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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

SUMMER SEMESTER, 2022-2023

DURATION: 1 HOUR 30 MINUTES

FULL MARKS: 75

Math 4643: Probability and Statistics II

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer all 3 (three) questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

1. a) Suppose that the joint probability density function of X and Y is:

$$f_{XY}(x, y) = \begin{cases} k \times 10^{-6} e^{(-0.001x - 0.002y)}, & \text{if } x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

9
(CO1)
(PO1)

- i. Show that X and Y are independent.
- ii. Determine the probability $P(X < 1000, Y > 1000)$.

- b) Suppose the random variables X, Y, and Z have the multivariate joint probability density function:

$$f_{XYZ}(x, y, z) = \begin{cases} 8xyz, & \text{if } 0 \leq x, y, z \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

8
(CO2)
(PO2)

Determine the following:

- i. $P(X \leq 0.5, Y = 0.5, Z = 0.7)$
- ii. Marginal probability distribution of X.
- iii. Conditional probability distribution of Y given $x = 0.5, z = 0.8$
- iv. $E(Z|X = 0.3, Y = 0.4)$

- c) The probability density function for the diameter of a drilled hole in millimeters is:

$$f_X(x) = \begin{cases} 10e^{-10(x-5)}, & \text{if } x > 5 \\ 0, & \text{otherwise} \end{cases}$$

8
(CO2)
(PO2)

Although the target diameter is 5 millimeters, vibrations, tool wear, and other nuisances produce diameters larger than 5 millimeters.

- i. Determine the mean and variance of the diameter of the holes.
- ii. Determine the probability that a diameter exceeds 5.1 millimeters.

2. a) Suppose that, the sodium content of twenty 300-gram boxes of organic cornflakes was determined. The data (in milligrams) are as follows: 131.15, 130.69, 130.91, 129.54, 129.64, 128.77, 130.72, 128.33, 128.24, 129.65, 130.14, 129.29, 128.71, 129.00, 129.39, 130.42, 129.53, 130.12, 129.78, 130.92. Now, answer the following questions:

12.5
(CO2)
(PO2)

- i. Can you support a claim that mean sodium content of this brand of cornflakes differs from 130 milligrams?
- ii. Check that sodium content is normally distributed.
- iii. Compute the power of the test if the true mean sodium content is 130.5 milligrams.

- b) The titanium content in an aircraft-grade alloy is an important determinant of strength. A sample of 20 test coupons reveals the following titanium content (in percent): 8.32, 8.05, 8.93, 8.65, 8.25, 8.46, 8.52, 8.35, 8.36, 8.41, 8.42, 8.30, 8.71, 8.75, 8.60, 8.83, 8.50, 8.38, 8.29, 8.46. The median titanium content should be 8.5 percent. Answer the following:
- Use the sign test with $\alpha = 0.01$ to investigate this hypothesis.
 - Use the normal approximation for the sign test to test $H_0 = 8.5$ versus $H_1 \neq 8.5$, with $\alpha = 0.01$.
3. a) Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature (x) and pavement deflection (y). Summary quantities were $n = 20$, $\sum y_i = 12.75$, $\sum x_i = 1478$, $\sum x_i^2 = 143,215.8$, and $\sum x_i y_i = 1083.67$. Answer the following:
- Calculate the least squares estimates of the slope and intercept. Graph the regression line.
 - Use the equation of the fitted line to predict what pavement deflection would be observed when the surface temperature is 85°F .
 - What is the mean pavement deflection when the surface temperature is 90°F ?
 - What change in mean pavement deflection would be expected for a 1°F change in surface temperature?
- b) Answer the following:
- Test for significance of regression using $\alpha = 0.01$. What is the P-value for this test? State the conclusions of the results from this test.
 - Estimate the standard errors of the slope and intercept.
 - Test the hypothesis $H_0 : \beta_1 = 10$ versus $H_1 : \beta_1 \neq 10$ using $\alpha = 0.01$. Find the P-value for this test.
 - Test $H_0 : \beta_0 = 0$ versus $H_1 : \beta_0 \neq 0$ using $\alpha = 0.01$. Find the P-value for this test and draw conclusions.

12.5
(CO2)
(PO2)

12.5
(CO3)
(PO2)

12.5
(CO3)
(PO2)

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.56	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64
28	12.46	13.58	15.31	16.93	18.94	37.92	41.34	44.46	48.28
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
32	15.13	16.36	18.29	20.07	22.37	42.58	46.19	49.48	53.49
34	16.50	17.79	19.81	21.66	23.96	44.90	48.60	51.97	56.06
38	19.29	20.69	22.88	24.88	27.34	49.51	53.38	56.90	61.16
42	22.14	23.65	26.00	28.14	30.77	54.09	58.12	61.78	66.21
46	25.04	26.66	29.16	31.44	34.22	58.64	62.83	66.62	71.20
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
55	31.73	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29
60	35.53	37.48	40.48	43.16	46.46	74.40	79.08	83.30	88.38
65	39.38	41.44	44.60	47.45	50.88	79.97	84.82	89.18	94.42
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43
75	47.21	49.48	52.94	56.05	59.79	91.06	96.22	100.84	106.39
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33
85	55.17	57.63	61.39	64.75	68.78	102.08	107.52	112.30	118.24
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12
95	63.25	65.90	69.92	73.52	77.82	113.04	118.75	123.86	129.97
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81

t Table

cum. prob one-tail two-tails	$t_{.50}$	$t_{.25}$	$t_{.20}$	$t_{.15}$	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$	
	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001	
df												
1	0.000	1.000	1.376	1.863	3.078	6.314	12.71	31.82	63.66	318.31	636.62	
2	0.000	0.816	1.061	1.385	1.888	2.920	4.303	6.965	9.925	22.327	31.599	
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924	
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610	
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.860	
6	0.000	0.715	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959	
7	0.000	0.705	0.896	1.119	1.415	1.895	2.353	2.948	3.499	4.785	5.403	
8	0.000	0.700	0.890	1.109	1.397	1.860	2.306	2.876	3.355	4.501	5.041	
9	0.000	0.700	0.889	1.100	1.383	1.833	2.262	2.821	3.250	4.257	4.761	
10	0.000	0.700	0.889	1.093	1.372	1.812	2.228	2.784	3.183	4.146	4.660	
11	0.000	0.697	0.886	1.088	1.363	1.795	2.201	2.718	3.106	4.025	4.437	
12	0.000	0.696	0.883	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318	
13	0.000	0.694	0.880	1.079	1.350	1.771	2.160	2.660	3.012	3.862	4.221	
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140	
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073	
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.692	4.015	
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.963	
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922	
19	0.000	0.688	0.861	1.065	1.328	1.729	2.093	2.538	2.861	3.579	3.883	
20	0.000	0.687	0.860	1.064	1.325	1.725	2.085	2.523	2.845	3.552	3.853	
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819	
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.505	2.819	3.505	3.782	
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768	
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745	
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725	
26	0.000	0.684	0.855	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707	
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.772	3.421	3.690	
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.765	3.408	3.674	
29	0.000	0.683	0.854	1.055	1.311	1.699	2.044	2.462	2.758	3.396	3.656	
30	0.000	0.683	0.854	1.055	1.310	1.697	2.041	2.457	2.750	3.385	3.646	
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551	
60	0.000	0.679	0.848	1.045	1.295	1.671	2.000	2.390	2.660	3.232	3.460	
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416	
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390	
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300	
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.961	2.326	2.576	3.090	3.291	
		0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
		Confidence Level										