ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2022-2023**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

1. a) Explain how numerical approaches differ from analytical approaches to solving a problem. 4 (CO4) (PO1)

   b) Derive the formula for the Secant method geometrically. 6 (CO4) (PO1)

   c) Figure 1 portrays the orbital geometry of planet Earth around the Sun, where S denotes the position of the Sun and E denotes the position of planet Earth.

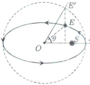


Figure 1: Geometry of the planetary orbit of Earth around the Sun for Question 1.c

Let θ denote the angle defined by $\angle E'OA$, measured in radians. The dotted circle is concentric to the ellipse and has a diameter equal to the major axis of the ellipse. Let T be the total period of planet Earth, and let t be the time it takes for Earth to go from A to E. The relationship between θ and t is dictated by Kepler's equation from orbital mechanics,

$$\theta - \epsilon \sin(\theta) = \frac{2\pi t}{T} \tag{1}$$

Here ϵ is the eccentricity of the elliptical orbit. In the case of planet Earth, the conventional value is $\epsilon = 0.017$.

   i. Formulate a function $f(\theta)$ such that its root will yield the value of the angle θ for a given value of t. 2.5 (CO1) (PO2)

   ii. Apply the Newton-Raphson method to estimate the value of θ corresponding to $t = \frac{T}{8}$. The initial guess is $\theta_0 = \pi$. Demonstrating the step-by-step mathematical procedure, conduct 4 iterations, and find the relative approximate error ($|\epsilon_a|\%$) and the number of significant digits that are at least correct (m) at the end of each iteration. Draw Table 1 and fill it out after performing the necessary calculations. 11.5 (CO2) (PO1)

Table 1: The relevant values obtained in the answer of Question 1.c

Iteration	θ_i	$f(\theta_i)$	$f'(\theta_i)$	θ_{i+1}	$\lvert\epsilon_a\rvert\%$	m
1	π					
2						
3						
4						

2. a) Define Truncation Error. What are the different avenues through which Truncation Error can be introduced? Explain with proper examples. 5 (CO4) (PO1)

b) The Taylor series for a function $f(x)$ is — 18 (CO4) (PO1)

$$f(x+h) = f(x) + f'(x)\frac{h}{1!} + f''(x)\frac{h^2}{2!} + f'''(x)\frac{h^3}{3!} + f''''(x)\frac{h^4}{4!} + f'''''(x)\frac{h^5}{5!} + \dots$$

Using the Taylor series (with at least the first 5 terms), derive the Maclaurin series of —

i. $\ln(1+x)$ ii. $\ln(1-x)$ iii. $\ln\left(\frac{1+x}{1-x}\right)$ iv. $\log_{(1-x)}(1+x)$

3. a) Compare and contrast the following interpolation methods: Direct method, Newton's Divided Difference method, and Lagrangian method. 4 (CO3) (PO1)

b) Suppose, there is an unknown function $f(x)$ and you are given 3 points $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$ through which $f(x)$ passes. Now, to approximate $f(x)$, you decide to use Newton's Divided Difference method of interpolation and obtain a quadratic interpolant $f_2(x)$. Derive the representation of 2^{nd} order Newton's Divided Difference polynomial $f_2(x)$. 9 (CO4) (PO1)

c) The specific heat capacity C of a substance is defined as the amount of heat that is required to raise the temperature of unit mass of that particular substance by 1 degree. Suppose, you are conducting an experiment to determine how much heat is required to bring some volume of water to its boiling point. The values of specific heat capacity C of water that you have calculated at different temperature values T are shown in Table 2. 6 (CO2) (PO1)

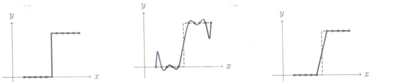
Table 2: Specific heat C of water as a function of temperature T for Question 3.c

Temperature, T ($°C$)	**Specific Heat, C ($Jkg^{-1}°C^{-1}$)**
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61°C$ using the Direct method of interpolation and a second order polynomial.

d) i. Why is $f_9(x)$ in Figure 2(b) demonstrably worse at capturing the pattern of $f(x)$ compared to $S(x)$ in Figure 2(c) ? Explain with necessary illustration(s). 4 (CO3) (PO1)

ii. Suppose you want to perform Spline Interpolation on $n+1$ data points $(x_0, f(x_0))$, $(x_1, f(x_1)), \dots$, and $(x_n, f(x_n))$ using Cubic splines of the form $f(x) = ax^3+bx^2+cx+d$. How many unknowns and how many equations would you have to deal with? Mention how you would obtain those equations and derive them. 5 (CO4) (PO1)



(a) A step function, $f(x)$. **(b)** A 9^{th} order polynomial interpolant, $f_9(x)$. **(c)** A linear spline interpolant, $S(x)$.

Figure 2: Predicting the nature of $f(x)$ for Question 3.d)i