d_2	D_3	D_4
2	1.880	1.128	0	3.267
3	1.023	1.693	0	2.575
4	.729	2.059	0	2.282
5	.577	2.326	0	2.115
6	.483	2.534	0	2.004
7	.419	2.704	.076	1.924
8	.373	2.847	.136	1.864
9	.337	2.970	.184	1.816
10	.308	3.078	.223	1.777
11	.285	3.173	.256	1.744
12	.266	3.258	.284	1.716
13	.249	3.336	.308	1.692
14	.235	3.407	.329	1.671
15	.223	3.472	.348	1.652

F chart



Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator															
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91

Chi square chart

Degrees of Freedom, *df*	Right-Tail Area			
	0.10	0.05	0.02	0.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.168	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141
15	22.307	24.996	28.259	30.578
16	23.542	26.296	29.633	32.000
17	24.769	27.587	30.995	33.409
18	25.989	28.869	32.346	34.805
19	27.204	30.144	33.687	36.191
20	28.412	31.410	35.020	37.566
21	29.615	32.671	36.343	38.932
22	30.813	33.924	37.659	40.289
23	32.007	35.172	38.968	41.638
24	33.196	36.415	40.270	42.980
25	34.382	37.652	41.566	44.314
26	35.563	38.885	42.856	45.642
27	36.741	40.113	44.140	46.963
28	37.916	41.337	45.419	48.278
29	39.087	42.557	46.693	49.588
30	40.256	43.773	47.962	50.892

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

$$UCL = D_4\bar{R}$$

$$LCL = D_3\bar{R}$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Capability Index, C_p= USL – LSL / 6 * σ
where $\sigma = \bar{R} / d_2$

CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

$$p \pm z\sqrt{\frac{p(1-p)}{n}}$$

CONFIDENCE INTERVAL FOR THE POPULATION MEAN, σ UNKNOWN

$$\bar{X} \pm t\frac{s}{\sqrt{n}}$$

SMALL SAMPLE TEST FOR MEAN

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

$$z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$n = \left(\frac{zs}{E}\right)^2 \quad n = p(1-p)\left(\frac{z}{E}\right)^2$$

TWO-SAMPLE TEST OF PROPORTIONS

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1-p_c)}{n_1} + \frac{p_c(1-p_c)}{n_2}}}$$

POOLED PROPORTION

$$p_c = \frac{X_1 + X_2}{n_1 + n_2}$$

PAIRED *t* TEST

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

$$s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$$

POOLED VARIANCE

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

TWO-SAMPLE TEST OF MEANS—SMALL SAMPLES

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

VARIANCE OF THE DISTRIBUTION OF DIFFERENCES IN MEANS

$$s_{\bar{X}_1 - \bar{X}_2}^2 = \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}$$

TEST STATISTIC FOR THE DIFFERENCE BETWEEN TWO MEANS

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

CORRELATION COEFFICIENT

$$r = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{(n-1)\, s_x\, s_y}$$

SLOPE OF THE REGRESSION LINE

$$b = r\frac{s_y}{s_x}$$

***Y*-INTERCEPT**

$$a = \bar{Y} - b\bar{X}$$

TEST STATISTIC FOR COMPARING TWO VARIANCES

$$F = \frac{s_1^2}{s_2^2}$$

ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*
Treatments	SST	$k-1$	SST/$(k-1)$ = MST	MST/MSE
Error	SSE	$n-k$	SSE/$(n-k)$ = MSE	
Total	SS total	$n-1$		

Bayes formula

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{\sum_{i=1}^{n}[P(B|A_i)P(A_i)}$$

Regression another formula:

$$a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n}$$

$$b = \frac{n\sum x_i y_i - [(\sum x_i)(\sum y_i)]}{n\sum x_i^2 - (\sum x_i)^2}$$

$$r^2 = \frac{[n\sum x_i y_i - (\sum x_i)(\sum y_i)]^2}{[n\sum x_i^2 - (\sum x_i)^2][n\sum y_i^2 - (\sum y_i)^2]}$$

CHI-SQUARE TEST STATISTIC

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

EXPECTED FREQUENCY

$$f_e = \frac{\text{(Row total)(Column total)}}{\text{Grand total}}$$